ENHANCING GNSS SIGNAL ACQUISITION THROUGH THE GRADIENT METHOD

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ABSTRACT

This paper describes an efficient algorithm, based on the Gradient method, for the fine estimate of the Doppler shift of Global Navigation Satellite System (GNSS) signals. The algorithm is a suitable solution for software radio implementations, as it requires a moderate computational burden. The new approach is based on a close-form expression that gives an effective way to refine the estimate of Doppler shift without using conventional Frequency Lock Loop (FLL) architectures. The algorithm has been assessed in simulation and tested with live GPS signals. Compared with the classical FLL-based architectures, a shorter transient time between the signal acquisition and the tracking phase is achieved.

***Index Terms*—** GNSS, acquisition, refinement, gradient, pull-in

1. INTRODUCTION

Modern digital GNSS receivers sample the analog signal at the output of the Intermediate Frequency (IF) filter and split the signal over different digital channels. The first step in GNSS processing is the signal acquisition: the satellites in view are detected and a first rough estimate of the Doppler shift and the code phase is performed. This operation is followed by the signal tracking. Over each channel, a Delay Lock Loop (DLL) is used to keep the synchronization between the incoming code and a locally generated replica, while a Phase Lock Loop (PLL) is employed to track the phase of the incoming carrier. The signal tracking relies on the signal correlation properties. It is fundamental to demodulate the navigation message and estimate the range between the user and the satellites. Such an estimate, performed on a set of at least four satellites, are used to compute the user's position through a triangulation procedure. The signal tracking has to perform a precise frequency estimation, as it is one of the most stringent constraints for GNSS receivers. The frequency estimation of the incoming carrier is usually performed by a two-step process, which includes a rough estimation, followed by a fine search [1][2][3]. Conventional receiver architectures generally include a Frequency Lock Loop (FLL) to refine the rough estimate performed by the signal acquisition. The FLL eases the PLL lock, reducing the transient time between the signal acquisition and the steady-state carrier/code tracking. Recently, new techniques based on digital signal processing have been developed in order to obtain higher precision and reduced computational load and improved robustness against noise and interference. For examples [4] and [5] are two of the most recent works emerging in the field.

This paper analyzes an efficient algorithm for the refinement of the initial estimate of the Doppler shift performed by the acquisition. Such an algorithm is based on the Gradient method [6] and is able to shorten the transient time between the signal acquisition and the tracking phase. Compared to the current state of the art, the method can be considered a suitable solution for software radio receivers, as it reduces the computational burden of classical FLL-based architectures.

The paper starts introducing the constraints a designer should face in real time software implementations and recalls the fundamentals of the Gradient method in sections II and III respectively. The new method is explained with the appropriated theory and validated in simulation. Section IV shows how the Gradient method is applied to GNSS signals. Section V shows the most significant results obtained processing real GPS signals and Section VI concludes the paper, giving some guidelines for navigation receiver design.

2. DIGITAL GNSS RECEIVER OVERVIEW

In a GNSS receiver, the signal at the antenna is amplified, filtered, down-converted to a lower intermediate frequency and finally sampled and converted to a digital format [7] [8]. In order to decode the navigation data transmitted by the satellites, the baseband receiver tracks the phase and the code of received signal by estimating and removing any Doppler shift [4]. Considering one digital channel, the samples at the front-end output are generally processed by the coupled loops composed by the DLL and the PLL. The DLL synchronizes the local and the incoming PRN codes, while the PLL generates an estimate of the phase and Doppler shift of the received RF carrier. Both the code and carrier tracking loops must successfully track their respective signals to allows the GNSS receiver to operate properly. The dilemma that a GNSS receiver tracking loop designer faces is the choice between good dynamic stress performance and precise measurements [9]. The wider the loop bandwidth, the more noisy is the estimate, but also the more dynamics it can accommodate. On the contrary, if the loop bandwidth is too narrow, the loop will never be able to follow dynamics of the signal without aiding.

Before tracking the incoming signal, all GPS receivers go through the acquisition phase. The signal acquisition is actually a two-dimensional search in time (code phase) and frequency [7] [10], in fact the correlation peak is detected, only when the Doppler shift on the incoming carrier is estimated. A large amount of literature proposes different techniques to perform the signal acquisition. Modern acquisition strategies are based on the Fast Fourier Transform (FFT) [1], which speeds up the acquisition process and represent an appropriated solution for software architectures. Among all, reference [12] deals with an efficient signal acquisition scheme considering new GNSS signal formats.

After the acquisition phase, the error on the frequency estimation is within the acquisition frequency bin. This directly depends on the integration period and is usually of the order of hundreds of Hz. The transient between the acquisition and the tracking has to be carefully designed, as the tracking loops have to lock the code and carrier phase starting from the coarse estimates of the signal acquisition. In many GPS receivers the carrier process is performed through two different steps. First, an FLL locks the frequency of the incoming carrier, providing a better estimate of the Doppler shift than the signal acquisition. Second, the architecture switches to a PLL to lock the phase of the incoming carrier. Once the PLL is locked, the receiver can decode the navigation data and perform carrier phase measurements on the basis of the local carrier. As mentioned, in this work we focus on the transient between the signal acquisition and tracking, proposing a new technique based on the Gradient method, which is quite different from the algorithms implemented in conventional GPS receivers.

3. GRADIENT METHOD THEORY

The Gradient method (often referred to as the method of steepest descent)[6][13] is one of the oldest and most widely known methods for maximizing (minimizing) a function of several variables $F(x_1, x_2, ..., x_n)$. In this work, the Gradient method is going to be applied for a maximization problem. The method is extremely important as it is one of the simplest with a comprehensive theory. Several other methods have been developed starting from Gradient method (see for example Newton's method, Conjugate Gradient method) [14].

In the following the mathematical treatment of the method addressed to the function we are interested in. A general optimization problem can be written as follows [15]:

$$\mathbf{x_{max}} = \arg\ \max\left\{F(\mathbf{x}) : \mathbf{x} \in \mathbf{\Omega}\right\} \tag{1}$$

where $\mathbf{x}$ is a n-dimensional vector, while $\mathbf{x_{max}}$ is a global maximum if $\mathbf{\Omega}$ is the domain of $F(\mathbf{x})$ or a local maximum if $\mathbf{\Omega}$ is a subset of the domain. Without going deep into theoretical aspects, it has to be noted that, essentially, all of the important local convergence characteristics of the method are notable when the method is applied to quadratic problems. Consider the following quadratic form [6]:

$$F(\mathbf{x}) = \frac{1}{2}\,\mathbf{x}^T\mathbf{A}\mathbf{x} - \mathbf{b}^T\mathbf{x} + c \tag{2}$$

where:$\mathbf{A}$ is a matrix, $\mathbf{b}$ is a vector and c is a constant. It is shown that if $\mathbf{A}$ is symmetric and negative-definite[1], $F(\mathbf{x})$ is maximized by the solution $\mathbf{A}\ \mathbf{x} = \mathbf{b}$. In fact, applying the gradient to Eq.(2), it is possible to obtain:

$$\nabla F(\mathbf{x}) = \mathbf{A}\ \mathbf{x} - \mathbf{b} \tag{3}$$

where:

$$\nabla F(\mathbf{x}) = \left[\frac{\partial}{\partial(x_1)}F(\mathbf{x}), \frac{\partial}{\partial(x_2)}F(\mathbf{x}), \ldots, \frac{\partial}{\partial(x_n)}F(\mathbf{x})\right] \tag{4}$$

Setting the gradient to zero $\nabla F(\mathbf{x}) = 0$ the resulting linear system to solve is: $\mathbf{A}\ \mathbf{x} = \mathbf{b}$. The solution of such a system rapresents the maximum of $F(\mathbf{x})$.

The method is often expressed in an iterative fashion:

$$\mathbf{x_{k+1}} = \mathbf{x_k} - a_k\mathbf{g_k} \tag{5}$$

where a_k is called *step length* and $\mathbf{g_k}$ is called *search direction.*

In order to find $\mathbf{x}$ that maximizes $F(\mathbf{x})$, the method starts from an arbitrary point $\mathbf{x_0}$ and converges to the maximum after several steps. At each steps, the algorithm computes a new value of $\mathbf{g_k}$ until the j-th is close enough (i.e.: difference $\mathbf{x_j} - \mathbf{x_{j-1}}$ is below a predefined treshold) to $\mathbf{x}$ that maximizes $F(\mathbf{x})$.

It is important to note that the primary differences among methods (Gradient method, Newton's method, etc.) resides in the definition of the vector $\mathbf{g_k}$. Once $\mathbf{g_k}$ is set, all the methods seek for the maximum of $F(\mathbf{x})$, following their own convergence trend.

If the Gradient method is implemented, $\mathbf{g_k}$ is as following:

$$\mathbf{g_k} = \left[\nabla F(\mathbf{x})\right]_{\mathbf{x}=\mathbf{x_k}} \tag{6}$$

The step length a_k can be chosen to maximize the following equation $F(\mathbf{x_k} - a_k\mathbf{g_k})$ in order to improve the converging time. Several techniques can be applied to define a_k. During the current analysis the Gradient method is used with constant search step for each iteration (i.e.: a_k =costant $\forall k$). A two-domensional function $F(\mathbf{x})$ and the iterative process is

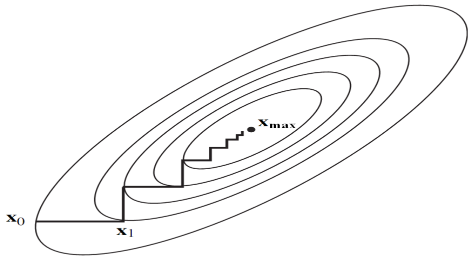

Fig. 1. Path obtained by Gradient method in a two-dimensional problem [6]

illustrated in Fig. 1 which shows contours of constant values of $F(\mathbf{x})$ and a typical sequence developed by the process. As mentioned, during the iterative process the successive direction vectors $\mathbf{g_k}$ are selected as successive gradients. Thus, the directions are not specified beforehand, but rather are determined sequentially at each step of the iteration. At step k the current gradient vector is evaluated. The result is multiplied by a_k and added to the previous point $\mathbf{x_k}$ to obtain a new point $\mathbf{x_{k+1}}$. Here, it's worth recalling that algorithms based on Newton's method and inversion of the Hessian using conjugate gradient techniques [16] are often a good alternative such a tradeoff between time consuming and convergence.

4. APPLICATIONS TO GNSS SIGNALS

A large amount of literature proposes different techniques to perform the signal acquisition. For example modern acquisition strategies are based on the Fast Fourier Transform (FFT), which speed up the acquisition phase for some receiver architectures. The signal acquisition is actually a two-dimensional search in time (code phase) $\overline{\tau}$ and frequency $\overline{f}_d$ [7] [10], in fact the correlation peak is detected, only when the Doppler shift on the incoming carrier is estimated. The Doppler shift is due to both the satellite motion and the frequency bias of the receiver clock. From a mathematical point of view, neglecting the noise for sake of simplicity, the correlation between the incoming and the local signals can be expressed as following:

$$R_{y,r}(\overline{\tau},\overline{f}_d) = \sum_{n=0}^{L-1} y_{IF}[n]c(nT_s - \overline{\tau})e^{j(2\pi(f_{IF}+\overline{f}_d)nT_s)} \quad (7)$$

where $y_{IF}[n]$ is the incoming signal to a IF (Intermediate Frequency), $c(nT_s - \overline{\tau})$ is the local replica of the code and $e^{j(2\pi(f_{IF}+\overline{f}_d)nT_s)}$ is a rotating vector so to obtain a bin of the search space for each $\overline{f}_d$ value and L is the digital integration window.

$R_{y,r}$ is also called Cross Ambiguity Function (CAF) [11]. The signal at the front end output is multiplied by the local replica of the PRN code, whose delay $\overline{\tau}$ is a variable used to evaluate the search space points. In a non-coherent system the resulting signal is split into two branches: the inphase and quadrature channels. In the inphase channel, the signal is multiplied by a local cosine, while in the quadrature branch it is multiplied by a local sine, whose frequency is the IF frequency plus a variable frequency shift, used to scan the frequency range of the search space. The use of the two branches is fundamental to recover all the signal power and obtain a result, which does not depend on the unknown phase of the incoming carrier. After the multiplication stage, the signal is integrated over a determined time window, which is generally multiple of the code period. The in-phase correlator output I and the quadrature correlator output Q components can be used to estimate the envelope $\sqrt{I^2 + Q^2}$, that is then compared to a threshold to determine the presence or the absence of the signal. Equivalently, it is also possible to evaluate the normalized $\left| R_{y,r}(\overline{\tau},\overline{f}_d)\right|^2$, that is: $S_{y,r}(\overline{\tau},\overline{f}_d) = \left|\frac{1}{L}R_{y,r}(\overline{\tau},\overline{f}_d)\right|^2$.

Assuming the local and the incoming codes aligned and supposing the local and the incoming carrier frequency synchronized, the correlation peak is detected. Fig. 2 shows the search space for GPS PRN 16 with C/N_0 of 45dBHz. In this case the integration window was set equal to 1 msec and the overall frequency range was 12KHz wide, centered on the IF. With an integration window of 1msec, the corresponding step in the frequency search was 500Hz.

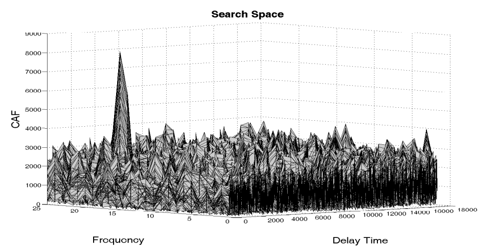

Fig. 2. CAF envelope in the search space (real data PRN 16)

It is evident that a first rough alignment between the incoming and the local signals has been found as the correlation peak rises above the noise floor. A decision logic determines if the correlation peak is detected and gives the first rough estimation of the code phase $\widehat{\tau}^{(A)}$ and the Doppler shift $\widehat{f}_d^{(A)}$ as output. In conventional receivers, a pull-in phase is often required to smooth such estimates and make the tracking subsystem able to lock the incoming signals. In this work the Gradient method is analyzed as an alternative technique to be used after the acquisition phase to refine both $\overline{\tau}$ and $\overline{f}_d$. $F(\mathbf{x})$ represents the CAF envelope, while $\mathbf{x}$ contains both the code phase $\overline{\tau}$ and the Doppler shift $\overline{f}_d$ thus, the evolution of the gradient on the CAF envelope would require the computation of the derivative on the the frequency and time domain.

However, in this work, the Gradient method is going to be applied to refine the estimate of the Doppler shift as it is the most significative part of the work. Therefore, a mono-

[1] If $\mathbf{A}$ is symmetric and negative-definite $F(\mathbf{x})$ is a convex paraboloid

dimensional problem will be faced, and x has only one component, which is the estimated frequency and $F(\mathbf{x})$ is the correlation function with respect to the frequency. x_0 represents the first frequency estimated $\widehat{f}_d^{(A)}$ from the signal acquisition, while the final value x_j is the refined frequency $\widehat{f}_d^{(R)}$ which will be directly used by the carrier tracking loop. At each step of the algorithm a new value of the Gradient is computed and evaluated for $\overline{f}_d = \overline{f}_{d_k}$. In case of a one-dimensional problem, the gradient corresponds to the CAF envelope derivative with respect to the frequency, which is:

$$\begin{aligned}\Gamma_{f_d}(\overline{\tau}, \overline{f}_d) \equiv \frac{\partial}{\partial(f_d)} S_{y,r}(\overline{\tau}, \overline{f}_d) = \\ 2\frac{1}{L}\sum_{n=0}^{L-1} y_c[n,\overline{\tau}]\cos(\Psi)\frac{1}{L}\sum_{n=0}^{L-1} y_c[n,\overline{\tau}]\sin(\Psi)(-\Psi) + \\ 2\frac{1}{L}\sum_{n=0}^{L-1} y_c[n,\overline{\tau}]\sin(\Psi)\frac{1}{L}\sum_{n=0}^{L-1} y_c[n,\overline{\tau}]\cos(\Psi)(\Psi)\end{aligned} \quad (8)$$

where $y_c[n,\overline{\tau}] \equiv y_{IF}[n]c(nT_s - \overline{\tau})$ and $\Psi[n,\overline{f}_d] \equiv 2\pi(f_{IF} + \overline{f}_d)nT_s$. Eq.(8) is the key-element of the iterative algorithm. Eq.(8) can be computated in a software receiver, with a moderate computational burden. In fact the term $\sum_{n=0}^{L-1} y_{IF}[n]c(nT_s - \overline{\tau})$ is simply the correlation between the incoming signals and the local code. Such term has to be evaluated either in the signal acquisition and in the signal tracking, as all the GNSS signal processing is based on correlation [11]. However, the advantage of the Gradient method with respect to a classic FFL is clear. At each step of the algorithm only one correlation has to be computed. On the contrary using the FLL, six correlations have to be evaluated at each integration time. Note that the use of only one correlation is suitable in software implementation, since the correlations between the incoming signal and the local codes are one of the most expensive operation of GNSS software receivers. Another advantage of Eq.(8) is that both the sine and the cosine terms already exist in any GNSS receivers, as they are the local carriers of the carrier tracking loop. Eq.(8) has been obtained in close form and gives an effective way to refine the Doppler shift estimate. When the difference between the last CAF envelope derivative $\Gamma_{f_d}(\overline{\tau}, \overline{f}_{d_k})$ and the previous value $\Gamma_{f_d}(\overline{\tau}, \overline{f}_{d_{k-1}})$ is below a predefined threshold, the algorithm stops the refinement and provides the last frequency estimate to the carrier tracking loop.

5. POST PROCESSING RESULTS

5.1. Simulation Environment

As first step of the analysis, the Gradient method has been assessed in simulation. A GPS like signal was generated with the FUll Educational Library of Signals for Navigation (N-FUEL) [17], which is a MATLAB based signal/disturbances generator developed by NavSAS group [18]. Such a generator is an extremely flexible software module, able to simulate the received signal samples for a wide range of system and receiver configurations. In this case, the signal included the PRN 3, modulated at Intermediate Frequency (IF) of 4.1304MHz, with a constant Doppler shift of 200Hz. The signal was generated considering a narrowband receiver front end (i.e.: 3.78MHz) and a sampling frequency of 16.3676MHz.

Fig. 3 shows a zoomed view of the CAF envelope with respect to the frequency and the corresponding derivative. The figure highlights the expected relationship between the convex function $F(\mathbf{x})$ and its derivative. The maximum of the CAF envelope is found in correspondence of the null of the derivative.

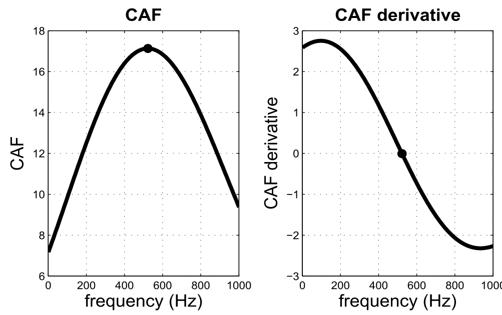

Fig. 3. CAF and his derivative envelope near the peak

The iterative process computes Eq.(5) until the following condition is verified:

$$\Gamma_{f_d}(\overline{\tau}, \overline{f}_{d_k}) - \Gamma_{f_d}(\overline{\tau}, \overline{f}_{d_{k-1}}) < \epsilon \quad (9)$$

where ϵ is the *decision threshold*. It represents the threshold within which the necessary condition $\Gamma_{f_d}(\overline{\tau}, \overline{f}_{d_k}) = 0$ is fulfilled. The step length a and the decision threshold ϵ play a key role in the design of the algorithm. If a is too small the method converges slowly, while if it is large the Gradient method might not converge. Likewise, if the decision threshold ϵ is large, the iterative process might stop far from the maximum without refining the initial value. A small value of ϵ might drastically increse the converging time.

Fig. 4 shows the Doppler shift and the derivative with respect to the number of steps required to achieve the condition of Eq.(9). The Doppler shift was about 530Hz (dot line) and the starting frequency values were $f_0 = 600$Hz. After few steps, the convergence is achieved and the frequency is very close to the correct Doppler shift value. The derivative value is near to the zero value. The same behaviour was obtained for a starting frequency values below $f_0 = 530$Hz.

5.2. Test with real data

The Gradient method has been tested with real GPS signals. Raw samples at the output of a commercial front end were

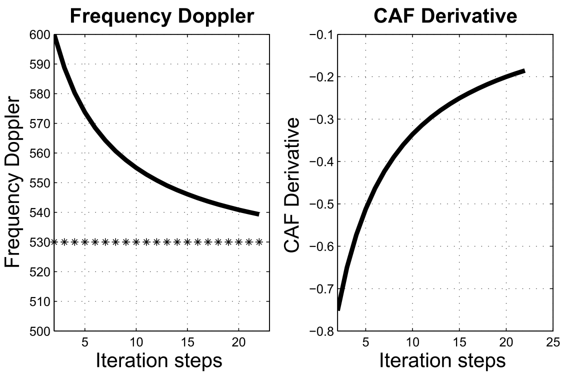

Fig. 4. Doppler shift and derivative behaviours

stored and processed off-line with a software receiver. The focus was on PRN 16 and PRN 24, that had a C/N_0 of 45dBHz and 47dBHz at the moment of the data collection.

Fig. 5 shows the estimated Doppler shift and the derivative with respect to the number of iterations of the Gradient method for PRN 16. It can be noted that the estimate of the Doppler shift converges to a well defined value. The initial swing depends on the starting point of the Gradient method $\widehat{f}_d^{(A)}$. In fact, we experienced that if the first frequency estimate was on the edge of the main lobe of the CAF envelope but far from the peak, the following estimate likely was on the opposite edge. After some iterations the estimate got close to the maximum of CAF envelope and converged to the target value. This trend can be smoothed tuning the step length. The problem of a smooth convergence has been faced, but it is not discussed in this paper. The focus of presented work is more on the feasibility and implementation of the Gradient method in a GPS receiver, rather than its optimization.

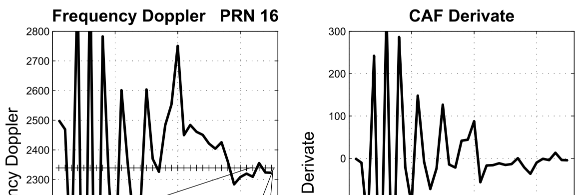

Fig. 5. PRN16 Estimated Doppler shift respect to the iteration steps

A possible implementation of the algorithm might foresee the computation of the gradient on the same segment of received data. Nonetheless, it must be observed that the real CAF maximum is noisy, as shown in Fig. 6. Computing the gradient on the same data set might induce the method to converge to a local maximum, resulting in a wrong estimate of the Doppler shift. A more appropriate implementation considers a new chunk of data at each k-th iteration. This allows to have a new realization of the noise process every time the method has to compute a new derivative. The result is that, on average, the noise contribution is reduced. This strategy has another advantage, which is particularly appreciated in software implementations. With a new chunk of data at each iteration, there is no need for any additional buffer for data storage, although a new correlations between the incoming signal $y_{IF}[n]$ and the local codes $c(nT_s - \overline{\tau})$ is required. The computation of a new correlation would have been required anyhow at each integration of the FLL. Therefore, the proposed implementation does not increase the computational burden with respect to current receiver architectures.

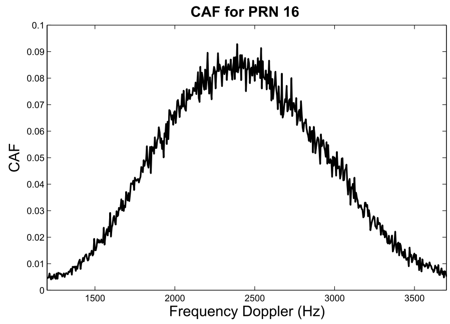

Fig. 6. CAF maximum in frequency domain for a real dataset

Note that with real GPS signals the Gradient method requires a slightly higher number of steps with respect to results obtained in simulation. This makes sense, because in the analysis reported in the previous section, no noise was added to the GPS signals.

Fig. 7 shows the estimated Doppler shift during the transient between the signal acquisition and the tracking phase, using either a classical FLL-based architecture (grey line) and the Gradient method (black line). Fig. 7 refers to 750 ms of tracking and considers the frequency estimate achieved in acquisition $\widehat{f}_d^{(A)}$ as reference (i.e.: 0Hz on the y-axis).

In this test, the FLL was a second order loop with a bandwidth of 15Hz. It took approximately 400 ms to refine the initial estimate of the Doppler shift and converge to -160Hz. After 400 ms the FLL switches to a PLL, that is able to phase lock (and thus frequency lock) the incoming carrier and keep the synchronization over time. One can note that the Gradient method is able to refine the frequency estimate as well. The Gradient method converges to -160Hz in a shorter time. After only 35 ms from the end of the signal acquisition, the PLL can start and synchronize the phase of the local and incoming carriers. In this case the Gradient method was set with $a_k = 1.5$ and $\epsilon = 1.8$. The effectiveness of the Gradient method was been tested for estimating Doppler shift of PRN 24. In this case the FLL took approximately 560 ms to refine the initial estimate of the Doppler shift and converge to -200Hz before switching the PLL. Also in this

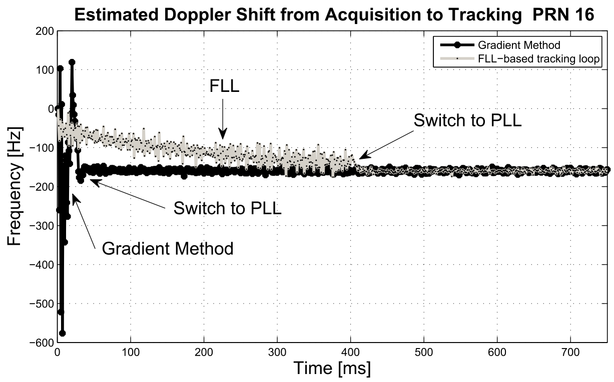

Fig. 7. PRN16 Estimated Doppler shift respect to the time

case, the Gradient method converges to -200Hz in a shorter time. After only 40 ms from the end of the signal acquisition, the PLL starts and synchronizes the phase of the local and incoming carriers. In this case the Gradient method was set with $a_k = 1.5$ and $\epsilon = 1.8$.

6. CONCLUSION

The paper introduces a novel algorithm, based on the Gradient method, for the refinement of the Doppler shift estimate of GNSS signals. The algorithm is thought for software radio receivers, as it requires a moderate computational burden. A comparison between the novel strategy and a classical FLL-based architectures has been assessed. The novel strategy eases the lock of the carrier tracking loop and a shorter transient time between the signal acquisition and the tracking phase is achieved. The theoretical analysis has been validated using real GPS signals. For both GPS PRN 16 and PRN 24, the novel algorithm is able to estimate the Doppler shift within few Hz of error. In both cases, the transient time between the signal acquisition and tracking is reduced of approximately one order of magnitude.

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