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ABSTRACT

Surface EMG signal modeling has important applications in the interpretation of experimental EMG data. Most models of surface EMG generation considered volume conductors homogeneous in the direction of propagation of the action potentials. However, this may not be the case in practice due to local tissue in-homogeneities or to the fact that there may be groups of muscle fibers with different orientations. This study addresses the issue of analytically describing surface EMG signals generated by bi-pinnate muscles, i.e., muscles which have two groups of fibers with two orientations. The approach will also be adapted to the case of a muscle with fibers inclined in the depth direction. Such muscle anatomies are in-homogeneous in the direction of propagation of the action potentials with the consequence that the system can not be described as space invariant in the direction of source propagation. In these conditions, the potentials detected at the skin surface do not travel without shape changes. This determines numerical issues in the implementation of the model which are addressed in this work. The study provides the solution of the non-homogenous, anisotropic problem, proposes an implementation of the results in complete surface EMG generation models (including finite length fibers), and shows representative results of the application of the models proposed.

1. INTRODUCTION

It is recognized that surface EMG signal modeling is fundamental for interpreting EMG recordings [19][20], designing and testing algorithms for information extraction [5], and for didactic purposes [24]. In the last decades, many surface EMG generation models have been proposed. Both numerical [17][23] and analytical [4][6][10][12][13][18] approaches were used to describe the electrical properties of the tissues separating the muscle fibers and the detecting electrodes. The volume conductor has been described as homogeneous in many past modeling approaches (e.g., [4][19]) and in-homogeneous in more recent developments [1][6][10]. In the latter case, layered descriptions of the tissues, either planar [6] or cylindrical [10][12][13], have been adopted.

In almost all the models proposed in the literature, the volume conductor is homogeneous in the direction of propagation of the intracellular action potentials. The latter assumption implies that the system, viewed as a spatial linear filter, is space invariant in the direction of propagation of the source. For infinite length fibers, the potential distribution over the skin can thus be described as a propagating plane wave, for which the spatial coordinate along the fiber direction x_{lo} and the time coordinate t are related by $x_{lo} = vt$, with v the constant velocity of propagation [10]. In case of infinite length fibers, the potential detected along the fiber direction at different spatial locations is thus simply the delayed version of a prototype shape. On the basis of these assumptions, it is possible to estimate a propagation delay from the surface EMG signals, and thus to establish the theoretical basis for methods aimed at the non-invasive estimation of muscle fiber conduction velocity (CV). The signal detection by spatial filtering has also its theoretical basis on the assumption that the EMG signals detected by electrodes placed along the spatial direction of the muscle fibers have equal shapes with a delay [16][21][22]. When considering finite length fibers, the proportional relation between space and time coordinates does not strictly hold since the source changes shape with time due to the generation and extinction of the action potentials at the end-plates and tendons. In these cases, important limitations of the simple theoretical approximations considered have a direct impact on, e.g., estimation of muscle fiber CV [11] and spatial filter selectivity [7].

Besides finite length fibers, the volume conductor may also introduce shape variations in the detected surface potentials due to in-homogeneities in the direction of propagation of the potentials [23]. Schneider et al. [23] showed that tissue in-homogeneities may significantly affect estimation of delay between surface detected potentials. In that study the in-homogeneities were included by local areas of conductivity different from that of the muscle. The muscle tissue may also be in-homogeneous due to not unique fiber orientation, e.g., muscles in which there are two groups of muscle fibers with two different directions (bi-pinnate muscles). The anatomy of muscles is very complex, presenting uni-, bi-, multi- pinnate, circum-pinnate or complex-pinnate distribution of bundles of fibers [2][3]. Models assuming fibers all parallel to each other may thus be not appropriate for the interpretation of particular experimental conditions.

In the case of bi-pinnate muscles (e.g., rectus femoris muscle), the muscle tissue is non-homogeneous and anisotropic since the two fiber groups have different conductivities in the direction parallel and perpendicular to the fibers. Thus, even in case of infinite length fibers, the surface potentials detected at different electrode locations along fiber direction change shape and the system is not space invariant in the direction of propagation of the action potentials.

No previous modeling works provided an analytical description of surface EMG signals generated by muscles with not unique fiber orientation. This study addresses this issue for the bi-pinnate case. Since the approach is analytical, simplifications of the experimental conditions will be introduced. The approach will also be adapted to the case of a muscle with fibers inclined in the depth direction. The study provides the solution of the non-homogenous, anisotropic problem, proposes an implementation of the results in complete surface EMG generation models (including finite length fibers), and shows representative results of the application of the models proposed.

2. METHODS

The top view of the investigated geometry is shown in Fig. 1a. A simplified model of a bi-pinnate muscle is considered. The two pinnation angles are generic. The muscle tissue is more conductive along the muscle fiber direction than in the perpendicular directions. The signal is detected directly over the muscle. No intermediate layer will be assumed since this would lead to important mathematical issues, as it will be discussed in the following. The solution of this problem will also provide the bases for the analysis of the generation of surface EMG signals from muscle fibers inclined in the depth direction, as shown in Fig. 1b (a lateral view of the system is provided in this case). In the latter case, a multi-layer geometry will be investigated, since with this geometry the addition of subcutaneous layers can be treated simply.

Figure 1 about here

2.1 Bi-pinnate Muscle

2.1.1 Mathematical model of a bi-pinnate muscle

The electrical field problems in physiology, in first approximation, can be considered as quasi-static [14]. Thus, from the bio-electrical point of view, the tissues can be described as a volume conductor. In these conditions, the electrical potential is the solution of the Poisson equation:

$$-\nabla\cdot(\underline{\underline{\sigma}}\nabla\varphi)=I \tag{1}$$

where φ is the potential (V), I is the current density source (A/m^3), and $\underline{\underline{\sigma}}$ the conductivity tensor (S/m). Different volume conductors can essentially be modeled choosing the proper conductivity tensor.

We will study a model of an infinite bi-pinnate muscle. The conductivity tensor is diagonal in the coordinate system (n,T,z), shown in Fig. 1a. n is the coordinate along the fibers, which are placed along the family of curves defined by the coordinates:

$$(x_0 \mp n\cos\theta^{\pm}, nsin\theta^{\pm}, z_0) \tag{2}$$

where the sign $\pm$ refers to the half space $y > 0$ and $y < 0$, the discontinuity of the conductivity is at $y = 0$, and $\theta^{\pm}$ indicates the two pinnation angles. T is orthogonal to the fiber direction and to z (Fig. 1a). In Fig. 1a the skin plane is *x-y* while in Fig. 1b the skin plane is *x-z*. Thus, in the first case a top view is provided while in the second a lateral view is reported in Fig. 1. The reason for this different choice of the reference axes will be clear later. The geometries reported in Figure 1 are rather simple and represents approximation of experimental conditions. However, the inclusion of more than one inclination angle makes the analytical solution very complex even with this simple geometry.

The system of orthonormal versors associated to (n, T, z) is

$$\begin{cases} a_n^{\pm} = \mp\cos\theta^{\pm}\vec{i} + sin\theta^{\pm}\vec{j} \\ a_T^{\pm} = -sin\theta^{\pm}\vec{i} \mp \cos\theta^{\pm}\vec{j} \\ a_z^{\pm} = \vec{k} \end{cases} \tag{3}$$

where $\vec{i}$, $\vec{j}$, and $\vec{k}$ are the versors associated to the coordinates (*x, y, z*).

2.1.2 Formulation of the mathematical problem and choice of the coordinate system

The mathematical problem of determining the potential distribution generated by a source in the volume conductor shown in Fig. 1a is to solve Eq. (1) with the boundary conditions of continuity of the potential and its flux at the intersection of the fibers ($y = 0$), and the vanishing of the solution at infinity.

The issue of imposing the interface conditions is not trivial in the (n, T, z) coordinate system (the system in which the conductivity tensor is diagonal). In this coordinate system, indeed, the interface presents a complex expression, crossing the characteristic lines of the coordinate system. The interface conditions are more easily described changing the coordinate system to (x, y, z). In this case, the interface is described as $y = 0$. Furthermore, the volume conductor is homogeneous in the direction parallel to the interface, which allows to consider the bi-dimensional Fourier transform of Eq. (1) in the homogeneous plane (x, z). In this case, for each pair of spatial frequencies (k_x, k_z), two ordinary differential equations of the variable y (one for the half space $y > 0$ and another for $y < 0$), with conditions at $y = 0$, are obtained.

2.1.3 Formulation of the Poisson equation in the system (x, y, z)

The versors associated to the x, y, and z axes of the Cartesian coordinate system are $(\vec{i}, \vec{j}, \vec{k})$. The differential operator of Eq. (1) becomes:

$$\nabla \cdot (\underline{\underline{\sigma}} \nabla \varphi) = \nabla \cdot ((\sigma_{nn} a_n{}^{\pm} a_n{}^{\pm} + \sigma_{TT} a_T{}^{\pm} a_T{}^{\pm} + \sigma_{zz} a_z{}^{\pm} a_z{}^{\pm}) \nabla \varphi) =$$

$$= \frac{\partial}{\partial x}\left(\vec{i} \cdot (\sigma_{nn} a_n{}^{\pm} a_n{}^{\pm} + \sigma_{TT} a_T{}^{\pm} a_T{}^{\pm}) \cdot \left(\vec{i}\,\frac{\partial \varphi}{\partial x} + \vec{j}\,\frac{\partial \varphi}{\partial y} \right) \right) +$$

$$+ \frac{\partial}{\partial y}\left(\vec{j} \cdot (\sigma_{nn} a_n{}^{\pm} a_n{}^{\pm} + \sigma_{TT} a_T{}^{\pm} a_T{}^{\pm}) \cdot \left(\vec{i}\,\frac{\partial \varphi}{\partial x} + \vec{j}\,\frac{\partial \varphi}{\partial y} \right) \right) + \sigma_{zz} \frac{\partial^2 \varphi}{\partial z^2} \qquad (4)$$

with $\vec{k}$ orthogonal to the vectors $a_n{}^{\pm}, a_T{}^{\pm}, \vec{i}, \vec{j}$ (Fig. 1a). Performing the scalar products in Eq. (4), we obtain:

$$\nabla \cdot (\underline{\underline{\sigma}} \nabla \varphi) = \alpha_{\pm} \frac{\partial^2 \varphi}{\partial y^2} + \beta_{\pm} \frac{\partial^2 \varphi}{\partial x \partial y} + \gamma_{\pm} \frac{\partial^2 \varphi}{\partial x^2} + \sigma_{zz} \frac{\partial^2 \varphi}{\partial z^2} \qquad (5)$$

with:

$$\begin{aligned} \alpha_{\pm} &= \sigma_{nn} \sin^2 \theta^{\pm} + \sigma_{TT} \cos^2 \theta^{\pm} \\ \beta_{\pm} &= (\mp \sigma_{nn} \pm \sigma_{TT}) \cos \theta^{\pm} \sin \theta^{\pm} \\ \gamma_{\pm} &= \sigma_{nn} \cos^2 \theta^{\pm} + \sigma_{TT} \sin^2 \theta^{\pm} \end{aligned} \qquad (6)$$

If $\theta^{\pm} = \pi/2$, which corresponds to a planar volume conductor without discontinuity in its conductivity, we obtain $x = T,\quad y = n,\quad \alpha_{\pm} = \sigma_{nn},\quad \beta_{\pm} = 0,\quad \gamma_{\pm} = \sigma_{TT}$, and the Poisson equation assumes the simpler form already investigated in the literature [6]. In the case $\sigma_{nn} = \sigma_{TT} = \sigma_{zz}$, we obtain an isotropic volume conductor and Eq. (5) represents the Poisson equation for an isotropic, homogeneous medium.

2.1.4 General solution of the Poisson equation for the bi-pinnate muscle

Being the problem linear and homogeneous in the plane (x,z), we can study the solution in the Fourier domain for these two coordinates:

$$\hat{\varphi}(y,k_x,k_z) = \iint \varphi(x,y,z) e^{-jk_x x} e^{-jk_z z} dxdz \qquad (7)$$

The response of the system to an impulsive source is given, in the spatial frequency domain, by the solution of the following equation:

$$\alpha_{\pm} \frac{d^2\hat{\varphi}}{dy^2} - jk_x \beta_{\pm} \frac{d\hat{\varphi}}{dy} - (\gamma_{\pm} k_x{}^2 + \sigma_{zz} k_z{}^2)\hat{\varphi} = -\delta(y-y_s) e^{-jk_x x_s} e^{-jk_z z_s} \qquad (8)$$

where (x_s, y_s, z_s) are the coordinates of the source. In the two-dimensional Fourier domain, the Poisson equation is a second order ordinary differential equation. With respect to the case of homogeneous planar tissues, investigated in [6], the Fourier transformation is performed in the depth muscle direction (z with the current notations). This does not allow to easily impose interface conditions in this direction, as it was done in [6], and thus it does not allow to easily add subcutaneous layers to the geometrical configuration. The only simple extension concerns the case of a semi-infinite muscle bounded by an insulating medium at a certain coordinate $z = z_b$; in such a case, the application of the image theorem allows to obtain the surface potential as twice that calculated for the case of infinite muscle, treated below. The solution of the problem with layers added above the bi-pinnate muscle requires to maintain the spatial coordinate z and solve a partial differential equation in two variables, instead of an ordinary differential equation, transforming only in the x coordinate. In this study we will limit our analysis of the bi-pinnate muscle to the case of only muscle tissue.

Eq. (8) can be studied as an eigenvalue problem [25]. Assuming that the solution has the form $\hat{\varphi} = e^{k_y y}$, the following expression for the exponent k_y is obtained:

$$k_{y\pm\pm} = \frac{jk_x\beta_\pm \pm \sqrt{(4\alpha_\pm\gamma_\pm - \beta_\pm{}^2)k_x{}^2 + 4\sigma_{zz}\alpha_\pm k_z{}^2}}{2\alpha_\pm} \qquad (9)$$

where the first $\pm$ symbol in $k_{y\pm\pm}$ refers to the plus or minus sign in front of the square root, the second refers to the two half spaces in which the discontinuity divides the domain ($y > 0$ and $y < 0$).

Substituting the expressions for $\alpha_\pm, \beta_\pm, \gamma_\pm$ [Eqs. (6)], the radicand in Eq. (9) can be rewritten as $[4\sigma_{nn}\sigma_{TT}k_x{}^2 + 4\sigma_{zz}(\sigma_{nn}\sin^2\theta + \sigma_{TT}\cos^2\theta)k_z{}^2]$, which is a positive number for any k_x and k_z and for the two regions divided by the discontinuity of the conductivity.

2.1.5 Boundary conditions and particular solution

Let's consider, without loss of generality, the source placed in the half space $y < 0$ and divide the space in three regions, i.e., $y < y_s$, $y_s < y < 0$, and $y > 0$. The solution in this three regions can be expressed as:

$$\begin{cases} \hat{\varphi}_1 = A_1(k_x, k_z)e^{k_{y+-}y} & y < y_s \\ \hat{\varphi}_2 = A_2(k_x, k_z)e^{k_{y+-}y} + B_2(k_x, k_z)e^{k_{y--}y} & y_s < y < 0 \\ \hat{\varphi}_3 = B_3(k_x, k_z)e^{k_{y-+}y} & y > 0 \end{cases} \qquad (10)$$

where it has been imposed the condition of limitation of the potential for $y \to \pm\infty$.

The particular solution of the problem under study can be obtained imposing the interface conditions for $y = 0$, and the source conditions for $y = y_s$ [10]. The source conditions are the continuity of the potential and the discontinuity of the flux given by the Dirac delta function at $y = y_s$ [10]. The following linear system is obtained:

$$\underline{\underline{A}}\,\underline{X} = \underline{b}, \qquad (11)$$

with:

$$\underline{\underline{A}} = \begin{bmatrix} -\alpha_{-} k_{y+-} e^{k_{y+-} y_s} & \alpha_{-} k_{y+-} e^{k_{y+-} y_s} & \alpha_{-} k_{y--} e^{k_{y--} y_s} & 0 \\ e^{k_{y+-} y_s} & -e^{k_{y+-} y_s} & -e^{k_{y--} y_s} & 0 \\ 0 & k_{y+-} & k_{y--} & -k_{y-+} \\ 0 & 1 & 1 & -1 \end{bmatrix}; \underline{X} = \begin{bmatrix} A_1 \\ A_2 \\ B_2 \\ B_3 \end{bmatrix}; \ \underline{b} = \begin{bmatrix} -e^{jk_x x_s} e^{jk_z z_s} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

Imposing the conditions in Eq. (11), A_1, A_2, B_2, and B_3 are obtained as functions of k_x and k_z. This provides, by Eq. (10), the solution in the transformed domain (k_x, k_z, y).

Analytically inverting the system of Eq. (11) we have:

$$\underline{X} = \begin{bmatrix} \dfrac{e^{-k_{y--} y_s}(k_{y-+} - k_{y--}) + e^{-k_{y+-} y_s}(k_{y-+} - k_{y+-})}{\alpha_{-}(k_{y+-} - k_{y--})(k_{y+-} - k_{y-+})} \\ \dfrac{e^{-k_{y--} y_s}(-k_{y--} + k_{y-+})}{\alpha_{-}(k_{y+-} - k_{y--})(k_{y+-} - k_{y-+})} \\ \dfrac{e^{-k_{y--} y_s}}{\alpha_{-}(k_{y+-} - k_{y--})} \\ \dfrac{e^{-k_{y--} y_s}}{\alpha_{-}(k_{y+-} - k_{y-+})} \end{bmatrix} e^{-jk_x x_s} e^{-jk_z z_s} \quad (13)$$

The solution in the space domain is obtained substituting Eq. (13) into Eq. (10) and inverting the Fourier transform.

2.1.6 Scale properties of the solution for propagating sources along a fiber

Since the system is not space invariant for sources travelling along the muscle fibers, the surface potential generated by an impulsive source should be computed, since it is different, for each source location. However, a relationship between solutions generated by sources at different locations can be obtained. Let's consider an impulsive source travelling along a fiber. Suppose, without loss of generality, that the reference system has its origin on the fiber considered, at the interface between the fiber bundles constituting the bi-pinnate muscle. Suppose that the source is at the position $\vec{r}_s = (x_s, y_s, 0)$.

The geometry of the system allows to express the instantaneous positions of the source while travelling along the fiber as vectors proportional to $\vec{r}_s$. Indeed, at time t, the position of the impulsive source is given by the following expression:

$$\vec{r}(t) = k(t)\vec{r}_s = \frac{|\vec{r}_s| - vt}{|\vec{r}_s|}\vec{r}_s \, . \qquad (14)$$

With the following change of variables:

$$(X,Y,Z) = \frac{1}{k(t)}(x,y,z) \quad , \quad \Phi = k(t)\varphi \qquad (15)$$

we obtain that the response to the source placed at $k(t)\vec{r}_s$ solves the same problem [for the variables (X,Y,Z,Φ)] as that for the source placed at $\vec{r}_s$ (with variables (x,y,z,φ)). Thus, once the solution is obtained for the source at a given location, the solution at any other point along the fiber is obtained by scaling the space variables by $k(t)$:

$$\Phi(X,Y,Z) = k(t)\varphi\left(\frac{x}{k(t)}, \frac{y}{k(t)}, \frac{z}{k(t)}\right) \qquad (16)$$

It is clear that this does not imply that the signal detected in time domain at different locations is only scaled during propagation (note for example that all three variables, including *z*, are scaled).

2.1.7 Numerical issues

As inverting analytically the Fourier transform of the solution is not possible, we adopted a numerical approach. The numerical implementation requires the sampling of the spatial frequencies k_x and k_z (see also [10]). Sampling of the angular frequencies k_x and k_z determines a periodic repetition in the spatial domain. The origin of the reference system, as reported in Fig. 1a, is placed at the pinnation interface and the source is placed in the half space $y < 0$. Since the potential is maximum in correspondence to the source and decreases when the distance from the source increases, it is important that the source is distant from the borders of the spatial region described to avoid the aliasing due to periodic repetition of the solution (refer to [10] for more details). Thus, it is preferable, when numerically solving the problem, to consider the source as located at the origin of the coordinate system, which implies that the pinnation interface is at a certain coordinate $y = y_p$ (which changes as the source moves since also the origin of the coordinate system "moves" with the source).

In this case, the solution can be expressed as

$$\begin{cases} \hat{\varphi}_1 = A_1(k_x, k_z)e^{k_{y+-}y} & y < 0 \\ \hat{\varphi}_2 = A_2(k_x, k_z)e^{k_{y+-}y} + B_2(k_x, k_z)e^{k_{y--}y} & 0 < y < y_p \\ \hat{\varphi}_3 = B_3(k_x, k_z)e^{k_{y-+}y} & y > y_p \end{cases} \tag{17}$$

The linear algebraic system for the determination of the arbitrary coefficients is given by Eq. (11) with:

$$\underline{\underline{A}} = \begin{bmatrix} -\alpha_- k_{y+-} & \alpha_- k_{y+-} & \alpha_- k_{y--} & 0 \\ 1 & -1 & -1 & 0 \\ 0 & k_{y+-}e^{k_{y+-}y_p} & k_{y--}e^{k_{y--}y_p} & -k_{y-+}e^{k_{y-+}y_p} \\ 0 & e^{k_{y+-}y_p} & e^{k_{y--}y_p} & -e^{k_{y-+}y_p} \end{bmatrix}; \quad \underline{X} = \begin{bmatrix} A_1 \\ A_2 \\ B_2 \\ B_3 \end{bmatrix}; \quad \underline{b} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{18}$$

and inverting the system we have:

$$\underline{X} = \begin{bmatrix} A_1 = \dfrac{k_{y+-}k_{y-+} - k_{y-+}k_{y--}}{\alpha k_{y+-}(k_{y+-} - k_{y--})(k_{y+-} - k_{y-+})} e^{(k_{y--} - k_{y+-})y_p} + \dfrac{1}{\alpha_-(k_{y+-} - k_{y--})} \\ A_2 = \dfrac{k_{y+-}(k_{y-+} - k_{y--})}{\alpha k_{y+-}(k_{y+-} - k_{y--})(k_{y+-} - k_{y-+})} e^{(k_{y--} - k_{y+-})y_p} \\ B_2 = \dfrac{1}{\alpha_-(k_{y+-} - k_{y--})} \\ B_3 = \dfrac{e^{(k_{y--} - k_{y-+})y_p}}{\alpha_-(k_{y+-} - k_{y-+})} \end{bmatrix} \tag{19}$$

It has to be noted that, with this reference coordinate system, the location of the pinnation interface changes when the source position changes. In the same way, the detection point "moves" as the source travels along the fiber.

Once the solution is obtained in the spatial frequency domain, it is necessary to compute the potential detected in a particular point in space (the detection point). To do so, a bi-dimensional inverse Fourier transform should be computed and the resultant function sampled at the spatial location of the detection point. However, since only one value of the inverse Fourier transform is needed, it may be obtained by a double integral in k_x and k_z. For a different source position, both the solution (since the medium is not space invariant) and the location of the detection point (since the origin of the axis always corresponds to the source) change.

2.2 Fibers inclined with respect to the detection surface

We will now consider the application of the concepts proposed above to the case of a planar volume conductor with different homogeneous layers. The muscle layer has the fibers inclined with respect to the detection surface in the depth direction; the other layers are considered isotropic (although this condition is not essential), and can model, for example, a fat, skin, or air layer. In this case, the system is again not space invariant in the direction along which the source propagates. The conductivity tensor for the muscle layer is diagonal with respect to the (n, T, z) reference system, defined above. For the other layers the conductivity tensor is given by a constant conductivity multiplied by the identity tensor.

In the following, we will discuss the geometry reported in Fig. 1b in the case of only one additional homogeneous layer. The extension to more layers is straightforward. The volume conductor is thus comprised of an infinite muscle extending in the half space $y < 0$, and an isotropic layer placed between the muscle and an insulating medium ($0 < y < y_d$).

2.2.1 Mathematical formulation and general solution

The solution for the muscle layer is the same as that obtained in the case of the bi-pinnate muscle of Fig. 1a, for the half space $y < 0$. For the isotropic layer, the solution is given by the following equation:

$$\sigma\left(\frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2}\right) = 0 \qquad (19)$$

Considering the Fourier transform in the (x, z) variables we have:

$$\frac{\partial^2 \hat{\varphi}}{\partial y^2} = (k_x{}^2 + k_z{}^2)\hat{\varphi} \tag{20}$$

whose general solution is:

$$\hat{\varphi}_3 = A_3 e^{\sqrt{k_x{}^2+k_z{}^2}\,y} + B_3 e^{-\sqrt{k_x{}^2+k_z{}^2}\,y} \tag{21}$$

The final solution is obtained considering the general expressions $\hat{\varphi}_1, \hat{\varphi}_2$ from Eq. (10) and $\hat{\varphi}_3$ from Eq. (21). In case of more than one isotropic layer, the expression of Eq. (21) is used for each layer.

2.2.2 Boundary conditions

The particular solution can be obtained imposing the continuity of the potential and its flux at the interface between the conducting media, the continuity of the potential and the discontinuity of the flux at the impulsive source, and the vanishing flux at the insulating interface.

The linear system of Eq. (11) is now defined by the following matrix of coefficients:

$$\underline{\underline{A}} = \begin{bmatrix} -\alpha_- k_{y+-} e^{k_{y+-} y_s} & \alpha_- k_{y+-} e^{k_{y+-} y_s} & \alpha_- k_{y--} e^{k_{y--} y_s} & 0 & 0 \\ e^{k_{y+-} y_s} & -e^{k_{y+-} y_s} & -e^{k_{y--} y_s} & 0 & 0 \\ 0 & \alpha_- k_{y+-} & \alpha_- k_{y--} & -\sigma\sqrt{k_x{}^2 + k_z{}^2} & \sigma\sqrt{k_x{}^2 + k_z{}^2} \\ 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & \sqrt{k_x{}^2 + k_z{}^2}\, e^{\sqrt{k_x{}^2+k_z{}^2}\, y_d} & -\sqrt{k_x{}^2 + k_z{}^2}\, e^{-\sqrt{k_x{}^2+k_z{}^2}\, y_d} \end{bmatrix};$$

$$\underline{X} = \begin{bmatrix} A_1 \\ A_2 \\ B_2 \\ A_3 \\ B_3 \end{bmatrix}; \quad \underline{b} = \begin{bmatrix} -e^{-jk_x x_s} e^{-jk_z z_s} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{22}$$

with $k_{y\pm\pm}$ and α_- defined in Eqs. (8) and (6), respectively, and (x_s, y_s, z_s) the coordinates of the impulsive source.

Solving the system in Eq. (11) and using the last two components of the vector $\underline{X}$, we obtain the analytical expression of the potential in the Fourier domain over the detection surface $y = y_d$:

$$\hat{\varphi}_3 = \frac{e^{\sqrt{k_x^2+k_z^2}\,y_d - k_{y--}y_s}\, e^{-jk_x x_s}\, e^{-jk_z z_s}}{\alpha\_k_{y+-} - \sigma\sqrt{k_x^{\,2}+k_z^{\,2}} + e^{2\sqrt{k_x^2+k_z^2}\,y_d}\,(\sigma\sqrt{k_x^{\,2}+k_z^{\,2}} + \alpha\_k_{y+-})} + $$

$$+ \frac{e^{-k_{y--}y_s}\, e^{-jk_x x_s}\, e^{-jk_z z_s}}{2\alpha\_k_{y+-}\cosh(\sqrt{k_x^{\,2}+k_z^{\,2}}\,y_d) + 2\sigma\sqrt{k_x^{\,2}+k_z^{\,2}}\,sinh(\sqrt{k_x^{\,2}+k_z^{\,2}}\,y_d)} \qquad (23)$$

Applying a bi-dimensional inverse Fourier transform, the desired solution is obtained.

In case of more than one isotropic layer, the conditions of continuity of the potential and its flux at each interface will be added. If one layer extends to infinity, the coefficient multiplying the corresponding diverging term in Eq. (21) will be set to zero.

As for the case of the bi-pinnate muscle, the system is not space invariant in the direction of propagation of the action potentials, thus the response of the system to a source should be computed for each source location, as described above for the bi-pinnate case.

The case of fiber inclined with respect to the *(x, z)* plane can be treated by considering the inclination of the detection system over the detection plane, as proposed in [6], thus the volume conductor of Fig. 1b may describe fibers inclined with any orientation. In particular, the detection system transfer function can include the inclination angle of system over the skin plane. This can be accounted for by rotating the transfer function of the system in the spatial or spatial frequency domain [6].

3. RESULTS

The concepts described above were included in a model for surface EMG signal simulation which accounts for the travelling of an action potential along a muscle fiber, its generation at the end-plate and its extinction at the tendon junctions. The model also includes the possibility of summing together motor unit action potentials with the correspondent firing patterns in order to describe the complete generation of the interference surface EMG signal [8][9]. In particular, once the single fiber action potentials have been generated, they are added to determine the motor unit action potential. Each motor unit is active in specific instants of time according to the specific recruitment strategy simulated. The summation of the motor unit action potential trains provides the interference surface EMG signal. In the following only a few representative results of the application of the developed models for the generation of single muscle fiber action potentials will be provided. Future work will focus on the systematic investigation of the effect of the specific volume conductor properties on the features of the detected surface EMG signals.

Fig. 2 shows the surface potential distribution generated by three impulsive sources located along a muscle fiber of a bi-pinnate muscle. The two pinnation angles are both equal to 15°, and the detection surface is at $z = 2$ mm. The conductivity of the muscle is 0.5 S/m in the fiber direction and 0.1 S/m in the directions perpendicular to the fibers. The origin of the axes corresponds to the source, as discussed in Section 2.1.7. The potential distribution changes depending on the source location.

Fig. 3 shows the surface potential generated by three impulsive sources located along a muscle fiber which is inclined by 15° in the depth direction with respect to the detection surface. The change in potential shape with changing the position of the source is evident. Figs. 2 and 3 show that the problems under consideration are not space invariant, thus the detected potential of a travelling source cannot be obtained as a convolution of an impulse response with a linear filter, as in previous works [6][10].

Figures 2 and 3 about here

Figs. 4 and 5 show examples of simulated signals from a bi-pinnate muscle. The traveling source is modeled by two current tripoles, generated at the end-plate and traveling in opposite directions along a fiber towards the tendons where they extinguish. Current source parameters are the same as in [18] and the way in which the generation and extinction of the intracellular action potentials is simulated is the same as in [19]. The shape of the surface detected action potentials changes in both the case of detection along fiber direction (Fig. 4) and in the other direction (Fig. 5). In both cases, indeed, the system is not space invariant in the direction of source propagation.

For bi-pinnate muscles, the signal recorded over the muscle is the summation of contributions of motor units with fibers having different orientations, thus, when detected along a certain direction, the interference signal will not show a constant delay at different muscle locations. Features of these signals can be investigated with the present model.

Figures 4 and 5 about here

4. DISCUSSION AND CONCLUSIONS

In this study we investigated the problem of analytically describing the surface EMG signal generation from volume conductors which are not space invariant along the direction of propagation of the sources. The case analyzed is that of a bi-pinnate muscle, which presents two fiber orientations. The model analyzed is a simplification of the real conditions, assuming a planar volume conductor with rectilinear fibers. However, it covers more complex anatomical conditions than previous works. Indeed, there are no other works in the literature which analytically addressed the issue of describing the surface EMG generation from muscle tissues with more than one different fiber direction. The availability of such a model may help in interpreting signals generated by muscles with more complex architectures than those comprised of fibers all parallel with respect to each other. In particular, the fact that the potentials detected over the skin along fiber direction may change in shape is important for practical reasons, such as the estimation of muscle fiber CV. Moreover, a model such as that proposed in this study may be used to test non-invasive methods for estimating the pinnation angle from, e.g., surface EMG topographical mapping [15].

The approach proposed for the solution of the problem is derived from previous works from our group [6][10]. The solution was investigated in the bi-dimensional Fourier domain, as previously done. With this approach, systems of partial differential equations are treated as systems of ordinary differential equations. However, the approach implies issues to be carefully addressed in the implementation of the solution. In case of space invariant systems, as those analyzed in [6][10], the solution is only shifted in space if the source is shifted. This implies that, in practice, the source can be considered as always located at the origin of the coordinate system. For systems which are not space invariant, the response to the source should be computed at each location and the center of the coordinate axis does not necessarily correspond to the source. We proposed, together with the general analytical solution with fixed coordinate axis, a solution for which the origin of the coordinates corresponds to the source. This significantly simplifies the numerical implementation.

The case of a muscle with fibers inclined in the depth direction was studied using similar concepts as those adopted for the bi-pinnate case. With this approach, the fibers may be described as oriented in a generic direction within the muscle.

The main contribution of this study is the derivation of the general solution of the problem of surface EMG signal generation from non-homogeneous muscle tissue when the in-homogeneity is due to the presence of two fiber orientations. The model developed may help in better understanding the generation of surface EMG signals from complex muscle architectures and in interpreting experimental results.

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FIGURE CAPTIONS

Fig. 1 a) Top view of the bi-pinnate muscle investigated. The muscle fibers are parallel to the skin plane and have two orientations (which can be, in general, different) defined by the angles $\theta^{\pm}$. The intracellular action potentials travel along the muscle fibers. The muscle tissue is in-homogeneous along the fiber direction and anisotropic. b) Lateral view of the second configuration considered, whose solution is derived from that obtained in the case of the bi-pinnate muscle. The muscle tissue is homogeneous and anisotropic with fibers inclined in the depth direction by the angle θ. An isotropic medium separates the muscle from the air layer. Note that the *x* and *y* axes are defined differently in the cases a) and b) (top view and lateral view, respectively).

Fig. 2 a) Top view of a volume conductor simulating a bi-pinnate muscle (pinnation angles 15°) with three locations of an impulsive source. The potential is detected at z = 2 mm. The potential distribution generated by the three sources is shown in b), c) and d), as contour plots. Note that the potential distribution changes with source position since the system is not space-invariant.

Fig. 3 a) A lateral view of a volume conductor simulating a muscle with fibers inclined in the depth direction (inclination angle 15°) with three locations of an impulsive source. The potential distribution generated by each of the three sources is shown in b), c) and d). The potential distribution changes with source position.

Fig. 4 Examples of simulated signals as generated by a fiber located in a bi-pinnate muscle and detected by an array of electrodes located along the direction of the muscle fiber. The locations of the end-plate and tendon junctions are schematically indicated by circles. The top view of the simulated muscle is shown in (a). Monopolar (b), single (c) and double differential (d) signals are reported.

Fig. 5 Examples of simulated signals as generated by a fiber located in a bi-pinnate muscle and detected by an array of electrodes placed in the other muscle fiber direction. The locations of the end-plate and the tendon junctions are schematically indicated by circles and are the same as in Fig.

4. The top view of the simulated muscle is shown in (a). Monopolar (b), single (c) and double differential (d) signals are reported.