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PITCH FUNCTION COMPARISON METHODOLOGY FOR SUPPORTING A SMART 3D SCANNER SELECTION

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Abstract

Working with 3D scanner devices one of the most critical problem normally is the quality of the points cloud that device is able to provide. This problem normally is composed by two aspects. The first one is surely the strategy that is implemented in order to acquire the object shape, that could be implemented with a selective sampling strategy, especially working with Free-Form surfaces, in order to provide high points density only on those regions that show high morphological complexity. The second aspect is the selection of the most "efficacy" 3D Scanner device in order to fulfil the specific application needs, that normally are correlated with the specific scenario in which the costumer/user works (resolution, accuracy, …). For what concerns this last point, actually the presence of many different acquisition technologies and solutions on the market, is creating a big confusion on the users, that sometimes risk to identify the wrong solution instead of finding one of the most efficient. So in order to support the potential user in this selection this paper propose a solution that integrating the morphological analysis of the object acquired, the costumer needs (resolution, accuracy,..) and the 3D scanner performances could help the user to identify the best solution.

Keywords: Reverse Engineering, Sampling Strategy, Scanner Uncertainty, Free-Form

1.0 Introduction

Reverse Engineering and 3D scanners are actually finding a wide number of applications. More than the traditional mechanical/manufacturing context the medical field is becoming one of the most involved sector working with 3D scanners. While the more traditional application context has developed a significant expertise with these tools, the others, as medicine, are approaching just in these years these tools and for these reason they have not significant expertise, from a technical point of view [1-3]. For this reason many experimentation have been developed in order to understand which solution could be better. Starting from the use of Computer Tomography (CT) that are employed in the facial trauma diagnosis [4], or the use of ultrasound systems for the volume estimation of human kidneys in vivo [5]. All these studies try to compare different solutions/strategies in order to understand which could the best one for their application. Other application of laser scanners are visible also in the agricultural context [6] where it is necessary to compare different possible solutions for measurement drift over time, the effect of material and colour on measurement accuracy, and the ability to map different surface patterns. While these studies have developed very vertical analysis for specific applications other studies

[7] have tried to show the results obtained with the evaluation of 3D (three dimensional) scanners to obtain 3D models or objects which can be used to build virtual reality systems. But while all these studies propose experimental validation of different possible solution for vertical or more horizontal applications it is necessary to find a more structured methodology, that working with quantitative parameters could be able to support the user in the selection of the best 3D scanner device for its application working on the morphology of the acquired object and on the 3D scanner performances.

2.0 **The "Optimal Pitch Function" concept**

While dealing with the approach considered in previous works [8,9], the Selective Sampling is a Point Clouds optimization strategy, based on considerations about the local morphological complexity of Surfaces, which the optimal sampled points density is supposed to depend on. In other words, the Selective Sampling Approach starts from the assumption that more complex surfaces, from the point of view of local morphology, always need more points, in order to be sampled with enough precision. On the other hand, simpler surfaces like cone-like, plane-like or cylinder-like ones need few points in order to be correctly sampled. In particular, the local morphological complexity of Surfaces has been once measured by the so called Discrete Gaussian Curvature parameter, which formulation for triangulated surfaces had been derived thanks to the exploitation of the Gauss-Bonnet Theorem. Thanks to the adoption of this Reference Parameter, a discrete value for local complexity could be measured in correspondence of every triangulated neighbourhood (consisting of those triangles sharing the same central node) the approximating surface is made up of. The whole set of Curvature values, associated to every point of the cloud (i.e. to every triangulated neighbourhood) has been called Curvature Map, showing the global behaviour of the approximating triangulation, for what concerns morphological complexity. The originally scanned object has a real surface, which is the result of both designing choices and manufacturing uncertainty. Furthermore, the Scanning Device, which the acquisition is carried out by, is a real measuring instrument too, so it is affected by its own uncertainty. The presence of non-Free-Form features on sampled surfaces causes the gap between originally scanned Surfaces and final rebuilt Models. All those causes for Surface Morphological local Complexity which survived the Pre-Processing points selection, even if they do not submit to the classical "Free-Form" definition (concerning smoothness and regularity of the surface), have been gathered under the name of "Disturbance". Thanks to a Geometrical and Statistical model for the contribution of the "Disturbance" to the local morphological complexity, it has been considered in order to translate the Curvature Map into a more practical instrument for Point Clouds optimization: the Pitch Map. The Pitch Map has been directly derived from the Curvature Map, considering the geometrical dependence of the Curvature Parameter from the assumed Scanning Pitch value, within the "Disturbance" model.

Since the influence of such Statistical characteristics of the Discrete Curvatures sampled population were also considered, a "Pitch Function" could be modelled, suggesting a precise "Optimal Pitch" value, based on:

- the local value assumed by the Discrete Gaussian Curvature (geometrical dependence from the Reference Parameter);
- the standard deviation estimated value, referred to the sampled points population the Cloud is made up of (dependence on the statistical characteristics of the whole Curvature population).

Finally, the obtained Pitch Map, based on the Pitch Function evaluation for every node of the

approximating Triangulated Surface, has been used in many cases in order to improve the points distribution of some Preliminary Point Clouds. The obtained results confirmed that using a Pitch Function, in order to perform Point Clouds improving, allows getting an adaptive sampling strategy, characterized by variable points density which goes with local surface morphological complexity.

The just mentioned Pitch Function can represent a sort of identity-card of the preliminary point cloud. In particular, when a Curvature Map is calculated, the "Optimal Pitch Function" will suggest the new local resolution (i.e. Pitch value) with respect to the detected morphological complexity.

It has to be considered that the Function itself depends on the preliminary Scanning Pitch, which is often supposed to be constant and arbitrary. In particular, the employed Scanning Device for the preliminary scanning session has been supposed to be unknown, so the Pitch value for the preliminary Point Cloud is the only parameter characterizing the previous, unknown measuring machine.

In fact, the Pitch Function has been derived from a geometrical and a statistical model of the Curvature Distribution, reflecting the Curvature Map characteristics, which had been already calculated. On the other hand, the Curvature Map itself depends on the Preliminary Scanning Pitch.

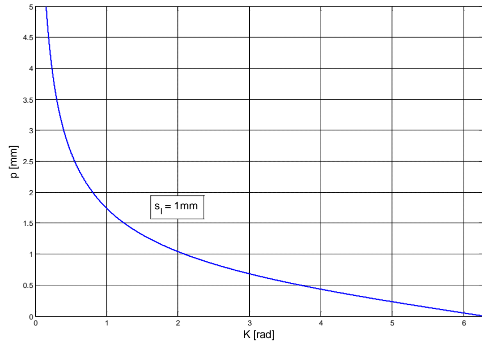
In other words, when dealing with sampled Surfaces, the new proposed Scanning Pitch is derived from the geometrical evaluation of the local Discrete Curvature (depending on the preliminary Scanning Pitch). Furthermore, the Optimal Pitch Function is calibrated through a single value of Standard Deviation, which has to be estimated from the original sampled Curvature population.

The Optimal Pitch for Sampled Surfaces, can be represented on a Cartesian plane as a single line, i.e. a by-univocal function of Discrete Curvature *K*. (Figure), for instance:

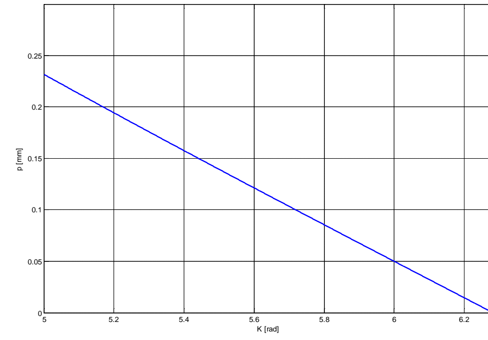
$$p = I\sqrt{\frac{1-\cos\left(\frac{2\pi-K}{4}\right)}{\cos\left(\frac{2\pi-K}{4}\right)}} = s_I(s_K)\sqrt{\frac{1-\sin\left(\frac{K}{4}\right)}{\sin\left(\frac{K}{4}\right)}} = p(K, s_K),\ \text{K>0.} \tag{1}$$

Where K indicates the local value of the Discrete Gaussian Curvature, and the Deviation s_I is referred to the position of the Points of the Cloud, on the z axis, which had been once modelled through the "Disturbance" Model [8]. In turn, it has also been proved that the value of the calibrating parameter, i.e. of the Deviation s_I, for Sampled Surfaces could be easily linked to the corresponding estimated Curvature deviation s_K. For this reason, the Optimal Pitch Function for sampled Surfaces is considered to be calibrated directly by a single value of Curvature Deviation, which is that estimated from the previously examined Point Cloud.

It has been defined in previous works [8,9] (Fig.1) as a continuous graph with vertical tendency when the local Discrete Curvature approaches small values. Furthermore, the Pitch existence interval is upper-bounded in correspondence of the maximum measurable Curvature $K = 2\pi$ rad. When the Curvature tends to such a limit, the corresponding theoretical Pitch value will be equal to zero.



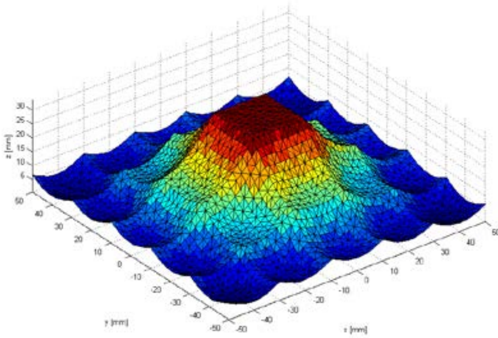
(a)



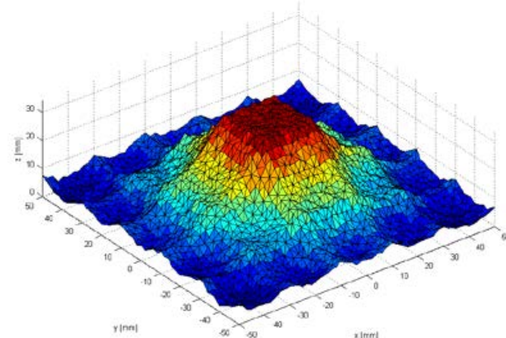
(b)

Figure 1: Optimal Pitch Function for a single Surface; a) Pitch Function for positive Curvature values; b) particular of the Curvature domain upper limit

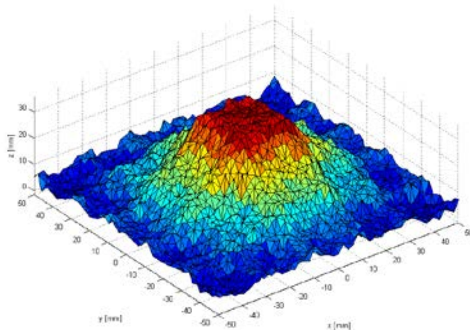
A benchmark Surface, which Equation is known, will be used to show the dependence of the Pitch Function on the estimated Standard Deviation s_K (Fig.2). In particular, the proposed virtual surface has been designed so that it is affected by a statistical noise which magnitude can be arbitrarily varied. In other words, it simulates the presence of what in some other works has been defined "Disturbance".



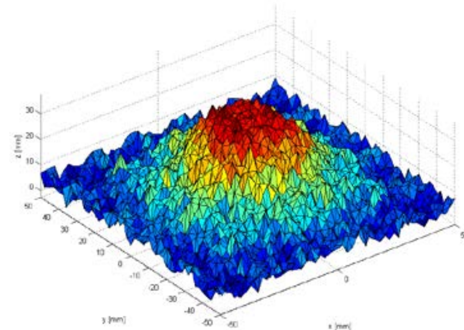
(a) $s_K \approx 0{,}05$ rad; $s_I = 0{,}1$ mm



(b) $s_K \approx 0{,}29$ rad; $s_I = 0{,}5$ mm



(c) $s_K \approx 0{,}7$ rad; $s_I = 1{,}0$ mm



(d) $s_K \approx 1{,}4$ rad; $s_I = 2{,}0$ mm

Figure 2: Ideal benchmark geometry with noise

All the just proposed surfaces share the same ideal shape. On the other hand, the noise amount makes the difference among the examples. In particular, four different values for the Curvature Standard Deviation have been calculated.

As a consequence, four different "Optimal Pitch" Functions (Fig.3) will be derived from the previous definition: one for every proposed Deviation value.

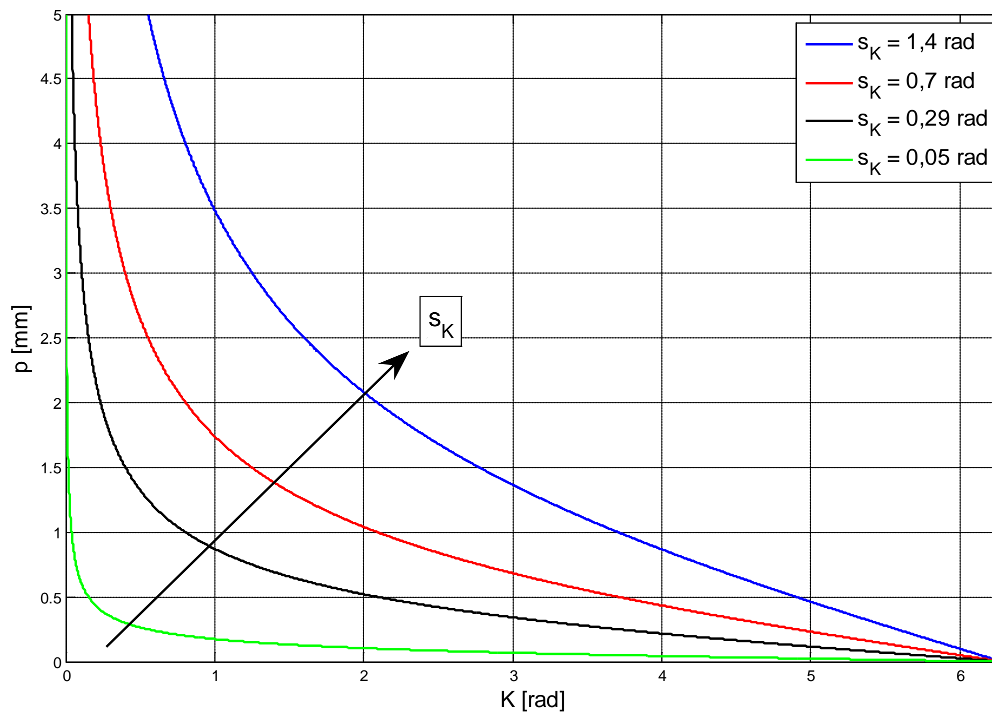


Figure 3: Graphical correlation between the acquisition pitch p an the curvature K varying the curvature standard deviation Sk

As it is shown in Figure, the Curvature Deviation amount influences the slope of the Pitch Function. In other words, since the Function suggests the proper Scanning Resolution in correspondence of a precise Curvature (i.e. morphological complexity) value, the estimated value of the Standard Deviation, related to the sampled Curvature Population, influences the distribution of the Pitch values, with respect to the Curvature amount. When the Deviation s_K comes to high values, the just mentioned Pitch Function will be characterized by regularity and it will tend to a line-like graph with negative slope.

On the other hand, if the Curvature Standard Deviation s_K becomes small and it tends to zero, the resulting Pitch Function will suggest to use very low Pitch values with almost all the detected Curvature amounts, from the smaller to the upper values: in fact, the Optimal Pitch Function will seem to be behind the Cartesian axes.

Physically speaking, in the first case the considered preliminary Point Cloud is affected by a relevant noise (or "Disturbance") which is assumed to "disturb" the future mathematical model correct reconstruction. If such a noising component of the morphological complexity is detected through the evaluation of the Standard Deviation, the resulting Pitch Function will suggest adopting relatively higher Pitch values, in correspondence of each calculated local Curvature amount: in fact, larger Pitches could help avoiding noise propagation within the

sampled point cloud.

On the other hand, if the estimated Deviation is very small, it will prove that the previously introduced noise or "Disturbance" is quite absent, and perhaps the very ideal Surface shape is dealt with. In such conditions, very large pitches should be adopted only if the calculated Curvature became very small. In fact, only plane-like, cylinder-like or cone-like geometries surely authorize the employment of scattered scanning grid, because they are characterized by negligible local morphological complexity.

Even if the originally derived Pitch Function is continuous, some limitations have to be kept for its utilization (Fig.4):

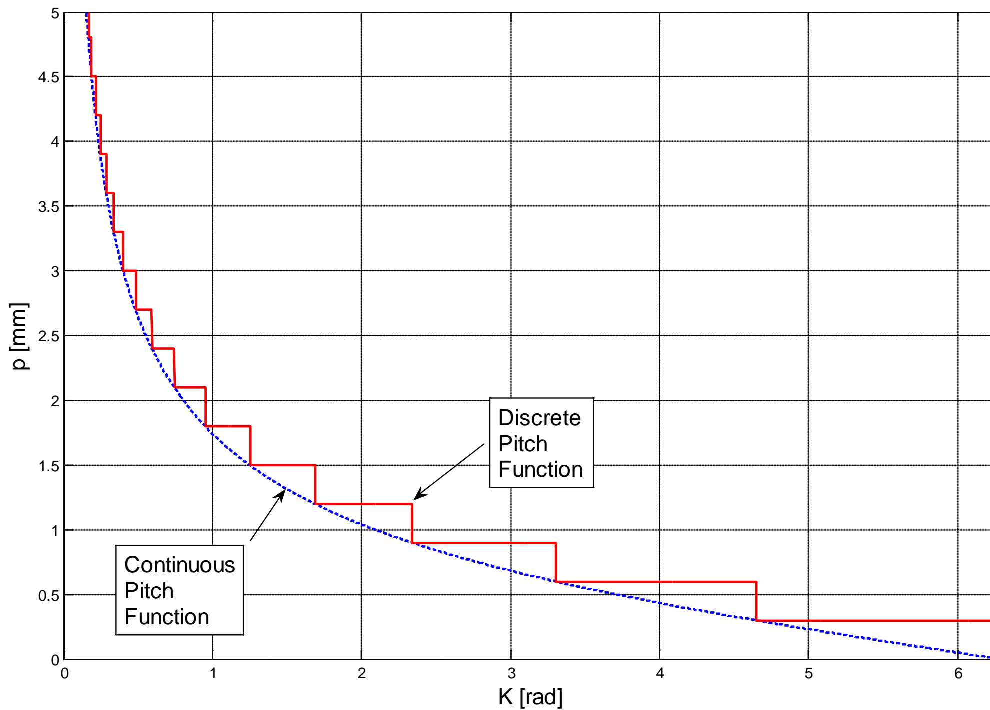


Figure4 : Optimal Pitch Function with arbitrarily subdivision (continuous to discrete). The increments are usually chosen basing on a preliminary evaluation of the available Scanners capabilities

- the "Surface Pitch Function" cannot be represented as a continuous graph. In fact, the optimization of the preliminary scanning grid works with discrete Pitch values. The discretization is often arbitrary, even if best results could be obtained employing very small Pitch variation.

  The choice of this "incremental resolution" for Pitch variation, in order to build the discrete Pitch Function, is often arbitrary, but it should be based on the performance of the Scanner which had been exploited for the preliminary scanning session.

- When the local Discrete Curvature amount tends to 2π , the corresponding Discrete Pitch value will be equal to zero no more. On the contrary, the smallest available Pitch value will correspond to the maximum possible Curvature value.

- The "Surface Pitch Function" will never be extended to the whole Curvature domain, from all the

negative values interval to the 2π upper limit. On the contrary, every examined surface is characterized by maximum and minimum Curvature values within the previous domain (Fig.5). In fact, the Pitch graph for sampled surface is often limited within a relatively small Curvature interval.

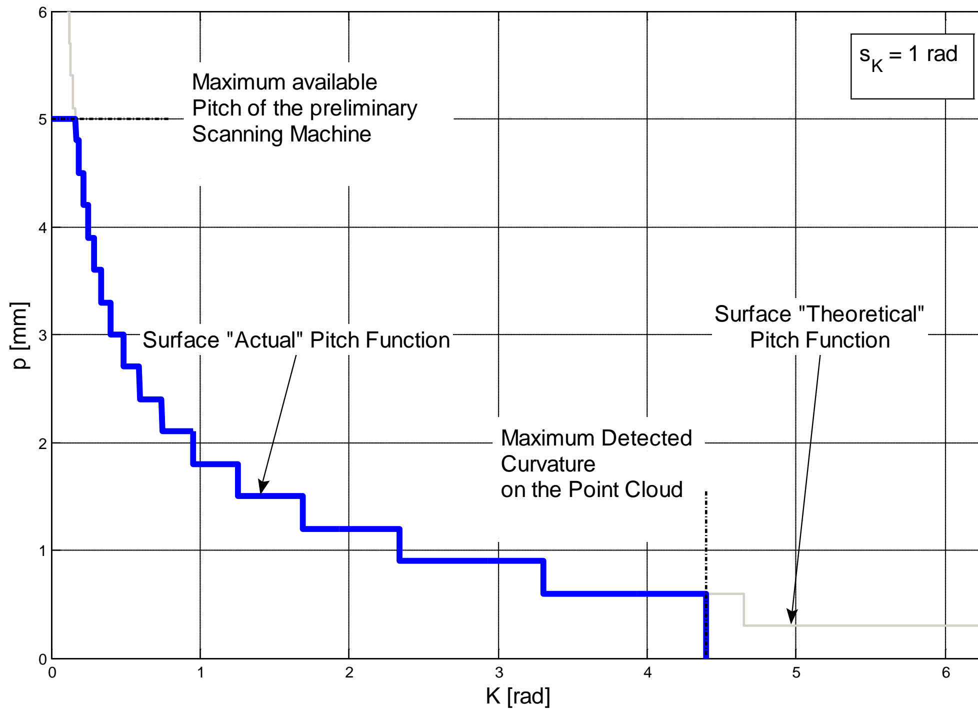


Figure 5: An example of discrete pitch *p* distribution

- Finally, the Pitch Function has to be also limited by a Pitch maximum amount. Even if the original, continuous Optimal Pitch Function tend to very high values, when plane-like surfaces are considered (infinite tendency of the Pitch Function when local Curvature becomes small), the Pitch enlargement must be coherent with at least two other factors. First, the proposed new pitch value must be suitable for the local geometry actual dimension. For example, a plane, square-like geometry cannot be sampled using too large Pitch values, but the maximum adoptable Pitch will have to suit the dimensions of the square. Furthermore, the performance of the new Scanner, employed for the Scanning Plan actuation, will have to be focused on. In fact, all Scanners can be characterized not only through the best resolution (i.e. the smallest available pitch), but also the worst one (i.e. the largest available Pitch). The worst working resolution is often linked to the Scanner maximum speed, when measuring relatively morphologically “simple” surfaces (Fig.6).

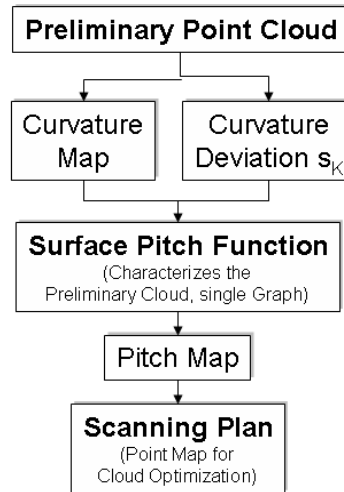


Figure 6 : Pitch function implementation methodology flow-chart

3.0 Evaluation of the Scanners Capability through "Pitch Functions" concept

Since the aim of this work is conveying an efficient Scanning Plan Management in order to perform the improvement of the Preliminary Point Clouds, even the suitability of the available Scanners for the task has to be taken into account.

This characteristic can be evaluated for Scanning devices through the comparison among the Optimal Pitch Functions which has been derived from the analysis of preliminary grids, and other analogue Pitch Functions which can be obtained from the analysis of measuring uncertainty of the scanning process, i.e. that related to the Measuring Device. In other words, Pitch Functions can be derived both from Point Clouds analysis and considerations about the measuring strategy, so they can characterize also Scanners. Both Function types focus on the local Discrete Curvature Evaluation and they exploit a Curvature Standard Deviation value in order to be properly calibrated.

In particular, an Optimal Pitch Function can be derived from consideration about the working conditions of a specific Scanning Machine too. Pitch values are calculated again with respect to the local Discrete Curvature amount. In fact, the Curvature model, based on the presence of Disturbance, is supposed to be the same when dealing with whatever considerable surface.

On the other hand, since the Pitch Function is known to be calculated basing on both geometrical and statistical considerations about the Curvature Distribution, dealing with measuring machines, particular remarks should be devoted to the derivation of the Curvature Standard Deviation s_K and the "Disturbance" Standard Deviation s_I .

In particular, the Curvature Model, which also the previously introduced Pitch Function had been derived from, will be considered again, for positive Curvature values:

$$K(I) = 2\pi - 4\arccos\left(\frac{I^2}{p_c{}^2 + I^2}\right). \tag{2}$$

Where K represents the Discrete Curvature Values, p_c stands for the "current" scanning Pitch, and I indicates the Points coordinates along the z-axis, which interprets the presence of Disturbance all over the sampled surface.

Starting from the just proposed Equation, such Scanners parameters as "Resolution of the Pointing System" (corresponding to the Scanning Pitch p), and as the "Resolution of the Scanning Sensor" (optical or piezo-electric, corresponding to the variable I in the previous Equation) are often listed in many Technical Handbooks as mere resolution intervals (Tab.1).

PARAMETERS FOR SCANNING DEVICE		Equivalent Standard Deviation [mm]
Pointing Resolution or Pitch value, x-y-axes [mm]	p_c	$s_p = \sqrt{\frac{p_c^2}{12}}$
Resolution of the Scanning Sensor, z-axis [mm]	I	$s_I = \sqrt{\frac{I^2}{12}}$

Table 1: 3D scanner parameters

Basing on this type of information, the evaluation of Curvature Deviation s_K , in order to calibrate a "Scanner Pitch Function", starts from considering the dependence of this parameter, on the employed scanning resolutions.

In particular, the Statistical Distribution of whatever measured values, within the resolution range $2a$ of a measuring machine, is usually interpreted as rectangular function. Since the Variance of such a distribution could be calculated as $a^2/3$, where a indicates the resolution half-interval, the Standard Deviation amount, due to the employed resolution will be calculated as:

$$s = \sqrt{\frac{a^2}{3}} . \tag{3}$$

In particular, for whatever available Scanner, the Equivalent Standard Deviations, for the resolutions of either the Pointing System or the Scanning Sensor, will be derived as shown in Table 1.

If all the other causes of uncertainty (beyond the pointing performance of the System, i.e. the currently employed Pitch value p_c, and the Scanning Sensor resolution) are supposed to be negligible, the Standard deviation of the K Discrete Curvature will be calculated following the instructions on the "ISO Guide to the Expression of Uncertainty in Measurement", for what concerns the Uncertainty propagation formula:

$$s_K \approx u_K = \sqrt{\left(\frac{\partial K}{\partial I}\right)^2 s_I^2 + \left(\frac{\partial K}{\partial p_c}\right)^2 s_{pc}^2} , \tag{4}$$

where K is the Discrete Gaussian Curvature, p_0 represents the preliminary Scanning Pitch and I stands for the measured z-coordinate amount.

If the previous Definition for Standard Deviations expression is considered (Tab.1), the Curvature Standard Deviation will be derived as follows:

$$s_K \approx u_K = \sqrt{\left(\frac{\partial K}{\partial I}\right)^2 \frac{I^2}{12} + \left(\frac{\partial K}{\partial p_c}\right)^2 \frac{p_c^2}{12}}\ . \tag{5}$$

Both p_c and I had been already considered in Table 1.

Since the resolution (Fig. 7) of the measuring instrument (i.e. the employed Scanning Pitch) can be varied within a specific interval from one Scanning Session to another, then also the corresponding Curvature Deviation s_K will be characterized by a variation interval.

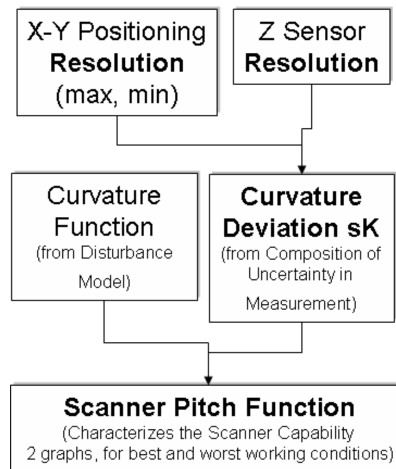


Figure 7: Proposed methodology Flow-Chart

In particular, maximum and minimum values for Curvature Standard Deviation will be individuated. As it has just been said, the Standard Deviation s_K is used for Pitch Function calibration, through the "Disturbance" Deviation $s_I(s_K)$.

For this reason, the lower and the upper bounds of the variation intervals for Standard Deviations will respectively correspond to two different Pitch Functions, which can be represented on the same Cartesian plane.

The area which is contained between the upper and the lower Pitch Function (Fig.8) represents the working "capability" of the Scanning machine.

Furthermore, for every "Scanner Pitch Function" which has been calibrated through specific resolution values, the minimum allowable Pitch (i.e. that corresponding to maximum Discrete Curvature amount) will correspond to the employed resolution itself. In order to calculate larger Pitch values, it has to be focused on the Scanner capability to vary its working resolution (Pitch Variation interval), as shown in Figure 9.

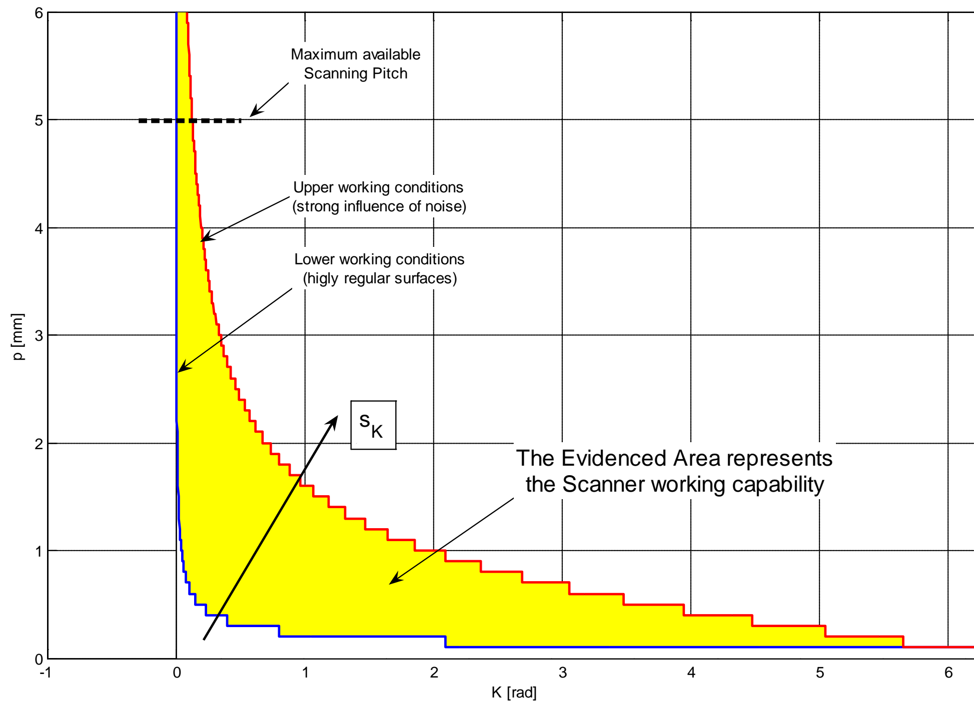


Figure 8: Area representing the working "capability" of the 3D Scanner device

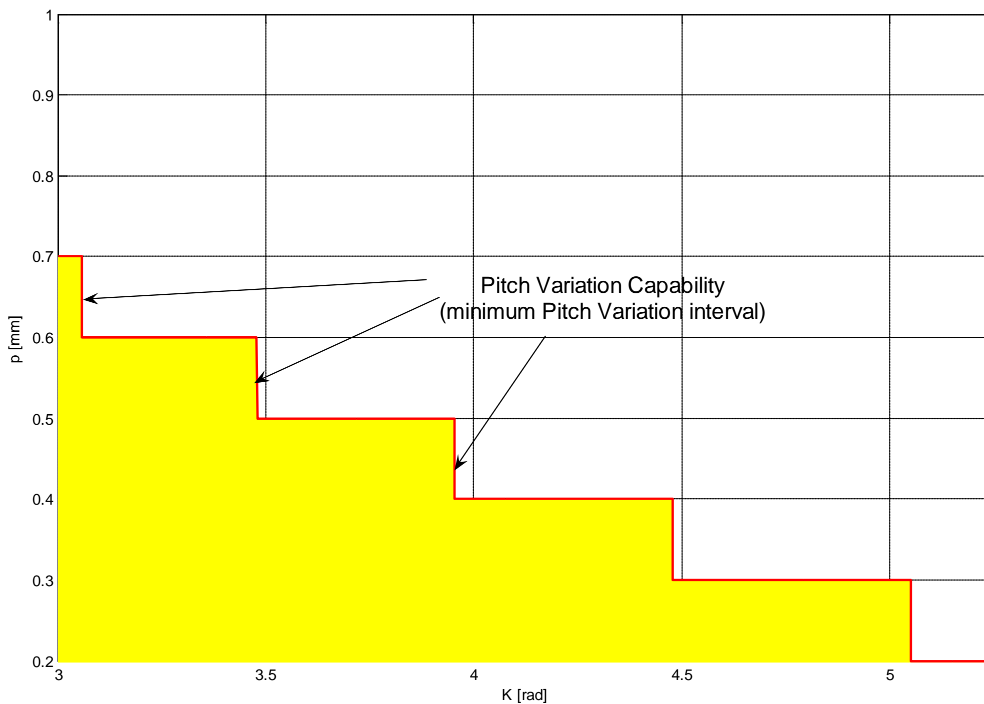


Figure 9: Area representing the real working "capability" of the 3D Scanner device

In other words, the just represented graph shows the Scanner suitability for analyzing different types of Surfaces, characterized by several degrees of noise. In particular, the upper, red function proposes Pitch values against measured Curvatures, in all those cases concerning strongly noised Surfaces.

On the other hand, the lower boundary Function is a “Pitch Function” too, but it corresponds to a smaller Curvature Deviation amount. For this reason, the lower Pitch Function represents all those working conditions, dealing with highly regular Surfaces, in which the noise is quite absent.

Both the upper and the lower boundary Pitch Functions represent anomalous working conditions for the considered Scanning device. In fact, in order to allow best exploitation of the Scanner performance, the examined Surfaces should be neither affected by great noise, nor too regular and smooth. In other words, the considered Scanning Device will allow best performances of the measuring process, only if the measured Surface is characterized by a Pitch Function which is contained within the Scanner Capability Area.

4.0 Comparison between “Surface Function” and “Scanner Function”: Capability Index

As it has just been said, the Pitch Function obtained from the Surface Analysis should be entirely contained in the Scanner Capability Area (Fig.10), in order to allow best performances of the considered Scanning Device.

In particular, the “middle” of the Capability Area is assumed to correspond to the best working conditions for the employed Measuring Machine.

If the Scanning Pitch is still considered to be a continuous function of Curvature (theoretical model), the “Middle” function is that Pitch Function corresponding to the mean value of the upper and the lower boundary Pitch Functions, as shown in Figure.

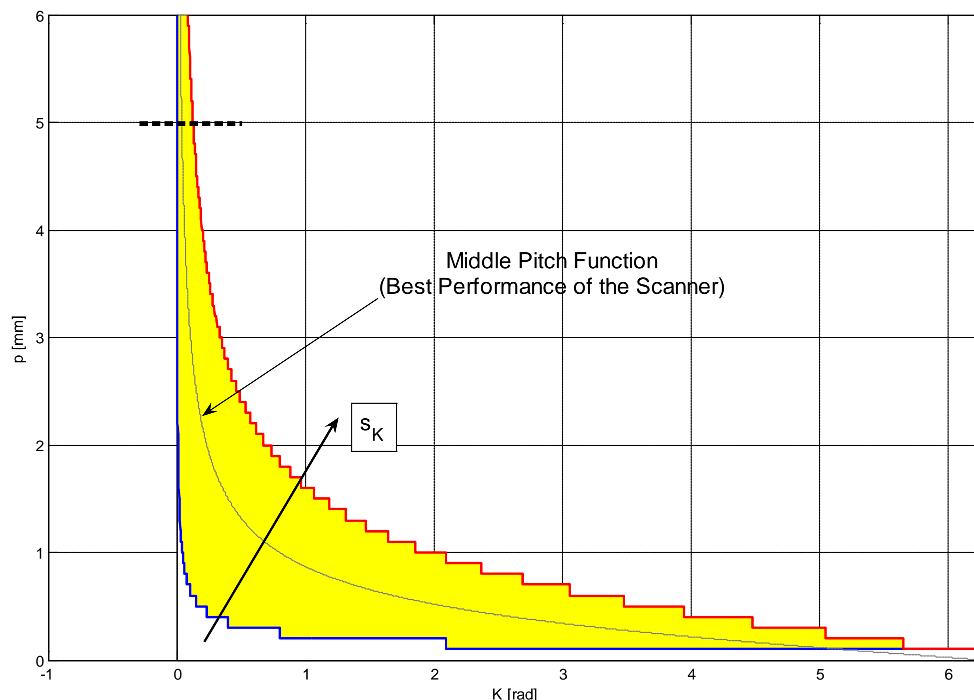


Figure 10: Area representing the working “capability” of the 3D Scanner device and the best 3D scanner performance

In particular, if the previously introduced Pitch Function for positive Curvature values is considered, the just mentioned "Mean Function" will be expressed as follows:

$$\overline{p} = \frac{p^+ + p^-}{2} = \frac{s_I^+ + s_I^-}{2}\sqrt{\frac{1-\sin\left(\frac{K}{4}\right)}{\sin\left(\frac{K}{4}\right)}} = \overline{s_I}\left(s_K^+, s_K^-\right)\sqrt{\frac{1-\sin\left(\frac{K}{4}\right)}{\sin\left(\frac{K}{4}\right)}}. \tag{6}$$

Where p represents the Pitch Functions, K is the Curvature amount, s_I^+ and s_I^- are the boundary values for the "Disturbance" Deviation, and finally s_K^+ and s_K^- are the corresponding boundary values for the Curvature Deviation.

When dealing Curvature amounts smaller than *3 rad*, the relation between the Curvature Deviation and the "Disturbance Deviation" can be approximated as follows [5,6]:

$$\frac{s_I}{s_K} \approx \frac{3}{2}\,\text{mm/rad}. \tag{7}$$

As a consequence, the influence of the Curvature Deviation could be directly evidenced in the previous Pitch Equation:

$$\overline{p} = \frac{p^+ + p^-}{2} = \frac{3}{2}\frac{s_K^+ + s_K^-}{2}\sqrt{\frac{1-\sin\left(\frac{K}{4}\right)}{\sin\left(\frac{K}{4}\right)}} = \frac{3}{2}\overline{s_K}\sqrt{\frac{1-\sin\left(\frac{K}{4}\right)}{\sin\left(\frac{K}{4}\right)}}. \tag{8}$$

Where the mean value for Curvature Deviation is focused on.

The expected quality of the Scanner performance, with respect to whatever analyzed Surface, can be evaluated through a sort of "Capability Index", which represents the "distance" of the examined Surface from the mean Pitch Function, which still represents the best possible performance of the considered machine (Fig.11).

The "distance" between whatever theoretical (i.e. without Pitch Discretization) Pitch Function from the mean one can be evaluated as follows:

$$d = \int_0^{2\pi}\left[p(K) - \overline{p}(K)\right]dK = \frac{3}{2}\left(s_K - \overline{s_K}\right)\int_0^{2\pi}\sqrt{\frac{1-\sin\left(\frac{K}{4}\right)}{\sin\left(\frac{K}{4}\right)}}\,dK. \tag{9}$$

Keeping the same formulation, the "Capability Index" can be expressed as:

$$C = \frac{1}{2} - \frac{\left| s_K - \overline{s_K} \right|}{s_K^+ - s_K^-}. \tag{10}$$

When the considered “Surface Pitch Function” coincides with the Mean Scanner Pitch Function, the corresponding “Capability Index” will be equal to 1 (100%, best performance), because the Measuring Machine is considered to work in best conditions.

When the Surface Pitch Function overlaps with the upper or lower boundary Functions of the Scanner, the Capability Index comes to zero, because the Scanning Device is considered to work in worst conditions.

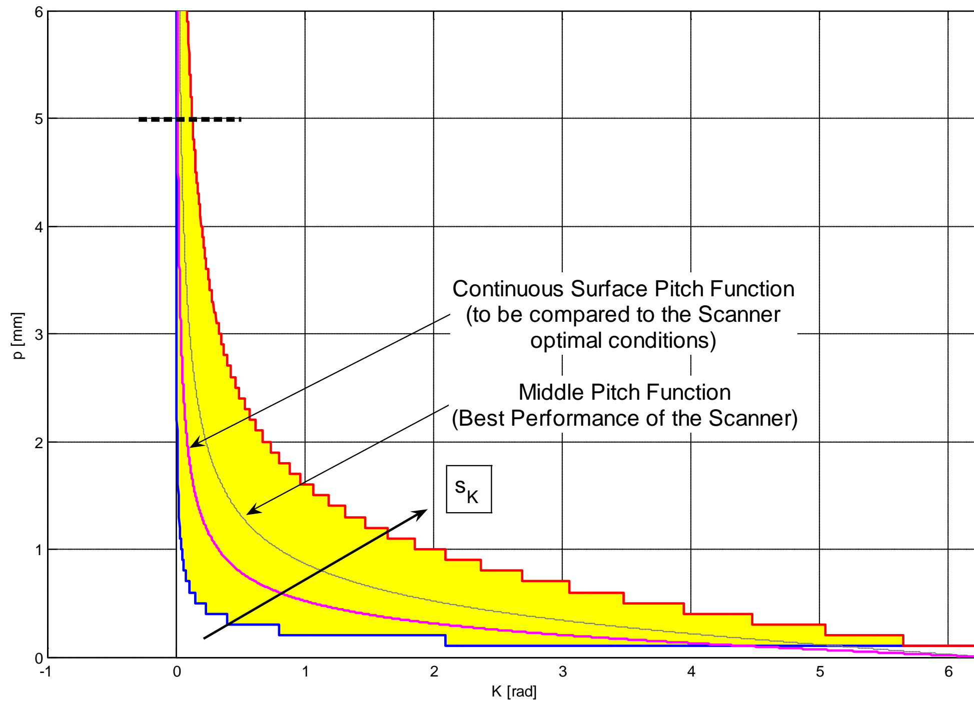


Figure 11: representation of the “Capability Index” as “distance” of the examined Surface from the mean Pitch Function

Obviously, the discretization of the considered Pitch Function has to be taken into account. In this case, the evaluation of the “distance” between two different Pitch Function is much more complex, even if it could be numerically carried out.

As an alternative, all the consideration about theoretical functions, which have just been dealt with, can be used in order to evaluate the Capability Index. Then, both the Scanner and the Surface Pitch Functions should be plotted on the same Cartesian Plane, employing Pitch discretization, in order to verify that the discretized Surface Function is really contained within the Scanner Capability Area.

As Figure 12 shows, two critical zones can be observed:

- the first one concerns the highest Curvature range. The Surface Pitch Function (in black) is characterized by a maximum Curvature amount which is often lower than the Curvature limit 2π. In correspondence of this Curvature amount, a Pitch value is suggested, which is a characteristic of the

analyzed Surface. The maximum Curvature amount, associated to the minimum suggested Pitch value for the considered Surface, will be here called "Minimum Detail". For example, for the Surface Pitch Function which had been proposed in Figure, a *0,3mm* Pitch value corresponds to about a *4,4rad* Curvature. As a consequence, the "Minimum Detail" amount will be defined as *0,3mm@4,4rad*.

On the other hand, the Minimum Detail for the Scanning Device can be obtained from the minimum available Pitch increment, associated to the upper Curvature limit, which is always equal to the upper limit 2π . For what concerns the example represented in Figure, the available Minimum Detail for the employed Scanner is *0,1mm@* 2π *rad*.

- The second critical feature is about the maximum available Pitch for Scanners, and the maximum suggested Pitch for Surfaces (Figure).

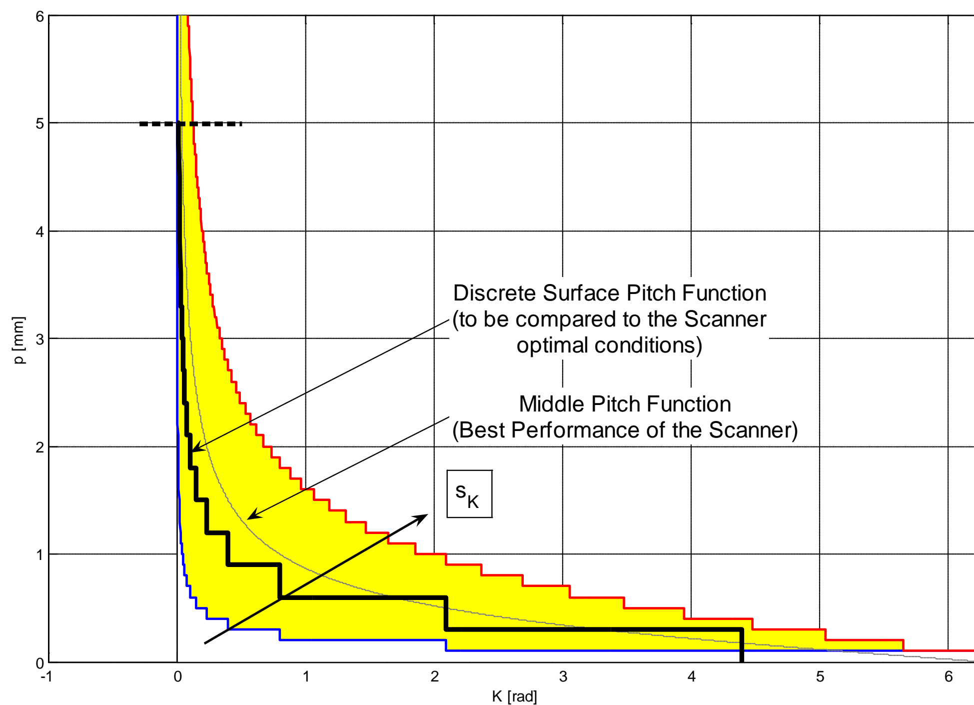


Figure 12: Critical conditions

The two most relevant critical zones have been evidenced, in order to compare the "Surface Pitch Functions" with "Scanner Pitch Functions". In particular, a Scanner model will result suitable for analysing a pre-determined surface only if:

- the Scanner "Minimum Detail" amount is lower or equal to that of the Surface (for what concerns both Pitch and Curvature amount);
- the maximum available Pitch value, for the considered Scanning Device, should be equal or higher than that suggested for the analyzed Surface.

5.0 Surface Functions and Scanner Functions: an experimental validation

In this section those Benchmark Surfaces which had been presented for Pitch Function description will be considered again as Analyzed Cases. The suitability a Measuring Machine will be tested, in order to Scan the just considered Surfaces. In order to so, the compatibility of the Scanner and the Surface Pitch Functions will be tested.

The employed scanning device (Roland Picza) [10] is characterized by the following data.

SCANNING PARAMETER	SYMBOL	VALUE (MAX/MIN)
Pointing Resolution [mm]	p	5/0,05
Scanning Resolution [mm]	I	0,025

Table 2: Technical data of the employed 3D scanner

The contributes of the Resolution values to the Curvature Deviation, which is employed in order to calibrate Pitch Functions, can be evaluated through the previously presented method, either for smallest or largest Scanning Pitches (Tab.3,4).

Source description	Uncertainty Type	Half Interval *a* [mm]	Equivalent Deviation, s_j [mm]	$\left(\frac{\partial K}{\partial x_j}\right)^2$	Contribution $\left(\frac{\partial K}{\partial x_j}\right)^2 \cdot s_j^2$
Pointing Resolution *p* [mm]	B	0,025	0,014	2731	0,57
Scanning Resolution *I* [mm]	B	0,0125	0,007	683	0,04
				SUM	0,60
				$\mathbf{s_K}$	0,8

Table 3: scanning device best available pointing resolution

Source description	Uncertainty Type	Half Interval *a* [mm]	Equivalent Deviation, s_j [mm]	$\left(\frac{\partial K}{\partial x_j}\right)^2$	Contribution $\left(\frac{\partial K}{\partial x_j}\right)^2 \cdot s_j^2$
Pointing Resolution *p* [mm]	B	2,5	1,4	$6,4\ 10^{-5}$	0,00013
Scanning Resolution *I* [mm]	B	0,0125	0,007	$2,0\ 10^{-9}$	negligible
				SUM	0,00013
				$\mathbf{s_K}$	0,012

Table 4: scanning device worst available pointing resolution

Since the lower and upper boundary values for Curvature Deviation have been calculated, the Scanner Pitch

Function can be plotted for the considered Device.

Once the Scanner Capability Area have been defined (Figure), the analyzed Surface Pitch Function can be plot on the same Cartesian plane.

As it has also been shown in previous Sections, four different Curvature Deviations correspond to the considered Surfaces. As a consequence, four different Pitch Functions will be tested, with respect to the Scanner Capability Area.

The Mean Curvature Deviation $\overline{s_K}$ can be easily calculated with the previous formula:

$$\overline{s_K} = \frac{s_K^+ + s_K^-}{2} \approx 0{,}41\,rad. \tag{11}$$

Furthermore, the Curvature Deviation corresponding to the previously proposed Surface Pitch Functions are here listed. (Tab.5)

s_K [mm]	Value	Capability Index [%]
$s_{K,1}$	0,05	5
$s_{K,2}$	0,29	35
$s_{K,3}$	0,7	13
$s_{K,4}$	1,4	-76

Table 5: Curvature deviations

The analysis of the Capability Indexes indicates that the considered Device will allow best performances while Scanning the second Surface. In fact, in this case the Capability Index assumes the highest value.

On the other hand, the fourth Surface is characterized by a negative Capability Index. In fact, as it is possible to see in Figure, the fourth graph is out of the Scanner Capability Area.

Some conclusions can be deduced from the Capability Analysis: the considered Scanner will work well, while sampling the second and the third Surfaces, while the first graph is too close to the lower boundary Pitch Function, so scanning the first Surface using this particular Device means working in anomalous conditions.

Finally, the Scanning Device results unable to properly Scan the fourth Surface, using a Scanning Plan obtained from the application of the considered Surface Pitch Function.

6.0 Discussions and Conclusions

It has also to be reminded that the adopted preliminary Scanning Pitch (which often does not represent the best performance of the Measuring Machine, or sometimes proves to be inadequate in order to properly describe some morphologically complex features) strongly influences the behaviour of the so called "Surface Pitch

Function”.

Since the Preliminary Point Cloud, which the Pitch Function has been calculated from, had been obtained employing an arbitrary Pitch value, the Function itself could be iteratively changed by improving the original distribution of the Points on the Preliminary Grid. As a consequence, the Optimal Pitch function building could be seen as an iterative cycle, consisting of few steps:

1) preliminary point cloud;
2) Curvature Map generation (Geometrical and Statistical Model of Curvature);
3) Pitch Map building;
4) Preliminary grid upgrade (Scanning Plan definition)

In a parallel way, the available Scanners can be characterized by Pitch Functions:

1) Upper and lower boundary Curvature Definition values for available Scanning Devices;
2) Evaluation of the Scanner Capability Area;

Finally, the Pitch Functions, which had been obtained from the Scanner evaluation and the Surface analysis, have to be compared in order to choose the most suitable Device to scan the considered Surface. Since both the Scanning Plan and the most suitable Scanning Machine have been defined, the new Scanning Session can be implemented.

The new Point cloud, which has been obtained from the whole optimizing process, can be used again as a “Preliminary Point Cloud”, in order to restart the optimizing process, thus performing a sort of “control cycle”. In fact, the previously defined cycle can be exploited until new Curvature Map does not convey significant advantages with respect to the previous ones, for what concerns surface reconstruction. In particular, the deviation between the previously optimized Point Cloud and the newly defined one should be even minimized at every iteration.

This strategy could be adopted for further verification that the proposed algorithm works well. Even if iterations could prove to be time consuming, the convergence speed of the proposed approach to a final solution is surely higher than that available in the case of a manually driven process (i.e. when an “expert” user chooses which points of Preliminary Clouds have to be erased, and which zones on the examined surface need more Points in order to be properly described).

Furthermore, the Scanning Plans resulting from the Curvature-driven approach are known to be more precise and univocal. In other words, using the Optimal Pitch Map, in order to improve the Points Distribution of Preliminary Point Clouds, grants the monotone convergence of the method. On the contrary, the manual intervention on the Scanning Plan definition could be affected by such inconvenient as subjective results or fitting and de-fitting incoherency.

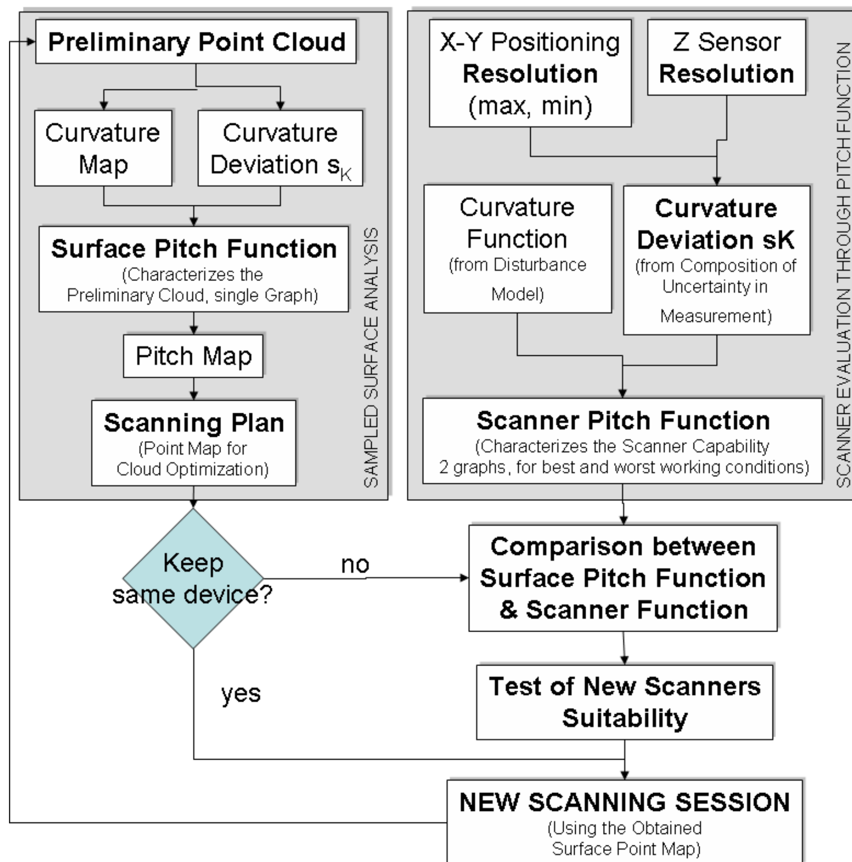


Figure 13: proposed methodology flow chart

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