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The capacitated transshipment location problem with stochastic handling utilities at the facilities

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Abstract The problem consists in finding a transshipment facilities location which maximizes the total net utility when the handling utilities at the facilities are stochastic variables, under supply, demand, and lower and upper capacity constraints. The total net utility is given by the expected total shipping utility minus the total fixed cost of the located facilities. Shipping utilities are given by a deterministic utility for shipping freight from origins to destinations via transshipment facilities plus a stochastic handling utility at the facilities, whose probability distribution is unknown. After giving the stochastic model, by means of some results of the extreme values theory, the probability distribution of the maximum stochastic utilities is derived and the expected value of the optimum of the stochastic model is found. An efficient heuristics for solving real-life instances is also given. Computational results show a very good performance of the proposed methods both in terms of accuracy and efficiency.

Keywords facilities location · stochastic utilities · asymptotic approximation · heuristics

1 Introduction

Let us consider a set of origins with a given supply, a set of destinations with a given demand, a set of potential transshipment locations with a deterministic fixed cost of location, lower and upper capacity constraints for the facilities, and stochastic shipping utilities from origins to destinations via transshipment facilities.

Each stochastic shipping utility is given by the sum of a deterministic utility for shipping freight from an origin to a destination via a transshipment facility plus a stochastic term, which represents the handling utility at the transshipment facility. The freight, when enters into a transshipment facility, is subject to handling operations which are typically organized in alternative handling operating scenarii. These scenarii represent sets of options for routing and processing the freight within the transshipment facility. Given the finite capacity of each handling operating scenario, congestion effects make the handling utility a stochastic variable, whose distribution is usually unknown.

The capacitated transshipment location problem with stochastic handling utilities at the facilities ($CTLP_s$) consists in finding a transshipment facilities location which maximizes the total net utility, given by the expected total shipping utility minus the total fixed cost of the located facilities, subject to supply, demand, and lower and upper facilities capacity constraints. In this paper we integrate the two main levels of a transshipment network, i.e. the network design (upper level), which leads to a network flow formulation with origins, transshipment facilities and destinations as nodes of the network, and the transshipment facilities management (lower level), where the management variables we consider are the stochastic handling utilities at the facilities. It is interesting to observe

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that the upper and lower level decisions are made in practice at different time instants: first the location decisions then the management decisions. Seeking for an optimal location of the facilities (first level), our model also takes into account the future management decisions (second level), which could affect the facilities optimal location.

The capacitated transshipment location problem with stochastic handling utilities at the facilities arises as a subproblem in several applications of logistics, and city logistics in particular, both at the strategic and the tactical decision levels [3] (in particular, when freight consolidation operations are performed at the transshipment facilities, e.g. for grocery and food shipping).

Only a few papers concerning location problems with stochastic utilities (or costs) are currently available. Among them, Ricciardi et al. [12] develop a heuristics for solving a p-median problem where the throughput costs are stochastic variables with a given probability distribution. Snyder et al. [13] consider a scenario-based stochastic version of a joint location-inventory model that minimizes the expected cost of locating facilities, allowing costs, lead times, demand, and some other parameters to be stochastic. Tadei et al. [14] consider a stochastic p-median problem where the costs for using the facilities are stochastic variables.

For the $CTLP_s$ we give the stochastic model. By means of some results of the extreme values theory, the probability distribution of the maximum stochastic utility from any origin i is derived and the expected value of the optimum of the stochastic model is found. An efficient heuristics for solving real-life instances is also given.

The remainder of the paper is organized as follows. Section 2 introduces the $CTLP_s$. In Section 3, the asymptotic approximation of the probability distribution of the maximum stochastic utility from any origin is derived, which allows to find the expected value of the optimum of the $CTLP_s$. Section 4 presents the heuristics for solving real-life instances. In Section 5, the computational results of the stochastic model and the heuristics are given. Finally, the conclusion of our work is reported in Section 6.

2 The problem

We consider the following parameters and data

- I: set of origins
- J: set of destinations
- K: set of potential transshipment locations
- P_i: supply at origin $i \in I$
- Q_j: demand at destination $j \in J$
- H_k: set of handling operating scenarii at transshipment facility $k \in K$, with $|H_k| = n_k$
- U_k: upper capacity of transshipment facility $k \in K$
- L_k: lower capacity of transshipment facility $k \in K$
- f_k: fixed cost of locating a transshipment facility $k \in K$
- v_{ij}^k: deterministic utility for shipping one unit of freight from origin $i \in I$ to destination $j \in J$ via transshipment facility $k \in K$

and the variables

- y_k: Boolean variable which is equal to 1 if transshipment facility $k \in K$ is located, 0 otherwise
- $\tilde{u}^{kl}$: stochastic variable with unknown probability distribution which represents the unit utility of handling operating scenario $l \in H_k$ at transshipment facility $k \in K$
- s_{ij}^k: deterministic variable which represents the flow from origin $i \in I$ to destination $j \in J$ via transshipment facility $k \in K$.

Let us assume:

- the system is balanced, i.e. $\sum_{i \in I} P_i = \sum_{j \in J} Q_j = T$. This is a standard assumption and it is straightforward to balance the system, if necessary
- $\{\tilde{u}^{kl}\}$ are independent and identically distributed (i.i.d.) stochastic variables with a common and *unknown* probability distribution

$$Pr\{\tilde{u}^{kl} \leq x\} = F(x). \tag{1}$$

The i.i.d. assumption for $\{\tilde{u}^{kl}\}$, which is necessary for deriving the asymptotic approximation of Section 3, can be justified as follows. The stochastic utility of a handling operating scenario at any facility k is extremely difficult to be measured in practice, then its probability distribution is generally unknown, and it would be rather arbitrary to assume a particular shape for it. Of course, the mildest hypothesis we can made for the shape of such unknown probability distribution (which is necessary for calculating the expected value) is that it does not vary within different scenarii and facilities, which corresponds to the 'identically distributed' assumption. Moreover, the alternative handling operating scenarii inside a facility do not obviously depend on the scenarii of the remaining facilities, and inside the same facility they slightly interact with each other in practice, allowing us to consider their stochastic utilities $\{\tilde{u}^{kl}\}$ as independent variables too.

Let $\tilde{v}_{ij}^{kl}(\tilde{u}^{kl})$ be the unit stochastic shipping utility from origin i to destination j via transshipment facility k in handling operating scenario l given by

$$\tilde{v}_{ij}^{kl}(\tilde{u}^{kl}) = v_{ij}^k + \tilde{u}^{kl}, \;\; i \in I, j \in J, k \in K, l \in H_k. \tag{2}$$

Let us define $\tilde{u}^k$ the maximum of the stochastic handling utilities $\{\tilde{u}^{kl}\}$ within the alternative handling operating scenarii $\{l\}$ at the transshipment facility k

$$\tilde{u}^k = \max_{l \in H_k} \tilde{u}^{kl}, \quad k \in K \tag{3}$$

which is still of course a stochastic variable with unknown probability distribution given by

$$B_k(x) = \Pr\left\{\tilde{u}^k \leq x\right\}. \tag{4}$$

As for (3) $\tilde{u}^k \leq x \Longleftrightarrow \tilde{u}^{kl} \leq x,\ l \in H_k$ and $\{\tilde{u}^{kl}\}$ are independent, using (1), (4) becomes

$$B_k(x) = \prod_{l \in H_k} \Pr\left\{\tilde{u}^{kl} \leq x\right\} = \prod_{l \in H_k} F(x) = [F(x)]^{n_k} \tag{5}$$

where $n_k = |H_k|$ is the total number of alternative handling operating scenarii at the transshipment facility k.

As far as the management level is considered, we assume that the facilities management policies are efficiency-based, so that, among the alternative handling operating scenarii $\{l\}$ at any transshipment facility k, the one which maximizes the stochastic shipping utility $\tilde{v}_{ij}^{kl}(\tilde{u}^{kl})$ will be selected, giving

$$\tilde{v}_{ij}^{k}(\tilde{u}^k) = \max_{l \in H_k} \tilde{v}_{ij}^{kl}(\tilde{u}^{kl}) = v_{ij}^k + \max_{l \in H_k} \tilde{u}^{kl} = v_{ij}^k + \tilde{u}^k, \quad i \in I, j \in J, k \in K \tag{6}$$

The $CTLP_s$ is formulated as follows

$$\max_y \mathbb{E}_{\tilde{u}^k}\left[\max_s \sum_{i \in I} \sum_{k \in K} \sum_{j \in J} \tilde{v}_{ij}^k(\tilde{u}^k) s_{ij}^k\right] - \sum_{k \in K} f_k y_k \tag{7}$$

subject to

$$\sum_{k \in K} \sum_{j \in J} s_{ij}^k = P_i, \quad i \in I \tag{8}$$

$$\sum_{i \in I} \sum_{k \in K} s_{ij}^k = Q_j, \quad j \in J \tag{9}$$

$$\sum_{i \in I} \sum_{j \in J} s_{ij}^k \leq T y_k, \quad k \in K \tag{10}$$

$$\sum_{i \in I} \sum_{j \in J} s_{ij}^k \leq U_k, \quad k \in K \tag{11}$$

$$\sum_{i \in I} \sum_{j \in J} s_{ij}^k \geq L_k, \quad k \in K \tag{12}$$

$$s_{ij}^k \geq 0, \quad i \in I, j \in J, k \in K \tag{13}$$

$$y_k \in \{0,1\}, \quad k \in K \tag{14}$$

where $\mathbb{E}_{\tilde{u}^k}$ denotes the expected value with respect to $\{\tilde{u}^k\}$.

The objective function (7) expresses the maximization of the total net utility given by the expected total shipping utility minus the total fixed cost of the located facilities. Constraints (8) and (9) ensure that supply at each origin i and demand at each destination j are satisfied. Constraints (10) prevent to ship freight through not located facilities. Constraints (11) and (12) ensure the upper and lower capacity restrictions at each transshipment facility k. Finally, (13) and (14) are the non-negativity and the integrality constraints, respectively.

Let us consider the Lagrangian relaxation of problem (7)-(14), obtained by relaxing constraints (9), (11), and (12) by means of the multipliers $\mu_j, j \in J$, $\lambda_k \geq 0, k \in K$, and $\eta_k \leq 0, k \in K$, respectively

$$\begin{aligned} \min_{\mu, \lambda \geq 0, \eta \leq 0} \max_y \ \mathbb{E}_{\tilde{u}^k}\left[\max_s \sum_{i \in I} \sum_{k \in K} \sum_{j \in J} \left(\tilde{v}_{ij}^k(\tilde{u}^k) + \mu_j - \lambda_k - \eta_k\right) s_{ij}^k\right] + \\ + \sum_{k \in K} \left[\lambda_k U_k + \eta_k L_k - f_k y_k\right] - \sum_{j \in J} \mu_j Q_j \end{aligned} \tag{15}$$

subject to (8), (10), (13), and (14).

The term $(\tilde{v}_{ij}^k(\tilde{u}^k) + \mu_j - \lambda_k - \eta_k)$ in (15) is named the unit "shadow" stochastic shipping utility from i to j via transshipment facility k, due to the fact that it contains the shadow prices μ_j, λ_k, and η_k.

Problem (15) subject to (8), (10), (13), and (14) gives an upper bound on the optimum of problem (7)-(14), but we know that, if the strong duality conditions are satisfied, the two problems are equivalent.

For any value of the multipliers $\{\mu_j\}$, $\{\lambda_k \geq 0\}$ and $\{\eta_k \leq 0\}$, a freight unit in i will be shipped towards the alternative (s,t), given by the transshipment facility s and the destination t (for the sake of simplicity, we assume that this alternative is unique) whose shadow stochastic utility $(\tilde{v}_{it}^s(\tilde{u}^s) + \mu_t - \lambda_s - \eta_s)$ is the maximum within those of the located transshipment facilities and destinations. So, the maximum unit shadow stochastic shipping utility from origin i becomes

$$\tilde{v}_i(\tilde{u}^s) = \tilde{v}_{it}^s(\tilde{u}^s) + \mu_t - \lambda_s - \eta_s = max_{k:y_k=1,j}\left(\tilde{v}_{ij}^k(\tilde{u}^k) + \mu_j - \lambda_k - \eta_k\right). \tag{16}$$

Problem (15) subject to (8), (10), (13), and (14), for any value of the multipliers $\{\mu_j\}$, $\{\lambda_k \geq 0\}$, and $\{\eta_k \leq 0\}$ and any transshipment facilities location $\{y_k\}$, gives the following trivial optimal flows

$$\begin{array}{l} s_{ij}^k = P_i, \;\; if \;\; \tilde{v}_{ij}^k(\tilde{u}^k) + \mu_j - \lambda_k - \eta_k = \tilde{v}_i(\tilde{u}^s) \\ s_{ij}^k = 0 \quad otherwise \end{array} \tag{17}$$

and, by (17) and the linearity of $\mathbb{E}_{\tilde{u}^s}$, the objective function (15) becomes

$$\begin{aligned} &\min_{\mu,\lambda\geq 0,\eta\leq 0}\max_{y}\mathbb{E}_{\tilde{u}^s}\left[\sum_{i\in I}P_i\tilde{v}_i(\tilde{u}^s)\right] + \sum_{k\in K}\left[\lambda_k U_k + \eta_k L_k - f_k y_k\right] - \sum_{j\in J}\mu_j Q_j = \\ &= \min_{\mu,\lambda\geq 0,\eta\leq 0}\max_{y}\sum_{i\in I}P_i\mathbb{E}_{\tilde{u}^s}\left[\tilde{v}_i(\tilde{u}^s)\right] + \sum_{k\in K}\left[\lambda_k U_k + \eta_k L_k - f_k y_k\right] - \sum_{j\in J}\mu_j Q_j. \end{aligned} \tag{18}$$

To calculate $\mathbb{E}_{\tilde{u}^s}\left[\tilde{v}_i(\tilde{u}^s)\right]$ in (18), we first need to know the probability distribution of $\tilde{v}_i(\tilde{u}^s)$, named $G_i(x)$

$$G_i(x) = Pr\{\tilde{v}_i(\tilde{u}^s) \leq x\} = Pr\left\{\max_{k:y_k=1,j}\left(\tilde{v}_{ij}^k(\tilde{u}^k) + \mu_j - \lambda_k - \eta_k\right) \leq x\right\}. \tag{19}$$

As

$$\max_{k:y_k=1,j}\left(\tilde{v}_{ij}^k(\tilde{u}^k) + \mu_j - \lambda_k - \eta_k\right) \leq x \Longleftrightarrow \tilde{v}_{ij}^k(\tilde{u}^k) + \mu_j - \lambda_k - \eta_k \leq x, \;\; k \in K : y_k = 1, j \in J \tag{20}$$

and the stochastic variables $\tilde{u}^k$ are independent (because $\tilde{u}^{kl}$ are independent), (19) becomes

$$\begin{aligned} G_i(x) &= \Pr\left\{max_{k:y_k=1,j}\left(\tilde{v}_{ij}^k(\tilde{u}^k) + \mu_j - \lambda_k - \eta_k\right) \leq x\right\} = \\ &= \Pr\left\{\tilde{v}_{ij}^k(\tilde{u}^k) + \mu_j - \lambda_k - \eta_k \leq x, \;\; k \in K : y_k = 1, j \in J\right\} = \\ &= \prod_{k\in K:y_k=1}\prod_{j\in J}\Pr\left\{\tilde{v}_{ij}^k(\tilde{u}^k) + \mu_j - \lambda_k - \eta_k \leq x\right\} = \\ &= \prod_{k\in K:y_k=1}\prod_{j\in J}\Pr\left\{v_{ij}^k + \tilde{u}^k + \mu_j - \lambda_k - \eta_k \leq x\right\} = \\ &= \prod_{k\in K:y_k=1}\prod_{j\in J}\Pr\left\{\tilde{u}^k \leq x - v_{ij}^k - \mu_j + \lambda_k + \eta_k\right\} = \\ &= \prod_{k\in K:y_k=1}\prod_{j\in J}B_k(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k) = \\ &= \prod_{k\in K:y_k=1}\prod_{j\in J}\left[F(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k)\right]^{n_k}. \end{aligned} \tag{21}$$

Unfortunately, the unknown probability distribution $F(.)$ in (21) still prevents the calculation of $G_i(x)$. A possible way to cope with this problem and get an explicit form for $G_i(x)$ is to consider an asymptotic approximation for it.

3 An asymptotic approximation for the maximum stochastic shipping utility from any origin $G_i(x)$

The method we use to derive an asymptotic approximation of the maximum stochastic shipping utility from any origin $G_i(x)$ is based on the following observation. Under mild conditions on the unknown probability distribution $F(x)$, the probability distribution $G_i(x)$ tends towards a specific functional form as the total number of alternative handling operating scenarii at transshipment facility k, n_k, becomes large.

Following Galambos [7] and using some results of the extreme values theory for i.i.d. stochastic variables, we will prove that the only condition requested for $F(x)$ is that it is asymptotically exponential in its right tail, i.e. there is a constant $\beta > 0$ such that

$$\lim_{y\to+\infty} \frac{1-F(x+y)}{1-F(y)} = e^{-\beta x}. \tag{22}$$

This is a very mild condition, as we observe that many probability distributions show such a behavior, among them the Gamma, Gumbel, Laplace, and Logistic distributions.

Firstly, let us consider the following aspect: the optimal solution of problem (15) subject to (8), (10), (13), and (14) does not change if any arbitrary constant is added or subtracted to the stochastic variables $\tilde{u}^k$.

Let us choose this constant as the root a_{n_k} of the equation

$$1 - F(a_{n_k}) = 1/n_k. \tag{23}$$

By replacing $\tilde{u}^k$ with $\tilde{u}^k - a_{n_k}$ in (21) one has

$$G_i(x \mid n_k) = \prod_{k\in K:y_k=1} \prod_{j\in J} \left[F(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k + a_{n_k})\right]^{n_k}. \tag{24}$$

Let us assume that $n_k, k \in K : y_k = 1$ are large enough to use $\lim_{n_k\to+\infty} G_i(x \mid n_k)$ as an approximation of $G_i(x)$.

The following theorem holds

Theorem 1 *Under condition (22), the probability distribution $G_i(x)$ becomes*

$$G_i(x) = \lim_{n_k\to+\infty} G_i(x \mid n_k) = \exp\left(-A_i e^{-\beta x}\right) \tag{25}$$

where

$$A_i = \sum_{k\in K:y_k=1} \sum_{j\in J} e^{\beta(v_{ij}^k + \mu_j - \lambda_k - \eta_k)}, \quad i \in I \tag{26}$$

is the "accessibility", in the sense of Hansen [10], of a freight unit from the origin i to the overall system of the located transshipment facilities and destinations.

Proof By (24) one has

$$\begin{aligned} G_i(x) &= \lim_{n_k\to+\infty} G_i(x \mid n_k) = \\ &= \lim_{n_k\to+\infty} \prod_{k\in K:y_k=1} \prod_{j\in J} \left[F(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k + a_{n_k})\right]^{n_k} = \\ &= \prod_{k\in K:y_k=1} \prod_{j\in J} \lim_{n_k\to+\infty} \left[F(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k + a_{n_k})\right]^{n_k}. \end{aligned} \tag{27}$$

As from (23), $\lim_{n_k\to+\infty} a_{n_k} = +\infty$, from (22) one obtains

$$\lim_{n_k\to+\infty} \frac{1 - F(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k + a_{n_k})}{1 - F(a_{n_k})} = e^{-\beta(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k)}. \tag{28}$$

By (28) and (23) one has

$$\begin{aligned} \lim_{n_k\to+\infty} F(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k + a_{n_k}) &= \lim_{n_k\to+\infty} \left(1 - [1 - F(a_{n_k})]e^{-\beta(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k)}\right) = \\ &= \lim_{n_k\to+\infty} \left(1 - \frac{e^{-\beta(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k)}}{n_k}\right) \end{aligned} \tag{29}$$

and, by reminding that $\lim_{n\to+\infty}(1 + \frac{x}{n})^n = e^x$

$$\lim_{n_k \to +\infty} \left[F(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k + a_{n_k})\right]^{n_k} = \lim_{n_k \to +\infty} \left[1 - \frac{e^{-\beta(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k)}}{n_k}\right]^{n_k} =$$
$$= \exp\left(-e^{-\beta(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k)}\right). \tag{30}$$

Substituting (30) into (27) and using (26), one finally gets

$$G_i(x) = \prod_{k \in K: y_k = 1} \prod_{j \in J} \exp\left(-e^{-\beta(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k)}\right) = \exp\left(-A_i e^{-\beta x}\right). \quad \square \tag{31}$$

It is interesting to observe that $G_i(x)$ in (25) becomes a Gumbel (or double exponential) distribution [9].

Having now an explicit form for $G_i(x)$, we can calculate $\mathbb{E}_{\tilde{u}^s}\left[\tilde{v}_i(\tilde{u}^s)\right]$ in (18) as follows

$$\overline{v}_i = \mathbb{E}_{\tilde{u}^s}\left[\tilde{v}_i(\tilde{u}^s)\right] = \int_{-\infty}^{+\infty} x dG_i(x) = \int_{-\infty}^{+\infty} x \exp\left(-A_i e^{-\beta x}\right) A_i e^{-\beta x} \beta dx, \;\; i \in I. \tag{32}$$

Substituting for $t = A_i e^{-\beta x}$, one gets

$$\begin{aligned}\overline{v}_i &= -1/\beta \int_0^{+\infty} \ln(t/A_i) e^{-t} dt = \\ &= -1/\beta \int_0^{+\infty} e^{-t} \ln t dt + 1/\beta \ln A_i \int_0^{+\infty} e^{-t} dt = \\ &= \gamma/\beta + 1/\beta \ln A_i = \\ &= 1/\beta(\ln A_i + \gamma)\end{aligned} \tag{33}$$

where $\gamma = -\int_0^{+\infty} e^{-t} \ln t \; dt \simeq 0.5772$ is the Euler constant.

By substituting (33) in (18) and disregarding the constant $\frac{\gamma}{\beta}\sum_{i \in I} P_i$, the $CTLP_s$ becomes the following non-linear deterministic mixed-integer problem, named $CTLP_d$

$$\min_{\mu, \lambda \geq 0, \eta \leq 0} \max_y 1/\beta \sum_{i \in I} P_i \ln A_i + \sum_{k \in K} \left[\lambda_k U_k + \eta_k L_k - f_k y_k\right] - \sum_{j \in J} \mu_j Q_j \tag{34}$$

subject to (8), (10), (13), and (14).

We denote by

$$x_{ij}^k = s_{ij}^k / P_i, \;\; i \in I, j \in J, k \in K \tag{35}$$

the probability that a freight unit in i is shipped towards the alternative (k, j), given by the transshipment facility k and destination j.

The following theorem holds

Theorem 2 *At optimality, the probability x_{ij}^k is given by*

$$x_{ij}^k = \frac{e^{\beta(v_{ij}^k + \mu_j - \lambda_k - \eta_k)}}{\sum_{k' \in K: y_{k'} = 1} \sum_{j' \in J} e^{\beta(v_{ij'}^{k'} + \mu_{j'} - \lambda_{k'} - \eta_{k'})}}, \;\; i \in I, j \in J, k \in K. \tag{36}$$

Proof At optimality, the probability that a freight unit in i is shipped towards the alternative (k, j) is equal to the probability that (k, j) is the alternative of maximum utility. Then, from the Total Probability Theorem [4], (22), and (26), one obtains

$$\begin{aligned}x_{ij}^k &= \int_{-\infty}^{+\infty} \prod_{u \neq k} \prod_{v \neq j} \exp\left[-e^{-\beta(x - v_{iv}^u - \mu_v + \lambda_u + \eta_u)}\right] d\left[\exp\left(-e^{-\beta(x - v_{ij}^k - \mu_j + \lambda_k + \eta_k)}\right)\right] = \\ &= e^{\beta(v_{ij}^k + \mu_j - \lambda_k - \eta_k)} \int_{-\infty}^{+\infty} \beta e^{-\beta x} exp(-A_i e^{-\beta x}) dx = \\ &= e^{\beta(v_{ij}^k + \mu_j - \lambda_k - \eta_k)} \int_0^{+\infty} e^{-A_i t} dt = \frac{e^{\beta(v_{ij}^k + \mu_j - \lambda_k - \eta_k)}}{A_i} = \\ &= \frac{e^{\beta(v_{ij}^k + \mu_j - \lambda_k - \eta_k)}}{\sum_{k' \in K: y_{k'} = 1} \sum_{j' \in J} e^{\beta(v_{ij'}^{k'} + \mu_{j'} - \lambda_{k'} - \eta_{k'})}} \;\; i \in I, j \in J, k \in K\end{aligned} \tag{37}$$

where $t = e^{-\beta x}$. $\square$

By (35) and (36), the optimal flows s_{ij}^k then become

$$s_{ij}^k = P_i x_{ij}^k = P_i \frac{e^{\beta(v_{ij}^k + \mu_j - \lambda_k - \eta_k)}}{\sum_{k' \in K: y_{k'}=1} \sum_{j' \in J} e^{\beta(v_{ij'}^{k'} + \mu_{j'} - \lambda_{k'} - \eta_{k'})}}, \quad i \in I, j \in J, k \in K \tag{38}$$

and it is trivial to check the satisfaction of constraints (8) and (13).

(36) represents a multinomial Logit model, which is widely used in choice theory [5]. In our case, it describes how the freight shipped from i is split among the alternatives $\{(k, j)\}$, due to the stochastic handling utilities at the transshipment facilities where the freight passes through.

It is interesting to note that (38), although it has been derived for located transshipment facilities, holds for all facilities k. So, for a located transshipment facility k, s_{ij}^k represents the actual flow from i to j via that located facility k. Vice versa, for a not located transshipment facility k, it represents the "potential" flow from i to j via that not located facility k. The potential flow will be used in our heuristics of Section 4.

Solving the $CTLP_d$ is much faster than solving the $CTLP_s$ as a stochastic programming model and it also has a good performance in terms of accuracy, showing a mean gap of 1.65% between the two optima (see Section 5). By using one of the best available stochastic commercial solvers, in one hour of computing time the $CTLP_s$ is just able to solve instances with up to a few origins and some decades of potential facilities locations and destinations, whereas in the same computing time the $CTLP_d$ can solve instances which are roughly three times larger. Unfortunately, if one wanted to consider even larger instances (hundreds of nodes for the total number of origins, destinations and potential facilities locations), which may appear in real-life applications, also the $CTLP_d$ becomes inefficient and some heuristics must be used.

One of such heuristics is given in the next section.

4 A heuristics for solving the $CTLP_d$

The heuristics for solving the $CTLP_d$ is based on three procedures which interact with each other. The first is a procedure for calculating the Lagrangian multipliers $\{\mu_j\}$, $\{\lambda_k \geq 0\}$, and $\{\eta_k \leq 0\}$ in (34), when a transshipment facilities location $\{y_k\}$ is already given, while the second and the third are, respectively, for locating and closing down facilities in order to improve the given facilities location.

Firstly, we present the three procedures, whereas the overall heuristics is given at the end of this section.

4.1 Lagrangian multipliers calculation

As previously assumed, a transshipment location $\{y_k\}$ is already given. We calculate the Lagrangian multipliers $\{\mu_j\}$, $\{\lambda_k\}$, and $\{\eta_k\}$ by an iterative method as follows.

Let us start with $\lambda_k = \eta_k = 0, k \in K$.

Calculate $\{\mu_j\}$ such that constraints (9), where s_{ij}^k are given by (38), are satisfied.
To do that, solve the following system of equations iteratively in $\{e^{\beta\mu_j}\}$, starting with any value for $\{e^{\beta\mu_j}\}$ (e.g., $e^{\beta\mu_j} = 1$, then $\mu_j = 0, j \in J$)

$$e^{\beta\mu_j} = Q_j / \sum_{i \in I} \sum_{k \in K: y_k=1} P_i \frac{e^{\beta v_{ij}^k} e^{-\beta\lambda_k} e^{-\beta\eta_k}}{\sum_{k' \in K} \sum_{j' \in J} y_{k'} e^{\beta v_{ij'}^{k'}} e^{\beta\mu_{j'}} e^{-\beta\lambda_{k'}} e^{-\beta\eta_{k'}}}, \quad j \in J. \tag{39}$$

Once $\{e^{\beta\mu_j}\}$ are calculated, the multipliers $\{\lambda_k\}$ and $\{\eta_k\}$ are updated as follows (note that these multipliers are also calculated for not located facilities).

Let $D_k(\lambda, \eta)$ be the throughput of facility k

$$D_k(\lambda, \eta) = \sum_{i \in I} \sum_{j \in J} s_{ij}^k(\lambda, \eta), \quad k \in K. \tag{40}$$

Let us note that $D_k(\lambda, \eta)$ is expressed only in the unknowns $\{\lambda_k\}$ and $\{\eta_k\}$, because $\{\mu_j\}$ have been just calculated (as a function of $\{\lambda_k\}$ and $\{\eta_k\}$).

Like flows $\{s_{ij}^k\}$, $\{D_k\}$ are also given for not located facilities. When facility k is located, D_k represents the actual throughput of that facility, whereas it represents its "potential" throughput when k is not located.

By (38), (40) becomes

$$D_k(\lambda,\eta) = e^{-\beta\lambda_k} e^{-\beta\eta_k} \sum_{i\in I} P_i \frac{\sum_{j\in J} e^{\beta v_{ij}^k} e^{\beta\mu_j}}{\sum_{k'\in K:y_{k'}=1} \sum_{j'\in J} e^{\beta v_{ij'}^{k'}} e^{\beta\mu_{j'}} e^{-\beta\lambda_{k'}} e^{-\beta\eta_{k'}}} = e^{-\beta\lambda_k} e^{-\beta\eta_k} \rho_k, \quad k\in K \tag{41}$$

where

$$\rho_k = \sum_{i\in I} P_i \frac{\sum_{j\in J} e^{\beta v_{ij}^k} e^{\beta\mu_j}}{\sum_{k'\in K:y_{k'}=1} \sum_{j'\in J} e^{\beta v_{ij'}^{k'}} e^{\beta\mu_{j'}} e^{-\beta\lambda_{k'}} e^{-\beta\eta_{k'}}}, \quad k\in K \tag{42}$$

is the current size of facility k (actual size if k is located or potential size if k is not located).

The updating of the multipliers $\{\lambda_k\}$ and $\{\eta_k\}$ is made as follows

- if $L_k \le \rho_k \le U_k$, leave $\lambda_k = \eta_k = 0$ (then $e^{-\beta\lambda_k} = e^{-\beta\eta_k} = 1$)
- if $\rho_k > U_k$, set $e^{-\beta\lambda_k} = U_k/\rho_k$ and $e^{-\beta\eta_k} = 1$
- if $\rho_k < L_k$, set $e^{-\beta\lambda_k} = 1$ and $e^{-\beta\eta_k} = L_k/\rho_k$

The rationale for the above updating mechanism is the following one. If the current size ρ_k of facility k does satisfy the lower and upper capacity constraints, the multipliers are kept like they are. Otherwise, if ρ_k is greater than the upper capacity U_k, $e^{-\beta\lambda_k} < 1$ and D_k will be reduced. If ρ_k is smaller than the lower capacity L_k, $e^{-\beta\eta_k} > 1$ and D_k will be augmented.

Given the updated multipliers $\{\lambda_k\}$ and $\{\eta_k\}$, the multipliers $\{\mu_j\}$ are then recalculated by (39) and the iterative procedure goes on until the upper and lower capacity constraints (11) and (12) are satisfied.

With the final values of $\{\mu_j\}$, $\{\lambda_k\}$, and $\{\eta_k\}$ one can calculate the optimal flows $\{s_{ij}^k\}$ for the given transshipment location $\{y_k\}$ by (38), and the optimum of the $CTLP_d$ by (34).

4.2 Locating a transshipment facility

The second procedure of our heuristics is devoted to locate facilities with the aim of improving the given facilities location.

By substituting (26) into (34), the optimum of the $CTLP_d$ can be written as follows

$$\min_{\mu,\lambda\ge 0,\eta\le 0} \max_y 1/\beta \sum_{i\in I} P_i \ln \sum_{k\in K:y_k=1} \sum_{j\in J} e^{\beta(v_{ij}^k+\mu_j-\lambda_k-\eta_k)} + \sum_{k\in K} [\lambda_k U_k + \eta_k L_k - f_k y_k] - \sum_{j\in J} \mu_j Q_j. \tag{43}$$

Let us consider in (43) the continuous relaxation of the binary variables $\{y_k\}$ in the interval [0,1] and the derivatives

$$\begin{aligned}
&\partial \left\{ 1/\beta \sum_{i\in I} P_i \ln \sum_{k\in K:y_k=1} \sum_{j\in J} e^{\beta(v_{ij}^k+\mu_j-\lambda_k-\eta_k)} + \sum_{k\in K} [\lambda_k U_k + \eta_k L_k - f_k y_k] - \sum_{j\in J} \mu_j Q_j \right\} / \partial y_k = \\
&= 1/\beta \sum_{i\in I} P_i \frac{\sum_{j\in J} e^{\beta(v_{ij}^k+\mu_j-\lambda_k-\eta_k)}}{\sum_{k'\in K:y_{k'}=1} \sum_{j\in J} e^{\beta(v_{ij}^{k'}+\mu_j-\lambda_{k'}-\eta_{k'})}} - f_k = \\
&= \frac{1}{\beta} e^{-\beta\lambda_k} e^{-\beta\eta_k} \rho_k - f_k.
\end{aligned} \tag{44}$$

(44) represents the impact on the optimum of a continuous variation of the location variable y_k. Let us consider only those not located facilities k for which $(\frac{1}{\beta} e^{-\beta\lambda_k} e^{-\beta\eta_k} \rho_k - f_k) > 0$, because only they could improve the current optimum (43) by "increasing" their y_k in the range [0,1]. Moreover, this improvement will be maximized when $y_k = 1$. So, if one wants to locate one of those facilities by improving the current optimum as much as possible, the transshipment facility with the highest positive term $(\frac{1}{\beta} e^{-\beta\lambda_k} e^{-\beta\eta_k} \rho_k - f_k)$ will be the candidate to be located, i.e. the facility r for which

$$r = argmax_{k:y_k=0} \left[\frac{1}{\beta} e^{-\beta\lambda_k} e^{-\beta\eta_k} \rho_k - f_k \right]^+.$$

If we define the "*profit*" of facility k as being the difference between its "*revenue*" $(\frac{1}{\beta} e^{-\beta\lambda_k} e^{-\beta\eta_k} \rho_k)$ and its "*cost*" f_k, the facility with the highest positive profit will be the candidate for locating.

Because of the Kuhn-Tucker conditions, we know that if the facility potential current size ρ_k is such that $L_k \le \rho_k \le U_k$, then $\lambda_k = \eta_k = 0$ and the facility profit is given by $(\frac{1}{\beta}\rho_k - f_k)$.

Otherwise, if $\lambda_k > 0$ then the potential current size of that facility is over its upper capacity and locating that facility would be highly recommended. In such a case, constraints (11) would be saturated, i.e. $D_k(\lambda, \eta) = e^{-\beta\lambda_k} e^{-\beta\eta_k} \rho_k = U_k$, and the facility profit would become $\left[\frac{1}{\beta} U_k - f_k\right]$.
If $\eta_k < 0$ then the potential current size of that facility is below its lower capacity and its locating should not be recommended at all. In such a case, constraints (12) would be saturated, i.e. $D_k(\lambda, \eta) = e^{-\beta\lambda_k} e^{-\beta\eta_k} \rho_k = L_k$, and the facility profit would become $\left[\frac{1}{\beta} L_k - f_k\right]$.

Let us note that the above criterion for locating facilities requires the possibility to calculate the size ρ_k for not located facilities too, but this can be easily done by (42), which holds for all k's.

4.3 Closing down a transshipment facility

The third and last procedure of our heuristics is for closing down facilities.
We observe that a mechanism similar to that of Section 4.2 can be adopted for finding, within the located facilities, the candidate to be closed down. In such a case, the transshipment facility q for which

$$q = argmin_{k:y_k=1} \left[\frac{1}{\beta} e^{-\beta\lambda_k} e^{-\beta\eta_k} \rho_k - f_k\right]^- \tag{45}$$

will be the candidate to be closed down (provided that the total upper capacity of the remaining located facilities is not less than the total flow T, otherwise closing down q would make the problem solution infeasible).

We are now ready to put the three procedures together and build up the overall heuristics to solve the $CTLP_d$.

4.4 The overall heuristics

As previously assumed, a transshipment location $\{y_k\}$ is given. Using the procedure developed in Section 4.1, we can calculate the Lagrangian multipliers and derive the optimum of the $CTLP_d$ and the optimal flows $\{s_{ij}^k\}$. Then, we try to improve the given transshipment location by locating and closing down facilities. This process calls the procedure for the Lagrangian multipliers calculation of Section 4.1 as a subroutine. We reiterate until no further improvements for the optimum can be found.

More in detail, the heuristics to solve the $CTLP_d$ acts as follows

- Problem Feasibility check.
  If the total upper capacity is less than the total flow, i.e. $\sum_{k\in K} U_k < T$ or the minimum lower capacity is greater than the total flow, i.e. $min_{k\in K} L_k > T$, then STOP, the problem is infeasible.
- While the number of iterations is not greater than MAXITER (maximum number of iterations) and the overall computing time is not greater than MAXTIME (maximum computing time), apply the Core heuristics (see Subsection 4.4.1) which builds a solution by locating and closing operations.
- Keep the best solution as the optimal one.

4.4.1 Core heuristics

The core heuristics builds a solution according to the following steps

1. Locate all facilities, i.e. $y_k = 1, k \in K$.
2. Compute the Lagrangian multipliers as in Section 4.1. Calculate the optimal flows and set the best solution $BestSol$ to the optimal flows.
3. Repeat the following steps
   (a) Decide whether to close down a transshipment facility or simultaneously close down and locate two different facilities. The decision is taken according to a rule based on a randomized process and a short term search memory. This rule, called the *operation choosing rule*, is described in detail in Subsection 4.4.2.
   (b) Let q be the facility to be closed down and r the facility to be located (if any). Close down q and locate r.
   (c) Compute the Lagrangian multipliers and the optimal flows. Set the current solution $CurrSol$ to the optimal flows.
   (d) If no locating operation has been performed and the value of $CurrSol$ is not better than that of $BestSol$, then exit from the heuristics and return the value $BestSol$. Otherwise, set $BestSol$ to $CurrSol$.

4.4.2 Operation choosing rule

In our heuristics, we can decide whether either to close down a facility (we remind that we start with all facilities located) or simultaneously close down and locate two different facilities. To take the above decision the operation choosing rule uses both a dynamic stochastic process guided by the search history and a short-term memory structure. The short-term memory structure is a list FL which forbids, after locating a facility, its closing down for a fixed amount m of iterations.

The rule works as follows

– If we are at the first iteration of the overall heuristics, initialize the locating probability step $\delta_O = 0$, otherwise set $\delta_O = 2/|K|$, where $|K|$ is the number of potential transshipment locations. Empty the list FL, set its size equal to MAXFL and put $v_O = 0$.
– While the solution is feasible
  – Get a random number $v \in (0, 1]$.
  – If $v \geq v_O$, find the candidate q to be closed down as in Section 4.3 and check that $q \notin FL$. Increment v_O by δ_O and, for all facilities in FL, decrement by one the number of iterations for which they cannot be closed down. Remove from FL the facilities for which the number of iterations is 0.
  – Otherwise, set $v_O = \delta_O$, find the candidate q to be closed down and the candidate r to be located, as in Sections 4.3 and 4.2, respectively. If their swapping (i.e. closing down q and locating r) is feasible and improves the current optimum, make the swapping, add r to FL and set to MAXFL the number of iterations for which r cannot be closed.

Let us note that, in the first iteration of the overall heuristics, we only apply closing operations, because we start with all facilities already located. The locating operations will be considered in next iterations of the heuristics. When a local optimum has been reached, a sort of local search is introduced by means of the facilities swapping mechanism described above.

5 Computational results

In this section we compare the $CTLP_s$, solved by a state-of-the-art stochastic solver, and the $CTLP_d$, solved both exactly, by means of a state-of-the-art nonlinear solver, and by our heuristics.

We consider three classes of instances, which contain identical instances except for their facility lower capacity. In particular, the lower capacity of Class 1 is set equal to zero. The lower capacity of Class 2 is set to 60% of the upper capacity, whilst for instances belonging to Class 3 such percentage rises up to 85%. For each class 50 instances are generated as follows

- the number of depots $|I|$ is drawn from $U[2, 3]$;
- the number of customers $|J|$ is drawn from $U[30, 40]$;
- the number of potential transshipment locations $|K|$ is drawn from $U[10, 20]$;
- supply P_i is drawn from $U[900, 1000]$;
- demand Q_j is drawn from $U[1, \sum_{i\in I} P_i/ \mid J \mid]$. If necessary, the demand of the last customer is adjusted so that the system is balanced;
- upper capacity U_k is drawn from $U[0.5avU, 3avU]$, where $avU = \sum_{i\in I} P_i/ \mid K \mid$;
- unit deterministic shipping utility v_{ij}^k is drawn from $U[1, 10]$, $k = 1, ..., |K|/2$, and $U[1, 5]$, $k = |K|/2+1, ..., |K|$;
- fixed cost $f_k = 0.3U_k \frac{TU}{|I||J||K|}$, $k = 1, ..., |K|/2$, $f_k = 0.03U_k \frac{TU}{|I||J||K|}$, $k = |K|/2 + 1, ..., |K|$, where $TU = \sum_{i\in I} \sum_{k\in K} \sum_{j\in J} v_{ij}^k$ is the total deterministic shipping utility;
- stochastic utility $\tilde{u}^k$ is drawn from a Gumbel probability distribution with mean equal to $\overline{v}^k$, where $\overline{v}^k$ is the arithmetic mean over i's and j's of the deterministic shipping utilities $\left\{v_{ij}^k\right\}$ associated to facility k. In order to have for $\tilde{u}^k$ values which are comparable with v_{ij}^k, while ensuring a sufficient impact of the stochastic component, $\tilde{u}^k$ is drawn from the range $[1, 20]$.

The rationale for choosing the above values for v_{ij}^k and f_k is to build two sets of potential transshipment locations such that in the first set the facility throughput will be close to the facility upper capacity and in the second set the throughput will be close to the facility lower capacity. In such a way, we guarantee that at least some of the constraints (11) and (12) will be activated in the optimal solution.

Both the $CTLP_d$ and the heuristics need to know a proper value of the positive parameter β. This is obtained by calibration as follows.

Let us consider the standard Gumbel distribution $G(x) = \exp\left(-e^{-x}\right)$. If one accepts an approximation error of 0.01, then $G(x) = 1 \Longleftrightarrow x = 4.60$ and $G(x) = 0 \Longleftrightarrow x = -1.52$. Let us consider the range $[m, M]$ where the stochastic utility $\tilde{u}^k$ is drawn from ($[1, 20]$ in our case). The following equations hold

$$\beta(m - \zeta) = -1.52 \tag{46}$$
$$\beta(M - \zeta) = 4.60 \tag{47}$$

where ζ is the mode of the Gumbel distribution $G(x) = \exp\left(-e^{-\beta(x-\zeta)}\right)$.

From (46) and (47) one gets

$$\beta = \frac{6.12}{M-m}.$$

In our case, as $M - m = 20 - 1 = 19$, we get $\beta = 0.32$.

For the calibration of β in real applications, more sophisticated methods would be required [8].

The solution of the $CTLP_s$ is generated by means of the stochastic programming module provided by XPress Optimization Suite [6]. The tests are performed by generating an appropriate number of scenarii for each instance. In order to tune this number, we start with 50 scenarii and increase them with step 50. Then we solve each instance 10 times, reinitializing every time the pseudo-random generator of the stochastic components with a different seed, and compute the standard deviation and the mean of the optima over the 10 runs. The appropriate number of scenarii is then fixed to the smallest value which ensures for each instance a maximum ratio between the standard deviation and the mean which is less than 1% [11]. According to our tests, this value is fixed to 200 scenarii, which shows a maximum ratio of 0.34%.

To solve the $CTLP_d$ we use the nonlinear solver BonMIN, release 1.3 [1, 2], within a time limit of 1000 seconds. The parameters are set to their default values, which show a satisfactory behavior both in accuracy and computational effort.

The heuristics presented in Section 4.4 is implemented in Matlab 2007. After a preliminary testing phase, the parameters of the heuristics are set as follows

– MAXITER= 50
– MAXTIME= 10000 seconds
– Size of the list FL= 3.

All the tests were performed on an Intel Core i7 8602.8 GHz with 4Gb of Ram. Each process of XPress and BonMIN is limited to one core only.

Table 1 compares the optima of the $CTLP_s$ and the $CTLP_d$. The table columns have the following meaning

- Column 1: instance class
- Column 2: mean optimum of the $CTLP_s$
- Column 3: mean optimum of the $CTLP_d$
- Column 4: percentage gap between the two optima with respect to the $CTLP_s$ optimum
- Column 5: mean computational time in seconds for the $CTLP_s$
- Column 5: mean computational time in seconds for the $CTLP_d$.

For each column, the mean results of the 50 generated instances in each class are given in the first three rows, whereas the last row gives the overall mean over the three classes.

According to the figures, the performance of the $CTLP_s$ is quite good. In fact, it shows a mean gap of 1.65%, while reducing the mean computing time of 17%. One can also notice that the percentage gap slightly increases when the facilities lower capacity does increase (from Class 1 to Class 3). Moreover, the computing time of the $CTLP_s$ highly increases from Class 1 to Class 3, whereas that of the $CTLP_s$ is less affected.

The good performance of the $CTLP_d$ is also confirmed by analyzing the number of located facilities, as well as the number of those facilities which are located by both models (the $CTLP_s$ and the $CTLP_d$).

Table 2 shows these results. In particular, the table columns have the following meaning

- Column 1: instance class
- Column 2: mean number of located facilities for the $CTLP_s$
- Column 3: mean number of located facilities for the $CTLP_d$
- Column 4: percentage of common located facilities. Such percentage is calculated as the ratio given by the number of common facilities over the number of located facilities for the $CTLP_s$.

The results show that the solutions of the $CTLP_s$ and the $CTLP_d$ share in average over 90% of the located facilities.

The results of our heuristics are summarized in Table 3, where the meaning of each column is as follows

– Column 1: instance class
– Columns 2 and 3: percentage gap between the heuristics (at the end of the first iteration and the best solution) and the $CTLP_d$ solved by BonMIN (we remind that the first iteration of the procedure starts with all facilities located and only closing down operations are performed)
– Columns 4, 5, and 6: computing time to find the optimum (in seconds) of the heuristics (at the end of the first iteration and the best solution) and the $CTLP_d$
– Columns 7 and 8: total computing time (in seconds) at the end of the heuristics and the $CTLP_d$.

The results show how the gap of the first iteration and the best solution increases from Class 1 to Class 3, giving a mean gap for the first iteration solution of 3.5%. On the other hand, after applying the closing down and locating operations (column $HeurBest$), the complete heuristics is able to reduce this overall gap to 1%. These results are even more impressive by considering that they can be obtained in a very short computing time. In fact, the mean total computing time of the heuristics is almost one third of that of the $CTLP_d$.

Moreover, if we consider the time when the optimum has been found, we discover that our heuristics needs only a mean computing time of 16.5 seconds, which is one fifth of that of the $CTLP_d$, 89.6 seconds (and almost one tenth of that of the $CTLP_s$, 153.4 seconds).

We note that, the computing time of the heuristics could be further reduced by implementing an ad hoc fixed point method to compute the Lagrangian multipliers in Section 4.1. In fact, the present implementation uses the standard Matlab $fsolve$ function, which becomes inefficient when the network size increases.

CLASS	**OPTIMUM**			**TIME (s)**	
	$CTLP_s$	$CTLP_d$	**GAP %**	$CTLP_s$	$CTLP_d$
1	20596.1	20741.2	0.70	24.2	91.3
2	21275.4	21696.7	1.98	141.7	139.1
3	21450.2	21935.0	2.26	293.2	150.6
MEAN	**21107.3**	**21457.6**	**1.65**	**153.4**	**127.0**

Table 1 Comparison between the $CTLP_s$ and the $CTLP_d$: optimum and computing time

CLASS	**LOCATED FACILITIES**		
	$CTLP_s$	$CTLP_d$	**COMMON (%)**
1	9.6	9.6	95.6
2	8.3	9.3	89.4
3	8.2	8.6	88.6
MEAN	**8.7**	**9.2**	**91.2**

Table 2 Comparison between the $CTLP_s$ and the $CTLP_d$: number of located facilities

CLASS	**GAP %**		**OPT TIME (s)**			**TOT TIME (s)**	
	HEURFIRST	**HEURBEST**	**HEURFIRST**	**HEURBEST**	$CTLP_d$	**HEURBEST**	$CTLP_d$
1	1.2	0.4	0.9	10.6	34.4	38.4	91.3
2	3.6	1.3	1.2	18.4	97.8	49.1	139.1
3	5.7	1.2	1.8	20.6	136.6	56.8	150.6
MEAN	**3.5**	**1.0**	**1.3**	**16.5**	**89.6**	**48.1**	**127.0**

Table 3 Comparison between the $CTLP_d$ and the heuristics: percentage gap and computing time

6 Conclusion

In this paper we have addressed the problem of locating transshipment facilities for freight transportation to maximize the total net utility, given by the expected total shipping utility minus the total fixed cost of the facilities. The main feature of this problem is that the handling utilities at the facilities are stochastic variables. This is due to the fact that the handling operations are organized in alternative scenarii and, given their finite capacity, congestion effects make the handling utilities stochastic variables, with unknown probability distribution.

The main contributions of the paper are

- integration into a comprehensive model of the two main levels of a transshipment network, i.e. the design level and the management level, which are made in practice at different time instants
- addressing the stochasticity of the handling utilities at the transshipment facilities, leading to a stochastic location problem.

Moreover, from a theoretical perspective, the paper shows that, under mild assumptions, the unknown probability distribution of the maximum stochastic shipping utility from any origin becomes a Gumbel distribution and the expected optimal flows are multinomial Logit functions.

Finally, from a computational point of view, the $CTLP_d$ shows a mean gap with the $CTLP_s$ less than 2% and the heuristics a mean gap with the $CTLP_d$ less than 1%. Moreover, the heuristics is able to reduce the computing time up to 90%, when compared to the $CTLP_s$.

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