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Free Space Optical System Performance for a Gaussian Beam Propagating Through Non-Kolmogorov Weak Turbulence

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Abstract—**Atmospheric turbulence has been described for many years by Kolmogorov's power spectral density model because of its simplicity. Unfortunately several experiments have been reported recently that show Kolmogorov theory is sometimes incomplete to describe atmospheric statistics properly, in particular in portions of the troposphere and stratosphere. It is known that free space laser system performance is limited by atmospheric turbulence. In this paper we use a non-Kolmogorov power spectrum which uses a generalized exponent instead of constant standard exponent value 11/3 and a generalized amplitude factor instead of constant value 0.033. Using this spectrum in weak turbulence, we carry out, for a Gaussian beam propagating along a horizontal path, analysis of** *long term beam spread, scintillation, probability of fade, mean signal to noise ratio* **and** *mean bit error rate* **as variation of the spectrum exponent. Our theoretical results show that for alpha values lower than $\alpha = 11/3$, but not for alpha close to $\alpha = 3$, there is a remarkable increase of** *scintillation* **and consequently a major penalty on the system performance. However when alpha assumes values close to $\alpha = 3$ or for alpha values higher than $\alpha = 11/3$** *scintillation* **decreases leading to an improvement on the system performance.**

Index Terms—**Atmospheric turbulence, beam spreading, bit error rate (BER), fade, non Kolmogorov spectrum, scintillation, signal to noise ratio (SNR), structure function.**

I. INTRODUCTION

FOR A long time, the structure function has been modeled according to Kolmogorov's power spectrum of refractive index fluctuations which is widely accepted and has been applied extensively in studies of optical and radio wave propagation in the atmosphere. However, recent experimental data from space-based stellar scintillation, balloon-borne *in-situ* temperature, and ground-based radar measurements indicate that turbulence in the upper troposphere and stratosphere deviates from predictions of the Kolmogorov model [5], [9], [10]. Further development of the turbulent theory of passive scalar transfer has shown that although the Kolmogorov spectrum is generally correct, it constitutes only one part of the more general behavior of passive scalar transfer in a turbulent flow [6]. In fact, Euler equation has two integrals of motion, and not simply energy. The second integral of motion proved to be the magnitude called helicity or mean helicity. It plays an important role in the non-Kolmogorov turbulent fluctuations; depending on the relationship between the parameters of the velocity field and of the passive scalar field various types of spectra with different alpha value can arise. Some anomaly behavior [3] seems to occur when the atmosphere is extremely stable because under such condition the turbulence is no longer homogeneous in three dimensions since the vertical component is suppressed. It has been shown [4] that for such two dimensional turbulence, the rate of the energy cascade from larger to smaller scales is reduced and Kolmogorov turbulence is not fully developed.

In addition anisotropy in stratospheric turbulent inhomogeneities has been experimentally investigated [5], [8], [11], [12] and turbulence spectrum was investigated by Lidar measurements [15]. The experimental results show the various strata and layers in the vertical turbulence profiles. It is shown that the power law exponent of the structure function is different from the cases of purely Kolmogorov. Finally, A. Tunick paper [18] showed recently experimental results on non-Kolmogorov turbulence even at low altitude (rooftop elevation) for lasercom systems that traverse complex inhomogeneous propagation paths.

We must accept de facto that turbulence is still an unsolved problem in classical physics, and the scientific community must persist in doing more simulations, measurements and experiments [7].

It is very important, therefore, to find new models more general than Kolmogorov spectrum in order to describe experimental data also in non-Kolmogorov turbulence. In this work, we present a theoretical spectrum model which reduces to one type of Kolmogorov only for a particular case of its exponent: the standard value 11/3. The exponent can assume all the values between the range 3 to 4. Using this new spectrum, following the same procedure already used from Andrews, Phillips, *et al.* [1], [2], we have analyzed the impact of the exponent's variation on *long term beam spread*, *scintillation index*, *probability of fade*, *mean signal to noise ratio* and *mean bit error rate* for horizontal path, that is for constant value of the refractive index structure parameter. We have done this analysis for a Gaussian beam propagating in weak turbulence. The same analysis both for plane wave and spherical wave has been reported in [13] and in [16]. Angle of arrival for plane wave and spherical wave in non Kolmogorov turbulence has been analyzed in [14].

We used in our approach the Rytov method because we supposed to be in weak turbulence, in fact the Rytov method, as re-

ported in [1], can not be used in strong turbulence conditions. In strong turbulence conditions we need to use parabolic equation method which leads to correct results for all turbulence conditions (weak, moderate and strong). However, parabolic equation method, in weak turbulence, leads to equivalent results with the Rytov method only for first-order and second-order field moments but only asymptotic results have been obtained at present for the fourth-order field moments. The paper [19]–[21] show a method based on parabolic equation and modal decomposition to carry out up to second-order moment of the field (two-frequency mutual coherence function), but they do not analyze the fourth-order field moment and the scintillation index.

II. NON KOLMOGOROV SPECTRUM

The basic power-law spectrum of Kolmogorov is defined by

$$\Phi_n(\kappa) = 0.033 \cdot C_n^2 \cdot \kappa^{-11/3} \tag{1}$$

where C_n^2 is the refractive-index structure parameter. The validity of the Kolmogorov spectrum is restricted to the inertial range although in some analyses it is extended to all spatial wave numbers. Here we examine a more general power spectrum model that describes non-Kolmogorov atmospheric turbulence in which the power law exponent 11/3 is allowed to deviate somewhat from this value.

We assume that in an atmosphere exhibiting non-Kolmogorov turbulence the structure function for the index of refraction is given by

$$D_n(r) = \beta \cdot C_n^2 \cdot r^\gamma \tag{2}$$

where γ is the power law which reduces to 2/3 in the case of conventional Kolmogorov turbulence. Here, β is a constant equal to unity when $\gamma = 2/3$, but otherwise has units $m^{-\gamma+2/3}$. Following same procedure reported in [1], the corresponding power-law spectrum associated with structure function takes the form

$$\Phi_n(\kappa, \alpha) = A(\alpha) \cdot \tilde{C}_n^2 \cdot \kappa^{-\alpha}, \quad \kappa > 0, \quad 3 < \alpha < 4 \tag{3}$$

where $\alpha = \gamma + 3$ is the spectral index or power law, $\tilde{C}_n^2 = \beta \cdot C_n^2$ is a generalized structure parameter with units $m^{-\gamma}$, and $A(\alpha)$ is defined by

$$A(\alpha) = \frac{1}{4\pi^2}\Gamma(\alpha - 1)\cos\left(\frac{\alpha\pi}{2}\right), \quad 3 < \alpha < 4 \tag{4}$$

and the symbol $\Gamma(x)$ in the last expression is the gamma function. When $\alpha = 11/3$, we find that $A(11/3) = 0.033$, and the generalized power spectrum reduces to the conventional Kolmogorov spectrum in (1). Also, when the power law approaches the limiting value $\alpha = 3$, the function $A(\alpha)$ approaches zero. Consequently, the refractive-index power spectral density vanishes in this limiting case.

III. LONG TERM BEAM SPREAD

The first important quantity that shows total average beam spot size radius on the receiver lens is the *long term beam spread*. It can be written as the sum of three terms: diffraction limited beam spreading, beam spreading due to small turbulence scales and beam wander which can be described by the variance of the instantaneous center of the beam in the receiver plane.

The analytical form of *long term beams spread* for a Gaussian beam wave is [1]

$$W_e^2 = \langle W_{LT}^2(\alpha) \rangle = W^2 \cdot [1 + \langle T(\alpha) \rangle] \tag{5}$$

where W is the diffraction limited spot size radius and $\langle T(\alpha) \rangle$ is the term which includes small scale beam spreading and beam wander atmospheric effects.

To carry out *long term beams spread* analysis we need to calculate the $\langle T(\alpha) \rangle$ term.

For horizontal path the parameter C_n^2 that appears inside the relation $\tilde{C}_n^2 = \beta \cdot C_n^2$ is constant. Following same formula reported in [1] but using the non Kolmogorov spectrum in (3), we carry out

$$
\begin{aligned}
T(\alpha) &= 4\pi^2 k^2 L \cdot \left(\int_0^1 \int_0^\infty \kappa \cdot \phi_n(\alpha, \kappa) d\kappa d\xi \right.\\
&\quad \left. - \int_0^1 \int_0^\infty \kappa \cdot \phi_n(\alpha, \kappa) \exp\left(-\frac{\Lambda L \kappa^2 \xi^2}{k}\right) d\kappa d\xi \right)\\
&= -2\pi^2 \cdot A(\alpha) \cdot \frac{1}{\alpha - 1} \cdot \Gamma\left(1 - \frac{\alpha}{2}\right)\\
&\quad \cdot \Lambda^{\alpha/2-1} \cdot \tilde{C}_n^2 \cdot k^{3-\alpha/2} \cdot L^{\alpha/2}\\
&= 0.25 \cdot \frac{\alpha}{\alpha - 1} \cdot \left[\sin\left(\alpha \cdot \frac{\pi}{4}\right)\right]^{-1} \cdot \tilde{\sigma}_R^2(\alpha) \cdot \Lambda^{\alpha/2-1} \tag{6}
\end{aligned}
$$

where $\xi = 1 - z/L$ (z is the propagation distance), $\Lambda = 2L/kW^2$ and we have defined a non Kolmogorov Rytov variance by the plane wave scintillation index in non Kolmogorov weak turbulence [17]

$$
\begin{aligned}
\tilde{\sigma}_R^2(\alpha) &= -8\pi^2 \cdot A(\alpha) \cdot \frac{1}{\alpha} \cdot \Gamma\left(1 - \frac{\alpha}{2}\right)\\
&\quad \cdot \sin\left(\alpha \cdot \frac{\pi}{4}\right) \cdot \tilde{C}_n^2 \cdot k^{3-\alpha/2} \cdot L^{\alpha/2}. \tag{7}
\end{aligned}
$$

It is interesting to observe that for $\alpha = 11/3$ we obtain the particular case of the Kolmogorov spectrum already reported in [1].

At this point, we plot in Fig. 1 the *long term beam spread* as a function of α for a particular horizontal case, we take

$$L = 1 \text{ km}; \ \tilde{C}_n^2 = 7 \cdot 10^{-14} m^{-\alpha+3}$$
$$\lambda = 1.55 \ \mu\text{m}; \ W_0 = 0.01 \text{ m}$$

where W_0 is the spot radius at the transmitter and we take a collimated beam at the transmitter.

We deduce from Fig. 1 that if alpha decreases from $\alpha = 11/3$ (excluding alpha values close to 3), then *long term beam spread* W_e increases up to a maximum value. At this point the curve changes its slope because of the term $A(\alpha)$ that assumes very low values. In addition it is shown that if alpha increases from $\alpha = 11/3$, then the *long term beam spread* W_e decreases down to a minimum value. At this point the curve changes its slope, because the term $\Gamma(1 - \alpha/2)$ assumes high values close to its asymptote as $\alpha \to 4$. The obvious physical interpretation of alpha approaching 3 is that turbulence tends to vanish. On the
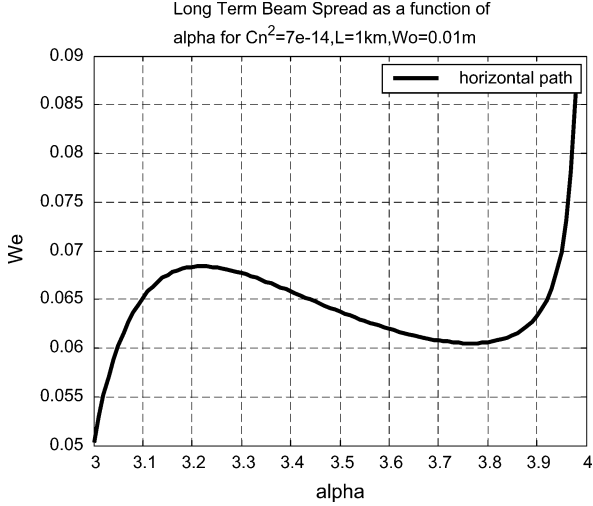
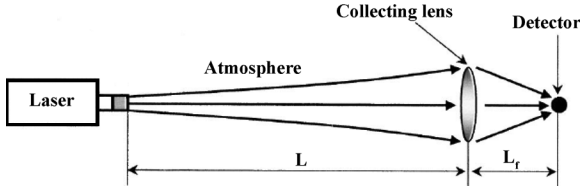Fig. 1. *Long term beam spread* as a function of *alpha* for horizontal path.

Fig. 2. Propagation geometry for a Gaussian beam originating at distance L to the left of a thin Gaussian lens.

other extreme, the physical interpretation of alpha approaching 4 is that phase effects dominate in the form of random tilts, which generate beam wander.

IV. SCINTILLATION

Another important parameter that is necessary to calculate the system performance is the scintillation index. In our analysis, we include aperture averaging effects of the receiver aperture, so we carry out the flux variance in the plane of the detector at a short distance L_f behind the collecting lens. We illustrate such a system below in Fig. 2.

To describe the beam characteristics at the input plane and at the front plane of the Gaussian lens, we use two sets of non-dimensional beam parameters. We assume the transmitted Gaussian beam at the input plane has finite radius W_0 and phase front radius of curvature given by F_0. Thus, we have

$$\text{at the transmitter } (z = 0): \Theta_0 = 1 - \frac{L}{F_0}$$

$$\Lambda_0 = \frac{2L}{kW_0^2}$$

$$\text{at the Gaussian lens } (z = L): \Theta_1 = \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2}$$

$$\Lambda_1 = \frac{\Lambda_0}{\Theta_0^2 + \Lambda_0^2}.$$

Following the same procedure as discussed in [1] for the standard Kolmogorov spectrum, but this time using a non Kolmogorov spectrum and introducing the collecting lens diameter

D_G, our analysis for a Gaussian beam leads to (neglecting beam wander effects)

$$
\begin{aligned}
\sigma_I^2 &(\alpha, D_G) \\
=&4\pi^2 \cdot k^{3-\alpha/2} \cdot L \\
&\cdot \text{Re} \int_0^1 \int_0^\infty \kappa \cdot \Phi_n(\kappa, \alpha) \cdot \exp\left\{ -\frac{L \cdot \kappa^2}{k(\Lambda_1 + \Omega_G)} \right. \\
&\cdot \left[(1 - \bar{\Theta}_1\xi)^2 + \Lambda_1\Omega_G\xi^2 \right] \Big\} \\
&\cdot \left\{ 1 - \exp\left[-j\frac{L\kappa^2}{k}\left(\frac{\Omega_G - \Lambda_1}{\Lambda_1 + \Omega_G} \right) \right. \right. \\
&\left. \left. \times \xi(1 - \bar{\Theta}_1\xi) \right] \right\} d\kappa d\xi \\
=&4\pi^2 \cdot k^2 \cdot L \cdot A(\alpha) \cdot \tilde{C}_n^2 \cdot \Gamma\left(1 - \frac{\alpha}{2}\right) \\
&\cdot \left[\frac{k}{L} \cdot (\Lambda_1 + \Omega_G) \right]^{1-\alpha/2} \cdot \\
&\times \left\{ \int_0^1 \left[(1 - \bar{\Theta}_1 \cdot \xi)^2 + \Lambda_1\Omega_G\xi^2 \right]^{\alpha/2-1} d\xi \right. \\
&- \text{Re} \int_0^1 \left[(1 - \bar{\Theta}_1 \cdot \xi)^2 + \Lambda_1\Omega_G\xi^2 \right. \\
&\left. \left. +j(\Omega_G - \Lambda_1) \cdot \xi(1 - \bar{\Theta}_1 \cdot \xi) \right]^{\alpha/2-1} d\xi \right\}
\end{aligned}
\tag{8}
$$

where $\xi = 1 - z/L$ (z is the propagation distance), $\Omega_G = 2L/kW_G^2$ is a non-dimensional parameter characterizing the spot radius of the Gaussian collecting lens, $W_G^2 = D_G^2/8$ and $\bar{\Theta}_1 = 1 - \Theta_1$.

At this point we plot $\sigma_I^2(1, D_G)$ as a function of alpha for a particular horizontal case. We take

$$L = 1000 \text{ m}; \ \tilde{C}_n^2 = 7 \cdot 10^{-14} m^{-\alpha+3}$$
$$\lambda = 1.55 \ \mu\text{m}; \ D_G = 0.1 \text{ m}$$
$$W_0 = 0.01 \text{ m}, \ \Theta_0 = 1.$$

The results are shown in Fig. 3.

We deduce from Fig. 3 that for alpha values lower than Kolmogorov value $\alpha = 11/3$, excluding alpha values close to 3, there is a increase of scintillation. Consequently, scintillation in this case leads to a larger penalty on system performance. We deduce also that there is a maximum value of scintillation where the curve changes its slopes because the term $A(\alpha)$ begins to decrease to zero. In addition for alpha values higher than $\alpha = 11/3$, scintillation slightly decreases and consequently it will lead to a slight gain in system performance. The physical interpretation of alpha approaching 4 is that the power spectrum contains fewer eddies of high wave numbers; therefore scintillation effect are reduced.

V. PROBABILITY OF FADE

Given a PDF model for irradiance fluctuations $p_I(I)$, the *probability of fade* describes the percentage of time the irradiance of the received signal is below some prescribed threshold
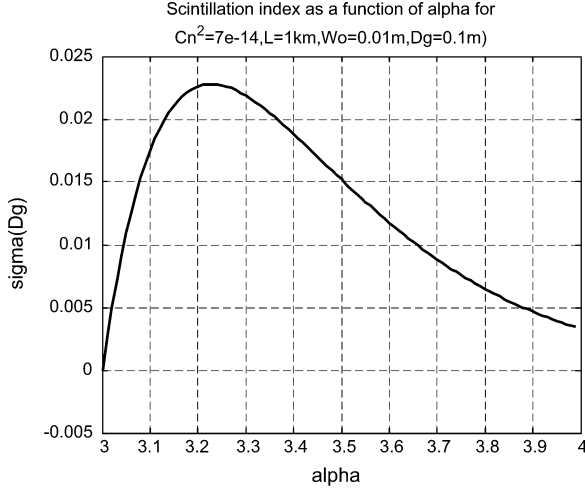
Fig. 3. Scintillation index as a function of alpha for horizontal path.

Fig. 4. *Probability of fade* as a function of alpha using the log-normal PDF.

value I_T. Hence, the *probability of fade* as a function of threshold level is defined by the cumulative probability [1]

$$p_I\left(I < I_T\right) = \int_0^{I_T} p_I\left(I\right) dI. \qquad (9)$$

The PDF most often used under weak irradiance fluctuations is the lognormal model and the resulting *probability of fade* leads to

$$p_I\left(I < I_T\right) = \frac{1}{2}\left\{1 + erf\left[\frac{\frac{1}{2}\cdot\sigma_I^2\left(\alpha, D_G\right) - 0.23\cdot F_T}{\sqrt{2}\cdot\sigma_I\left(\alpha, D_G\right)}\right]\right\} \qquad (10)$$

where $erf(x)$ is the error function. In arriving at this expression we have introduced the fade threshold parameter

$$F_T = 10\cdot\log_{10}\left(\frac{\langle I\rangle}{I_T}\right)[dB]. \qquad (11)$$

The fade parameter F_T, given in decibels $[dB]$, represents the dB level below the on-axis mean irradiance that the threshold I_T is set.

Using scintillation index (8) into (10), we calculate the *probability of fade* as a function of alpha for a fixed fade threshold parameter for a particular horizontal case.

We take

$$L = 1\text{ km};\ \tilde{C}_n^2 = 7\cdot10^{-14}m^{-\alpha+3}$$
$$\lambda = 1.55\ \mu\text{m};\ D_G = 0.1\text{ m},\ F_T = 3\text{ dB}$$
$$W_0 = 0.01\text{ m},\ \Theta_0 = 1.$$

The plot is shown in Fig. 4.

VI. MEAN SIGNAL TO NOISE RATIO

In this paragraph is shown the *mean signal to noise ratio* in presence of atmospheric turbulence using non Kolmogorov power spectrum. The received irradiance over long measurement intervals must be treated like random variable because of
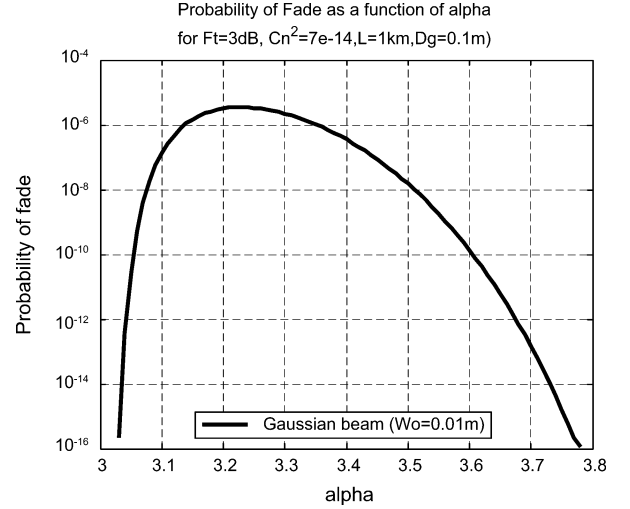
the turbulence. Based on [1], [2], the *mean signal to noise ratio* $\langle\text{SNR}\rangle$ at the output of the detector in the case of a shot-noise limited system assumes the form

$$\langle\text{SNR}\rangle = \frac{SNR_0}{\sqrt{1 + \sigma_I^2\left(\alpha, D_G\right)\cdot SNR_0^2}} \qquad (12)$$

where $\sigma_I^2\left(\alpha, D_G\right)$ has been defined before, SNR_0 is the signal to noise ratio in absence of turbulence.

We plot in dB units *mean signal to noise ratio* $\langle\text{SNR}\rangle$ as a function of *signal to noise ratio without turbulence* SNR_0 for several alpha values, using Gaussian beam model for scintillation. We take the following parameters:

$$L = 1\text{ km};\ \tilde{C}_n^2 = 7\cdot10^{-14}m^{-\alpha+3}$$
$$\lambda = 1.55\ \mu\text{m};\ D_G = 0.1\text{ m}$$
$$W_0 = 0.01\text{ m},\ \Theta_0 = 1.$$

The plot, shown in Fig. 5, illustrates the impact of the alpha variation on the $\langle\text{SNR}\rangle$ performance. For alpha values lower than $\alpha = 11/3$, excluding alpha values close to 3, there is a penalty on the system performance with respect to the case of Kolmogorov $\alpha = 11/3$. For alpha values higher than $\alpha = 11/3$ there is a gain on the system performance with respect to the case of Kolmogorov $\alpha = 11/3$. Finally also there is a gain on system performance with respect to Kolmogorov $\alpha = 11/3$ when alpha assumes values very close to $\alpha = 3$ because the amplitude factor $A\left(\alpha\right)$ assumes very low values and consequently the *scintillation* reported before in Fig. 2 approaches zero.

VII. MEAN BIT ERROR RATE

In the presence of optical turbulence, the probability of error is considered a conditional probability that must be averaged
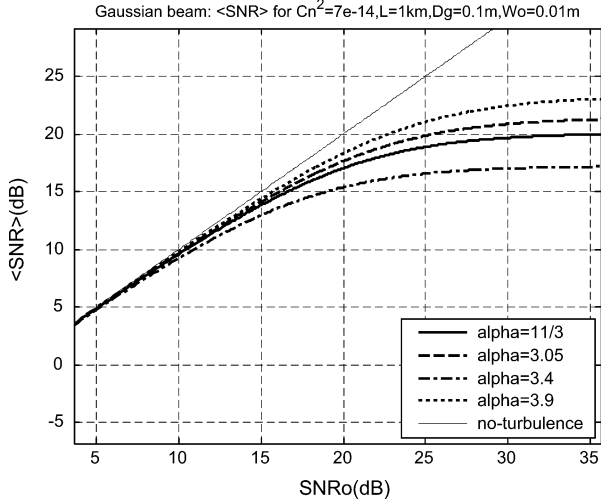
Fig. 5. *Mean signal to noise ratio* as a function of *signal to noise ratio without turbulence* for several alpha values, using Gaussian beam model for scintillation.
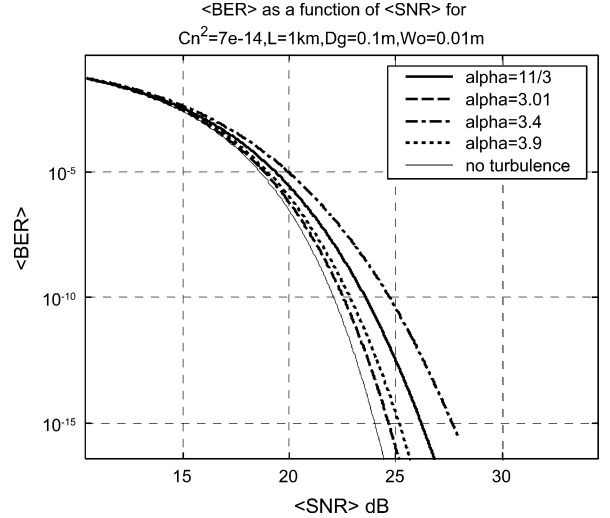

Fig. 6. *Mean bit error rate* as a function of *mean signal to noise ratio* for several alpha values.

over the PDF of the random signal to determine the unconditional mean BER. In terms of a normalized signal with unit mean, this leads to the expression [1], [2]

$$
\begin{aligned}
\Pr(E) &= \langle \text{BER} \rangle \\
&= \frac{1}{2} \cdot \int_0^\infty p_I(u) \cdot erfc\left(\frac{\langle \text{SNR} \rangle \cdot u}{2 \cdot \sqrt{2}}\right) du \quad (13)
\end{aligned}
$$

where $p_I(u)$ is taken to be the log normal distribution with unit mean

$$
\begin{aligned}
p_I(u) &= \frac{1}{u \cdot \sigma_I(D_G, \alpha) \cdot \sqrt{2 \cdot \pi}} \\
&\cdot \exp\left\{-\frac{\left[\ln(u) + \frac{1}{2} \cdot \sigma_I^2(D_G, \alpha)\right]^2}{2 \cdot \sigma_I^2(D_G, \alpha)}\right\}, \ u > 0. \quad (14)
\end{aligned}
$$

We plot the *mean bit error rate* $\langle \text{BER} \rangle$ as a function of *mean signal to noise ratio* $\langle \text{SNR} \rangle (dB)$ for several alpha values using Gaussian beam model for scintillation. We take the same parameters

$$
\begin{aligned}
L &= 1 \text{ km}; \ \tilde{C}_n^2 = 7 \cdot 10^{-14} m^{-\alpha+3} \\
\lambda &= 1.55 \ \mu\text{m}; \ D_G = 0.1 \text{ m} \\
W_0 &= 0.01 \text{ m}, \ \Theta_0 = 1.
\end{aligned}
$$

The plot is shown in Fig. 6.

It shows the impact of the alpha variation on $\langle \text{BER} \rangle$ performance. Also in this analysis, when alpha is lower than $\alpha = 11/3$, excluding alpha alpha values close to 3, there is a penalty, but for alpha higher than $\alpha = 11/3$, there is a improvement on the system performance. However, when alpha assumes values close to $\alpha = 3$, there is a gain on the $\langle \text{BER} \rangle$ performance with respect to $\langle \text{BER} \rangle$ value corresponding to $\alpha = 11/3$, because the scintillation approaches zero.

VIII. CONCLUSION

In this paper we introduced a non Kolmogorov power spectrum which uses both a generalized exponent and a generalized amplitude factor instead of a constant standard exponent value $\alpha = 11/3$ and a constant amplitude factor 0.033 associated with the conventional Kolmogorov spectrum. This non-Kolmogorov spectrum has been developed from a generalized structure function. It has been shown, for a Gaussian beam wave propagating along horizontal link, the *long term beam spread, scintillation, probability of fade, mean SNR* and *mean BER* as variations depending on the alpha exponent lead to results somewhat different than those obtained with the standard value of Kolmogorov $\alpha = 11/3$.

It is shown that for alpha values lower than $\alpha = 11/3$, but not for alpha close to $\alpha = 3$, there is a remarkable increase of scintillation and consequently a major penalty on the system performance. However, when alpha assumes a value close to $\alpha = 3$, the amplitude factor $A(\alpha)$ assumes a very low value and consequently the *long term beam spread* and *scintillation* decrease, leading to an improvement on the system performance. Finally, also for higher alpha values than $\alpha = 11/3$ the *scintillation* decreases and consequently it improves system performance.

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