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Some Relations Among Entropy Measures

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Abstract

Article body

Some Relations Among Entropy Measures

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Abstract: Several entropies are generalizing the Shannon entropy and have it as their limit as entropic indices approach specific values. Here we discuss some relations existing among Rényi, Tsallis and Kaniadakis entropies and show how the Shannon entropy becomes the limit of Kaniadakis entropy.

Keywords: Entropy, Generalized Entropies.

In information theory, measures of information can be obtained from the probability distribution of some events contained in a sample set of possible events. These measures are the entropies. In 1948 [1], Claude Shannon defined the entropy H of a discrete random variable X as $H(X) = \Sigma_i\, p(x_i) I(x_i) = -\Sigma_i\, p(x_i) \log_b p(x_i)$. In this expression, the probability of i-event is p_i and *b* is the base of the used logarithm. Common values of the base are 2, Euler's number e, and 10.

Besides Shannon entropy, several other entropies are used in information theory; here we will discuss a few of them, stressing how they are linked. These generalized entropies are Rényi, Tsallis and Kaniadakis (also known as κ-entropy) entropies [2-4]. In the following formulas we can see how these entropies are defined, with a corresponding choice of measurement units equal to 1:

$$(1)\quad \text{Shannon}: \; S = -\sum_i p_i \ln p_i \qquad (2)\quad \text{Rényi}: \; R = R_q = \frac{1}{1-q}\ln\sum_i p_i^q$$

$$(3)\quad \text{Tsallis}: \; T = T_q = \frac{1}{q-1}\left(1-\sum_i p_i^q\right)$$

$$(4)\quad \text{Kaniadakis}\ (\kappa\text{-entropy}): \; K_\kappa = -\sum_i \frac{p_i^{1+\kappa} - p_i^{1-\kappa}}{2\kappa}$$

In (2)-(4) we have entropic indices q and κ, and:

$$\lim_{q\to 1} R = S; \quad \lim_{q\to 1} T = S; \quad \lim_{\kappa\to 0} K = S$$

Let us consider the joint entropy H(A,B) of two independent subsystems A,B. We have:

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$$(5)\quad S(A,B) = S(A) + S(B) \qquad (6)\quad R(A,B) = R(A) + R(B)$$

$$(7)\quad T(A,B) = T(A) + T(B) + (1-q)T(A)T(B)$$

$$(8)\quad K(A,B) = K(A)\Im(B) + K(B)\Im(A) \quad \text{with} \quad \Im = \sum_i \frac{p_i^{1+\kappa} + p_i^{1-\kappa}}{2}$$

Note that for the generalized additivity of κ-entropy, we need another function containing probabilities (see [5] and references therein). Let us find relations between R and T entropies. From (3), we have:

We aim to suit all browsers, but recommend Firefox particularly:

$$(1-q)T = \sum_i p_i^q - 1 \rightarrow \sum_i p_i^q = 1 + (1-q)T$$

$$(9)\qquad R = \frac{1}{1-q}\ln\sum_i p_i^q = \frac{\ln[1+(1-q)T]}{1-q}$$

Moreover:

$$(1-q)R = \ln[1+(1-q)T] \rightarrow \quad \exp[(1-q)R] = 1 + (1-q)T$$

$$(10)\qquad T = \frac{\exp[(1-q)R]-1}{(1-q)}$$

We can find these relations in [3]. Let us consider, instead of Tsallis T, κ-entropy K. We have that:

$$(11)\quad K = \frac{T_{1+\kappa} + T_{1-\kappa}}{2},$$

$$\text{where}: \quad T(q = 1+\kappa) = -\frac{1}{\kappa}\sum_i p_i^{1+\kappa} + \frac{1}{\kappa}; \qquad T(q = 1-\kappa) = \frac{1}{\kappa}\sum_i p_i^{1-\kappa} - \frac{1}{\kappa}$$

$$K = \frac{1}{2}\left\{-\frac{1}{\kappa}\sum_i p_i^{1+\kappa} + \frac{1}{\kappa} + \frac{1}{\kappa}\sum_i p_i^{1-\kappa} - \frac{1}{\kappa}\right\} = -\frac{1}{2\kappa}\left\{\sum_i p_i^{1+\kappa} - \sum_i p_i^{1-\kappa}\right\}$$

Moreover:

$$(12)\quad \Im = \frac{\kappa}{2}\left(-T_{1+\kappa} + T_{1-\kappa} + \frac{2}{\kappa}\right)$$

$$\Im = \frac{\kappa}{2}\left\{\frac{1}{\kappa}\sum_i p_i^{1+\kappa} - \frac{1}{\kappa} + \frac{1}{\kappa}\sum_i p_i^{1-\kappa} - \frac{1}{\kappa} + \frac{2}{\kappa}\right\} = \sum_i \frac{p_i^{1+\kappa} + p_i^{1-\kappa}}{2}$$

Since Tsallis entropy T is related to Rényi entropy, we can easily find the relation of κ- to Rényi entropy too.

$$(13)\quad K = \frac{T_{1+\kappa} + T_{1-\kappa}}{2}$$

$$= \frac{1}{2}\left\{\frac{\exp[(1-1-\kappa)R_{1+\kappa}]-1}{1-1-\kappa} + \frac{\exp[(1-1+\kappa)R_{1-\kappa}]-1}{1-1+\kappa}\right\}$$

$$= \frac{\exp(\kappa R_{1-\kappa}) - 1 - \exp(-\kappa R_{1+\kappa}) + 1}{2\kappa} = -\frac{\exp(-\kappa R_{1+\kappa}) - \exp(\kappa R_{1-\kappa})}{2\kappa}$$

$$(14)\quad \Im = \frac{\kappa}{2}\left(-T_{1+\kappa} + T_{1-\kappa} + \frac{2}{\kappa}\right)$$

$$= \frac{\kappa}{2}\left\{-\frac{\exp[(1-1-\kappa)R_{1+\kappa}]-1}{1-1-\kappa} + \frac{\exp[(1-1+\kappa)R_{1-\kappa}]-1}{1-1+\kappa} + \frac{2}{\kappa}\right\}$$

$$= \frac{1}{2}\{\exp(\kappa R_{1-\kappa}) - 1 + \exp(-\kappa R_{1+\kappa}) - 1 + 2\} = \frac{\exp(\kappa R_{1-\kappa}) + \exp(-\kappa R_{1+\kappa})}{2}$$

Of course, we can find (13) and (14) directly from the expression of Rényi measure:

$$R_q = \frac{1}{1-q}\ln\sum_i p_i^q \rightarrow (1-q)R = \ln\sum_i p_i^q \rightarrow \sum_i p_i^q = \exp[(1-q)R_q]$$

$$K = -\frac{1}{2\kappa}\left\{\sum_i p_i^{1+\kappa} - \sum_i p_i^{1-\kappa}\right\} = -\frac{1}{2\kappa}\left\{\exp(-\kappa R_{1+\kappa}) - \exp(\kappa R_{1-\kappa})\right\}$$

Let us note that, when κ→0, R becomes Shannon entropy. In this manner we can easily see that:

$$(15)\quad \lim_{\kappa\to 0} K = \lim_{\kappa\to 0}\frac{1}{2\kappa}\{\exp(\kappa S) - \exp(-\kappa S)\} = \lim_{\kappa\to 0} S\frac{\sinh(\kappa S)}{\kappa S} = S$$

$$(16)\quad \lim_{\kappa\to 0} \Im = \lim_{\kappa\to 0}\frac{1}{2}\{\exp(\kappa S) + \exp(-\kappa S)\} = \lim_{\kappa\to 0}\cosh(\kappa S) = 1$$

Let us consider also:

$$(17)\quad K^* = \frac{\sinh(\kappa S)}{\kappa}$$

$$(18)\quad \Im^* = \cosh(\kappa S)$$

These two equations are telling that Kaniadakis functions must be even functions of index κ.

Let us conclude with an example. We can consider the histogram h of the grey tone pixels of an image, for using it as an experimental probability p distribution. From it, we can calculate κ-entropy of an image and the function required by the generalized additivity (8), for entropic index κ ranging from 0 to 1. Comparing them to (17) and (18), we obtain plots given in the Figures 1 and 2. It is interesting to note that κ-entropy, for low values of its entropic index, is a hyperbolic sine. The function, which appears in the generalized additivity, behaves as an hyperbolic cosine.

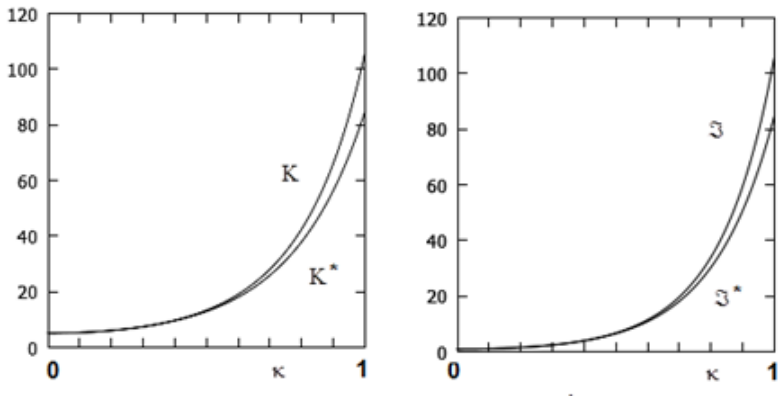

Figure 1: Comparison of κ-entropy functions to (19) and (20).

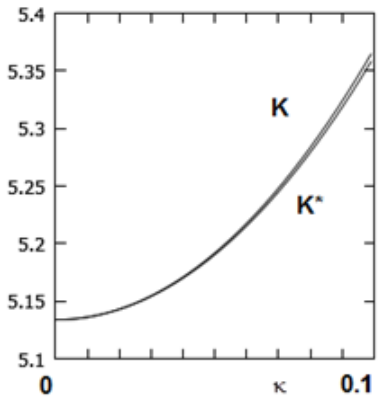

Figure 2: Entropy K and K* for low values of the entropic index.

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