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Identification of a low-order model for thermal stress monitoring

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Abstract

In mechanical and aeronautical applications, assessment of fatigue damage accumulation due to thermal transients is currently performed by means of on-line fatigue monitoring systems.
The algorithms for on-line calculation of thermal stresses are one of the main components of these systems and are often based on the Green's Function Technique (GFT). The GFT allows thermal stresses to be determined from inputs (e.g. fluid temperatures, pressures, flow rates or metal boundary temperatures) by numerically solving a set of convolution integrals. Since each convolution integral involves a large number of operations per time unit, the GFT may become very time consuming for applications characterised by several inputs and outputs.
In this paper a Low-Order Model (LOM) is developed to perform on-line calculation of thermal stresses. The model is able to replace the GFT. Convolution integrals which characterise the GFT are converted in a system of uncoupled first-order differential equations. Unknown coefficients are evaluated by means of the Prony identification method, taken from the structural dynamics and applied to thermo-elasticity.
Two study cases are presented; results show that the LOM leads to a significant reduction in the number of operations per time unit with respect to the GFT, without losing accuracy.

Keywords: reduced model, thermal stress, on-line calculation, Prony method.

1. Introduction

In mechanical and aeronautical applications characterised by high safety requirements and expensive maintenance costs (e.g. aircraft engines [1], nuclear power plants [2-4]) assessment of fatigue damage accumulation due to thermal transients is performed by on-line fatigue monitoring systems. This allows the evaluation in real time of damage accumulation in component critical locations, that is where a fatigue crack is expected to appear.
Fatigue monitoring systems are made of several modules assembled together. Each module is used to evaluate one of the parameters which affect the fatigue damage of the component (i.e. temperature and thermal stresses) and it is based on ad-hoc algorithms, since FE commercial codes are too time consuming for on-line applications.
In the literature (Refs. [2]-[5]) the Green's Function Technique (GFT) is one of the most widespread technique used to perform on-line calculation of thermal stresses. The GFT allows calculating the time histories of thermal stress from the inputs time histories by means of convolution integrals. The only preliminary step necessary to employ the GFT is the calculation of the Green's functions of the system, i.e. the responses of the system to unit step inputs.
The GFT requires a number of operations per time unit (N_{op}) proportional to the decay time t_D of the Green's functions involved in the calculation and to the number of both the inputs (N_{in}) and the outputs (N_{out}) of the system.
As a consequence, in applications characterised by a large number of outputs (i.e stress to monitor) and inputs (i.e. fluid temperatures or heat flows) and by Green's functions with large decay times (which affect the length of vectors involved in the convolution products) the GFT may become very time consuming for on-line monitoring.
In [6] a novel methodology has been proposed to perform on-line calculation of thermal stresses. Convolution integrals which characterise the GFT are replaced by time integration of a Low-Order Model (LOM) made of uncoupled first-order differential equations.
The key idea in developing the LOM was the approximation of any Green's function necessary for thermal stress calculation with a series of exponential terms.
In [6] coefficients of the series (amplitudes and exponents) were calculated by means of a procedure based on the non-linear least square method. Initial guess values and iterative calculations were needed.
In this paper it is proposed to build the LOM for thermal stress monitoring by means of the Prony's identification method [7], taken from the structural dynamics and extended to thermo-mechanics. It allows calculating the coefficients of the LOM through the solution of two linear systems and the calculation of the roots of a real polynomial. Neither initial guess values nor iterative calculations are needed.
In detail, as shown in Fig. 1, the necessary steps are:
1. Application of unit step inputs to a detailed FE thermal model; evaluation of the thermal transients within the whole model; application of the thermal transients to a detailed FE thermo-elastic model, evaluation of the thermal stress transients of the whole model; selection of the Green's functions corresponding to thermal stresses to monitor;

2. Approximation of the Green's functions with a series of exponential terms; identification of the amplitudes and the exponents of the series by means of the Prony method;
3. Use of amplitudes and exponents identified in step 2 to build a set of elementary models of uncoupled first-order differential equations; assembly of the elementary models to form the LOM.
Once the LOM is built, thermal stresses can be obtained by time integration, given the time histories of the inputs.
The resulting reduced model is similar to that described in [8], developed as a tool for feedback control in thermal applications. Both are based on the idea of developing a reduced order model, starting from the approximation of the Green's functions of the system with a series of exponential terms.
In [8] the elementary models assembled to form the complete reduced model are SIMO (Single Input Multi Output) models. As a consequence, linear identification technique cannot be applied and their coefficients are necessarily estimated by means of an iterative identification procedure based on the non-linear least-square method.
On the contrary, in this paper the elementary models, assembled to form the LOM, are SISO (Single Input Single Output) models, because each of them replaces a Green's function. In this case the Prony linear identification method can be successfully used.

2. Development of the low-order model

If the GFT is used for on-line calculation of thermal stress $\sigma(t)$ at any point P(x,y,z) of a thermo-elastic model with I inputs, equation

$$\sigma(t) = \bar{\sigma} + \sum_{i=1}^{I} \int_0^t G_i(t-\tau)\cdot\frac{dF_i(\tau)}{d\tau}d\tau = \sum_{i=1}^{I}\left(\bar{G}_i\cdot\bar{F}_i + \int_0^t G_i(t-\tau)\cdot\frac{dF_i(\tau)}{d\tau}d\tau\right) \tag{1}$$

has to be numerically solved (Ref. [2]), where

$\bar{\sigma}$: initial steady-state value of thermal stress,
$F_i(t)$: i^{th} input at time t,
$\bar{F}_i$: i^{th} input at time t=0,
$G_i(t)$: Green’s function of i^{th} input at time t,
$\bar{G}_i$: Green’s function of i^{th} input at steady-state.

An analogous equation could be written for time derivative $d\sigma(t)/dt$. In this case the time derivative of $G_i(t)$ is the Green's function. The following equation can be obtained

$$\frac{d\sigma}{dt}(t) = \frac{d\sigma}{dt}(0) + \sum_{i=1}^{I}\int_0^t H_i(t-\tau)\cdot\frac{dF_i(\tau)}{d\tau}d\tau, \tag{2}$$

with

$$H_i(t) = \frac{dG_i}{dt}(t) \text{ and } \frac{d\sigma}{dt}(0) = 0 \cdot$$

If the i^{th} Green's function $G_i(t)$ is written as a sum of exponential terms

$$G_i(t) = \sum_{j=1}^{J_i} g_{ij}(t) = \sum_{j=1}^{J_i}\eta_{ij}\cdot(1-e^{\lambda_{ij}\cdot t}), \tag{3}$$

according to equations (1) and (2) thermal stress and its time derivative are respectively series made of terms $\hat{\sigma}_{ij}(t)$ and $d\,\hat{\sigma}_{ij}(t)/dt$ having the form

$$\hat{\sigma}_{ij}(t) = \eta_{ij}\cdot\bar{F}_i + \int_0^t \eta_{ij}\cdot(1-e^{\lambda_{ij}\cdot(t-\tau)})\cdot\frac{dF_i(\tau)}{d\tau}d\tau, \tag{4}$$

and

$$\frac{d\hat{\sigma}_{ij}}{dt}(t) = -\int_0^t \eta_{ij}\cdot\lambda_{ij}\cdot e^{\lambda_{ij}\cdot(t-\tau)}\cdot\frac{dF_i(\tau)}{d\tau}d\tau \tag{5}$$

respectively. Each term of equations (4) and (5) represents the contribution of the j^{th} exponential term to the response due to the time history of the i^{th} input.
Before continuing it is worth stating that the hypothesis made at equation (3) is reasonable. In fact, Green’s functions of thermal stress at any point P of a FE thermo-elastic model are characterised by a transient which leads to a steady-state asymptotic value. The analytical expression of any Green's function is the sum of a constant term (steady-state value) and a series of exponential terms. The exponents of the series are the thermal eigenvalues of the thermal model. The

amplitude of any exponential term is the contribution of the corresponding thermal eigenvector to thermal stress at that point.
As a consequence, the choice of exponential terms employed in equation (3) to approximate the Green's functions, is fully justified from a physical point of view.
Now equation (4) has to be managed in order to obtain an expression comparable to equation (5). In detail, the integral in equation (4) can be managed in the following way:

$$\begin{aligned}&\int_0^t \eta_{ij}\cdot\left(1-e^{\lambda_{ij}\cdot(t-\tau)}\right)\cdot\frac{dF_i(\tau)}{d\tau}d\tau=\\&=\int_0^t \eta_{ij}\cdot\frac{dF_i(\tau)}{d\tau}d\tau-\eta_{ij}\cdot\int_0^t e^{\lambda_{ij}\cdot(t-\tau)}\cdot\frac{dF_i(\tau)}{d\tau}d\tau=\\&=\eta_{ij}\left(F_i(t)-F_i(0)\right)-\eta_{ij}\cdot\int_0^t e^{\lambda_{ij}\cdot(t-\tau)}\cdot\frac{dF_i(\tau)}{d\tau}d\tau\end{aligned}, \tag{6}$$

so that equation (4) becomes

$$\hat{\sigma}_{ij}(t)=\eta_{ij}\cdot F_i(t)-\int_0^t \eta_{ij}\cdot e^{\lambda_{ij}\cdot(t-\tau)}\cdot\frac{dF_i(\tau)}{d\tau}d\tau\cdot \tag{7}$$

If equation (5) is compared to equation (7) the following relationship can be written

$$\frac{d\hat{\sigma}_{ij}}{dt}(t)=\lambda_{ij}\cdot\sigma_{ij}(t)-\lambda_{ij}\cdot\eta_{ij}\cdot F_i(t)\cdot \tag{8}$$

Equation (8) represents the core of the LOM and is valid for any term of the series of equation (3). As a consequence, if the i^{th} Green's function $G_i(t)$ is written as a series of J_i exponential terms, the elementary model

$$\dot{\hat{\boldsymbol{\sigma}}}_{\mathbf{i}}=\boldsymbol{\alpha}_{\mathbf{i}}\cdot\boldsymbol{\sigma}_{\mathbf{i}}+\boldsymbol{\beta}_{\mathbf{i}}\cdot F_i, \tag{9}$$

made of J_i uncoupled first-order differential equations follows, where

$\hat{\boldsymbol{\sigma}}_{\mathbf{i}}$: column vector having $\tilde{\sigma}_{ij}(t)$ at the j^{th} position;

$\dot{\hat{\boldsymbol{\sigma}}}_{\mathbf{i}}$: column vector having $d\tilde{\sigma}_{ij}(t)/dt$ at the j^{th} position;

$\boldsymbol{\alpha}_{\mathbf{i}}$: $(J_i \times J_i)$ diagonal matrix having λ_{ij} at the j^{th} position;

$\boldsymbol{\beta}_{\mathbf{i}}$: $(J_i \times 1)$ column vector having the j^{th} term equal to $-\lambda_{ij}\cdot\eta_{ij}$.

If the above procedure (equations (3)-(9)) is extended to a model with a number I of inputs, I elementary models similar to that of equation (9) can be written. When they are assembled together they form the LOM

$$\begin{cases}\dot{\hat{\boldsymbol{\sigma}}}=\mathbf{A}\cdot\hat{\boldsymbol{\sigma}}+\mathbf{B}\cdot\mathbf{F}\\ \sigma=\mathbf{1}\cdot\hat{\boldsymbol{\sigma}}\end{cases}, \tag{10}$$

with

$$\hat{\boldsymbol{\sigma}}=\begin{Bmatrix}\hat{\boldsymbol{\sigma}}_{\mathbf{1}}\\ \hat{\boldsymbol{\sigma}}_{\mathbf{2}}\\ \vdots\\ \hat{\boldsymbol{\sigma}}_{\mathbf{I}}\end{Bmatrix};\quad \mathbf{A}=\begin{bmatrix}\boldsymbol{\alpha}_{\mathbf{1}}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\boldsymbol{\alpha}_{\mathbf{2}}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\ddots&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\boldsymbol{\alpha}_{\mathbf{I}}\end{bmatrix};\quad \mathbf{B}=\begin{bmatrix}\boldsymbol{\beta}_{\mathbf{1}}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\boldsymbol{\beta}_{\mathbf{2}}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\ddots&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\boldsymbol{\beta}_{\mathbf{I}}\end{bmatrix};\quad \mathbf{F}=\begin{Bmatrix}F_1\\ F_2\\ \vdots\\ F_I\end{Bmatrix};\quad \mathbf{1}=\begin{Bmatrix}1\\ 1\\ \vdots\\ 1\end{Bmatrix}^T. \tag{11}$$

The second equation of the system (10) means that total thermal stress is the sum of all the terms $\sigma_{ij}(t)$.
If relationship

$$\boldsymbol{\sigma}'=\hat{\boldsymbol{\sigma}}+\mathbf{A}^{-1}\cdot\mathbf{B}\cdot\mathbf{F}, \tag{12}$$

is applied to equation (10), the LOM assumes its final shape

$$\begin{cases}\dot{\boldsymbol{\sigma}}'=\mathbf{A}\cdot\boldsymbol{\sigma}'+\mathbf{C}\cdot\dot{\mathbf{F}}\\ \sigma=\mathbf{G}\cdot\mathbf{F}+\mathbf{1}\cdot\boldsymbol{\sigma}'\end{cases} \tag{13}$$

with

$\mathbf{C}=\mathbf{A}^{-1}\cdot\mathbf{B}$

$\overline{\mathbf{G}}=-\mathbf{1}\cdot\mathbf{A}^{-1}\cdot\mathbf{B}$: vector whose i^{th} term is equal to the steady-state value of the i^{th} Green function $G_i(t)$.

3. Identification of the low-order model parameters

According to section 2, in order to build LOM's matrices [A] and [C] of equation (13) it is necessary to approximate any Green's function with a series made of exponential terms. The LOM accuracy in performing the on-line calculation of thermal stress $\sigma(t)$, depends on the accuracy of the exponential series in fitting the Green's functions.

In this paper, the identification of the most suitable series for any Green's function is performed by means of the Prony method (see Appendix), applied to the thermo-mechanical field. It is a method developed at the end of the XVIII century and subsequently modified, currently used (Ref. [7]) for the identification of linear vibrating systems.

The method is simple and straightforward. It requires two linear systems to be solved according to the linear least square method and the roots of a real polynomial to be found. Its application to the current case requires 5 steps.

Step 1: Sampling of the continuous Green's function $G_i(t)$ with a constant sampling time Δt.

Step 2: Definition of function $G'_i(t)$ defined as

$$G'_i(t) = \overline{G}_i - G_i(t) = \sum_{j=1}^{J_i} \eta_{i,j} \cdot e^{\lambda_{i,j} \cdot t}, \tag{14}$$

which represents the variable part of the i^{th} Green's function, as shown in Fig. 2.

Step 3: Evaluation of coefficients χ_j ($0 \le j \le J_i-1$) solving the linear system

$$\begin{bmatrix} h_0 & h_1 & h_2 & \cdots & h_{J_i-1} \\ h_1 & h_2 & h_3 & \cdots & h_{J_i} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ h_{Q-J_i} & h_{Q-(J_i-1)} & h_{Q-(J_i-2)} & \cdots & h_{Q-1} \end{bmatrix} \cdot \begin{Bmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \vdots \\ \chi_{J_i-1} \end{Bmatrix} = - \begin{Bmatrix} h_{J_i} \\ h_{J_i+1} \\ \vdots \\ \vdots \\ h_Q \end{Bmatrix} \tag{15}$$

being $h_q = G'_i(q \cdot \Delta t)$ and Q (> J_i) the total number of samples.

Step 4: Evaluation of the roots V_j of the polynomial

$$\chi_0 + \chi_1 \cdot V_1 + \chi_2 \cdot V_2 + \ldots + \chi_{J_i-1} \cdot V_{J_i-1} + V_{J_i} = 0 \tag{16}$$

and then the exponents λ_{ij} of equation (3) through the relationship

$$V_j = e^{\lambda_{i,j} \cdot \Delta t}. \tag{17}$$

Step 5: Calculation of the amplitudes η_{ij} of equation (3) by solving with the least squares method the linear system:

$$\begin{cases} h_0 = \eta_{i,1} + \eta_{i,2} + \ldots + \eta_{i,J_i} \\ h_1 = V_1 \cdot \eta_{i,1} + V_2 \cdot \eta_{i,2} + \ldots + V_{J_i} \cdot \eta_{i,J_i} \\ h_2 = V_1{}^2 \cdot \eta_{i,1} + V_2{}^2 \cdot \eta_{i,2} + \ldots + V_{J_i}{}^2 \cdot \eta_{i,J_i} \\ \vdots \\ h_Q = V_1{}^Q \cdot \eta_{i,1} + V_2{}^Q \cdot \eta_{i,2} + \ldots + V_{J_i}{}^Q \cdot \eta_{i,J_i} \end{cases}, \tag{18}$$

where $V_j{}^q = e^{\lambda_{i,j} \cdot q \cdot \Delta t}$, adding the following relationship among the η_{ij}:

$$\sum_{j=1}^{J_i} \eta_{i,j} = \overline{G}_i, \tag{19}$$

which forces the approximated Green's functions to match exactly the steady-state values evaluated by FEM. Introduction of equation (19) in equation (18) implies that the first line is identically null and unknowns are reduced by one. The new system to solve is

$$\begin{cases} h_1 - \overline{G}_i \cdot V_{J_i} = (V_1 - V_{J_i}) \cdot \eta_{i,1} + (V_2 - V_{J_i}) \cdot \eta_{i,2} + \ldots + (V_{J_i-1} - V_{J_i}) \cdot \eta_{i,J_i-1} \\ h_2 - \overline{G}_i \cdot V_{J_i}{}^2 = \left(V_1{}^2 - V_{J_i}{}^2\right) \cdot \eta_{i,1} + \left(V_2{}^2 - V_{J_i}{}^2\right) \cdot \eta_{i,2} + \ldots + \left(V_{J_i-1}{}^2 - V_{J_i}{}^2\right) \cdot \eta_{i,J_i-1} \\ \vdots \\ h_Q - \overline{G}_i \cdot V_N{}^Q = \left(V_1{}^Q - V_{J_i}{}^Q\right) \cdot \eta_{i,1} + \left(V_2{}^Q - V_{J_i}{}^Q\right) \cdot \eta_{i,2} + \ldots + \left(V_{J_i-1}{}^Q - V_{J_i}{}^Q\right) \cdot \eta_{i,J_i} \end{cases}, \tag{20}$$

At step 4 of the identification procedure exponents λ_{ij} are evaluated from the roots of a real polynomial; as a consequence, roots can be real (positive or negative) or complex conjugate. Not all roots are acceptable. The physical behaviour of the system implies that admissible exponential terms have real and negative exponents, because they represent transients without any periodic oscillation (no complex numbers) which expires after a certain amount of time (no positive real components). The following criterion is proposed for each exponent:

- real and negative exponent: accepted;
- real and positive exponent: discarded;
- two complex conjugate exponent:
    - Positive real part: discarded;
    - Negative real part: accepted; the exponent is set equal to the real part; the imaginary term is discarded.

4. Application to a thick pipe

In order to test the identification procedure and to show the features and the capabilities of the LOM the case of a thick pipe is discussed. Although it is not one of the applications where the GFT needs to be replaced because of the amount of calculation, it allows a clear comprehension of the methodology proposed in this paper.

The pipe has constant material properties. Two different cases are presented, characterised by different boundary conditions:

1. constant convective coefficients,
2. variable convective coefficients.

Geometry and material properties of the pipe are shown in Fig. 3, where also time histories of fluid temperatures T_{fl1} and T_{fl2} are plotted. The output of on-line calculations is the hoop stress at the inner radius of the pipe with the assumption of perfectly radial temperature gradient.

Time step for on-line calculation of thermal stress is $\Delta\tau = 0.5$ s for both the GFT and the LOM.

4.1 Constant convective coefficients

The thermo-mechanical FE model is linear. Inner and outer convective coefficients are h_i=1200 W/m^2/K and h_o=500 W/m^2/K respectively. Time histories of fluid temperatures can be directly used to evaluate thermal stresses by means of the GFT.

First of all, a detailed FE thermo-mechanical model is used to evaluate the Green's functions $G_{fl1}(t)$ and $G_{fl2}(t)$ (Fig. 4) due to unit step inputs of $T_{fl1}(t)$ and $T_{fl2}(t)$ respectively.

Once the Green's functions are evaluated, the identification of the LOM coefficients described in section 3 is performed. The sampling time Δt is chosen equal to 0.5 s. In this way the stress peak, which characterises the function $G_{fl1}(t)$ at t = 40 s, can be described with a reasonable number of samples.

The identification procedure starts with a series made of 1 exponential term (J_i=1). Then the number of exponential terms J_i is increased till:

$$\varepsilon_G = \max\left(\left|G_i(t) - \sum_{j=1}^{J_i} \eta_{ij} \cdot \left(1 - e^{\lambda_{ij} \cdot t}\right)\right|\right) \leq \frac{1}{100} \cdot \overline{G}_i, \qquad (21)$$

i.e. till the maximum error ε_G is lower than 1% of the steady-state value of the Green's function.

The result of the identification procedure is shown in Tab. 1, where amplitudes and exponents for both the Green's function are plotted. By means of these coefficients the LOM can be built. and then used to evaluate thermal stress on-line.

The LOM is finally used to evaluate on-line thermal stress due to time histories of fluid temperatures plotted in Fig. 3b. Time integration is performed by means of the Euler implicit method.

In Fig. 5 time history of thermal stress evaluated by FEM and both error due to the GFT and the LOM are plotted. The accuracy of the LOM is proved to be very good and even better than the GFT's.

The comparison between the GFT and the LOM involves also the number of operations per time unit.

The LOM requires only 70 operations per time unit, whilst the GFT needs more than 10000 operations to be performed. The LOM allows a 99% reduction with respect to the GFT.

4.2 Variable convective coefficients

In the second case the thick pipe is characterised by variable convective coefficients along the inner and the outer radius of the pipe. Time histories of convective coefficients are listed in Table 2.

For this class of applications it has been already shown in [5] that the GFT is still applicable even if the model is non linear. Time histories of fluid temperatures are to be used to evaluate time histories of model boundary temperatures by time integration of a reduced thermal model. Then boundary temperatures can be used as inputs for thermal stress calculation by means of the GFT.

In fact, since the non-linearity of the model is confined over the boundary of the thermal model, it does not affect thermal stress calculation once time history of boundary temperatures is known.

In this study case, the boundary metal temperatures are the temperatures at the inner and at the outer radius of the pipe, called $T_i(t)$ and $T_o(t)$ respectively. The detailed description of the monitoring methodology necessary to evaluate $T_i(t)$ and $T_o(t)$ is beyond the scopes of this paper. For more details, ref. [5] is recommended. Here time histories of $T_i(t)$ and $T_o(t)$, plotted in Fig. 6 are given as data.
Since the GFT is admissible, a LOM can be derived. In detail the following procedure has been followed:

- Evaluation by means of detailed thermo-mechanical FE model of Green's functions $G_i(t)$ and $G_o(t)$ which represent hoop stress at the inner radius due to unit step input of $T_i(t)$ and $T_o(t)$ respectively (Fig. 7).
- Approximation of $G_i(t)$ and $G_o(t)$ until ε_G <1% (see equation (21) for details about ε_G) using sampling time Δt = 0.1s, getting exponential series listed in Tab. 3.
- LOM assembly and evaluation of thermal stress using $T_i(t)$ and $T_o(t)$ as inputs (Fig. 8).

Also in this study case, the accuracy of the LOM is very good, even if slightly worse than the GFT's. The comparison of the number of operations require by the two methodologies shows that the LOM requires only 52 operations per time unit, whilst the GFT needs 1600 operations to be performed. The LOM allows a 97% reduction with respect to the GFT.

5. Conclusions

A low-order model (LOM) has been proposed to perform on-line calculation of thermal stress in critical locations of fatigue critical components.
The procedure used to identify the unknown parameters of the LOM is based on the Prony method, taken from structural dynamics. The method is linear. Neither iterations nor initial guess values are needed.
On the basis of the proposed study cases the following conclusions are drawn:

1. The Prony method allows one to identify properly the LOM.
2. The LOM can be used in place of the Green's Function Theory (GFT) both in linear models and in models characterised by variable convective coefficients. In the former case fluid temperatures can be used as the LOM inputs, in the latter metal boundary temperatures are to be used.
3. Employment of the LOM for on-line calculations allows:
   - a reduction in the number of operations per time unit up to the 99% with respect to the GFT;
   - evaluation of thermal stress with an accuracy comparable to the GFT.

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Appendix - Prony method

Given a function

$$h(t) = \sum_{r=1}^{N} \eta_r \cdot e^{\lambda_r \cdot t} , \tag{A-1}$$

described with a sampling period Δt, if the following substitutions are performed

$$\begin{aligned} h_q &= h(q \cdot \Delta t) \\ V_r &= e^{\lambda_r \cdot \Delta t} \end{aligned}, \tag{A-2}$$

the n^{th} sample can be written as

$$h_n = \sum_{r=1}^{N} \eta_r \cdot V_r{}^n \cdot \tag{A-3}$$

If the previous relationship is extended to q samples the set of equations

$$\begin{aligned} h_0 &= \eta_1 + \eta_2 + \ldots + \eta_N \\ h_1 &= V_1 \cdot \eta_1 + V_2 \cdot \eta_2 + \ldots + V_N \cdot \eta_N \\ h_2 &= V_1{}^2 \cdot \eta_1 + V_2{}^2 \cdot \eta_2 + \ldots + V_N{}^2 \cdot \eta_N \\ &\vdots \\ h_q &= V_1{}^q \cdot \eta_1 + V_2{}^q \cdot \eta_2 + \ldots + V_N{}^q \cdot \eta_N \end{aligned} \tag{A-4}$$

follows.
If any equation is multiplied by a coefficients χ_i of the equation

$$\chi_0 + \chi_1 \cdot V_1 + \chi_2 \cdot V_2 + \ldots + \chi_q \cdot V_q = 0\,, \tag{A-5}$$

adding all the equations gives

$$\sum_{i=0}^{q} \chi_i \cdot h_i = \sum_{j=1}^{N} \left(\eta_j \sum_{i=0}^{q} \chi_i \cdot V_j{}^i \right) \cdot \tag{A-6}$$

If equation (A-5) is valid, then any term on the right hand side of equation (A-6) is zero and therefore it is

$$\sum_{i=0}^{q} \chi_i \cdot h_i = 0 \tag{A-7}$$

If q is set equal to N and χ_N is set equal to unity, the set of equations

$$\begin{bmatrix} h_0 & h_1 & h_2 & \cdots & h_{N-1} \\ h_1 & h_2 & h_3 & \cdots & h_N \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ h_{Q-N} & h_{Q-(N-1)} & h_{Q-(N-2)} & \cdots & h_{Q-1} \end{bmatrix} \cdot \begin{Bmatrix} \chi_0 \\ \chi_1 \\ \vdots \\ \vdots \\ \chi_{N-1} \end{Bmatrix} = -\begin{Bmatrix} h_N \\ h_{N+1} \\ \vdots \\ \vdots \\ h_Q \end{Bmatrix}, \tag{A-8}$$

can be written, where Q is the total number of available samples.
The unknown coefficients χ_i can be obtained solving the linear system with the least squares method.
Roots of equation (A-5) can be then evaluated and exponents λ_r can be obtained by means of relationship

$$V_r = e^{\lambda_r \cdot \Delta t}\,.$$

The solution can be completed by deriving amplitudes η_r solving with the least square method the system (A-4) with q $\in$ [N,Q].

Figures & Tables

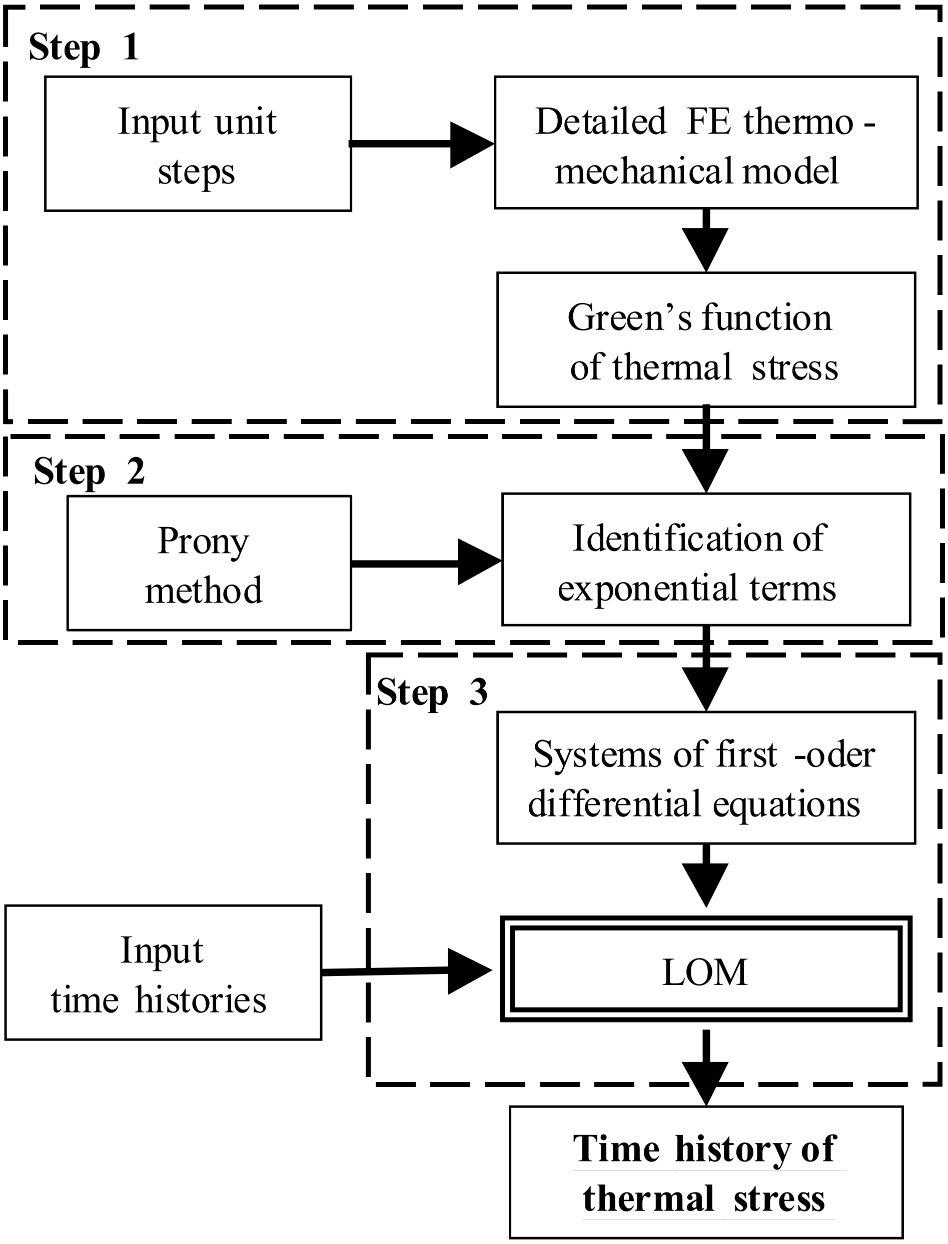


Fig. 1 - Flow-chart of the LOM development and employment.

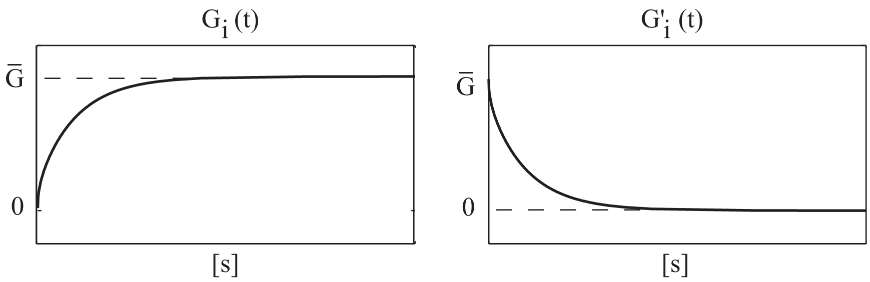
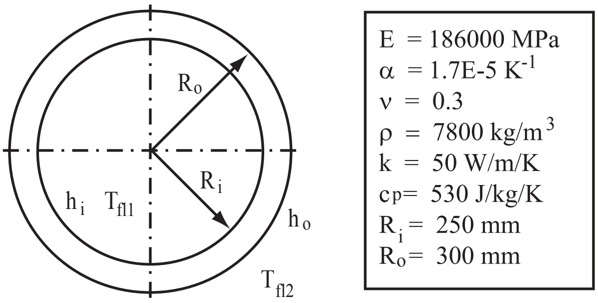
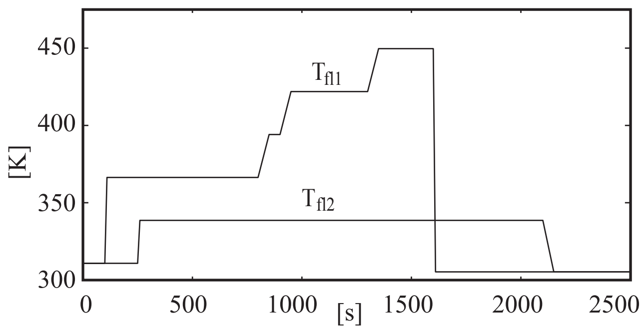


Fig. 2 - A Green's function for thermal stress (a) and its variable part (b).



a) Geometry and material properties



b) Time histories of fluid temperatures

Fig. 3 - Thick pipe data (a) and time history of fluid temperatures (b).

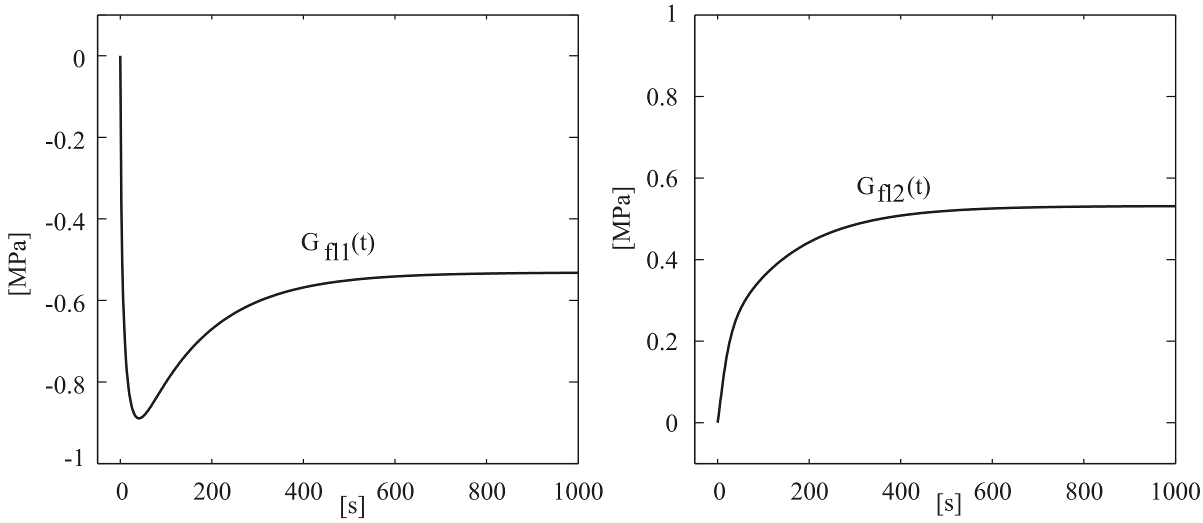


Fig. 4 - Study case 1: Green's functions due to the inner and the outer fluid temperature.

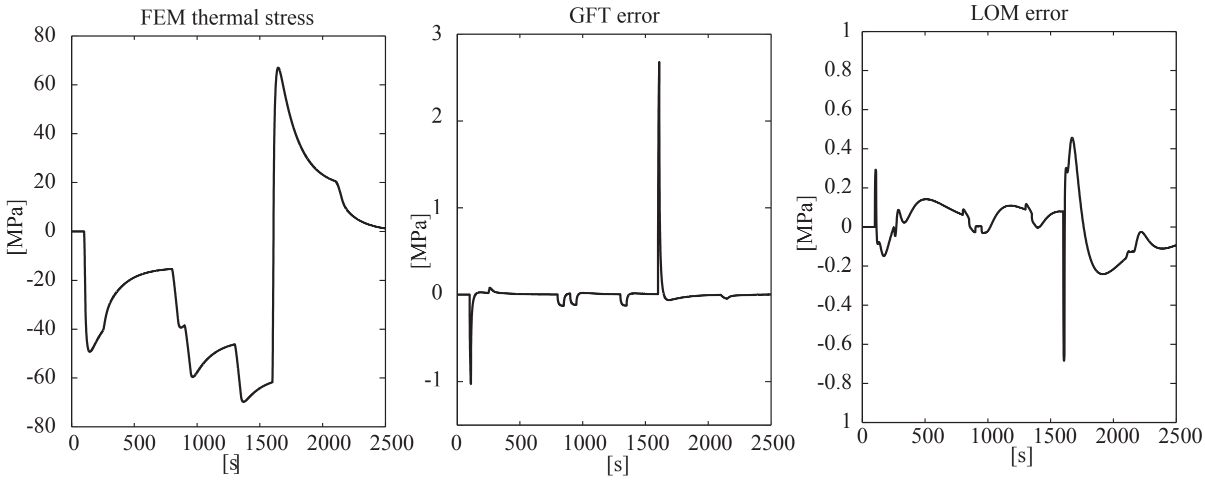


Fig. 5 - Study case 1: FEM results and accuracy of both the GFT and the LOM.

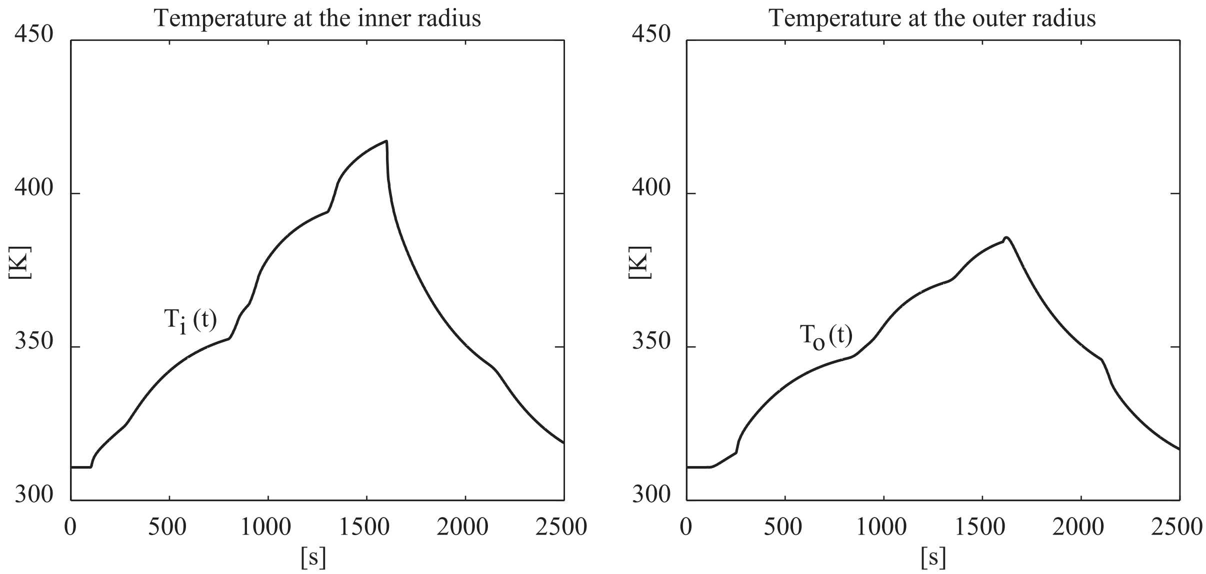


Fig. 6 - Study case 2: Time histories of metal temperatures at the inner and at the outer radius.

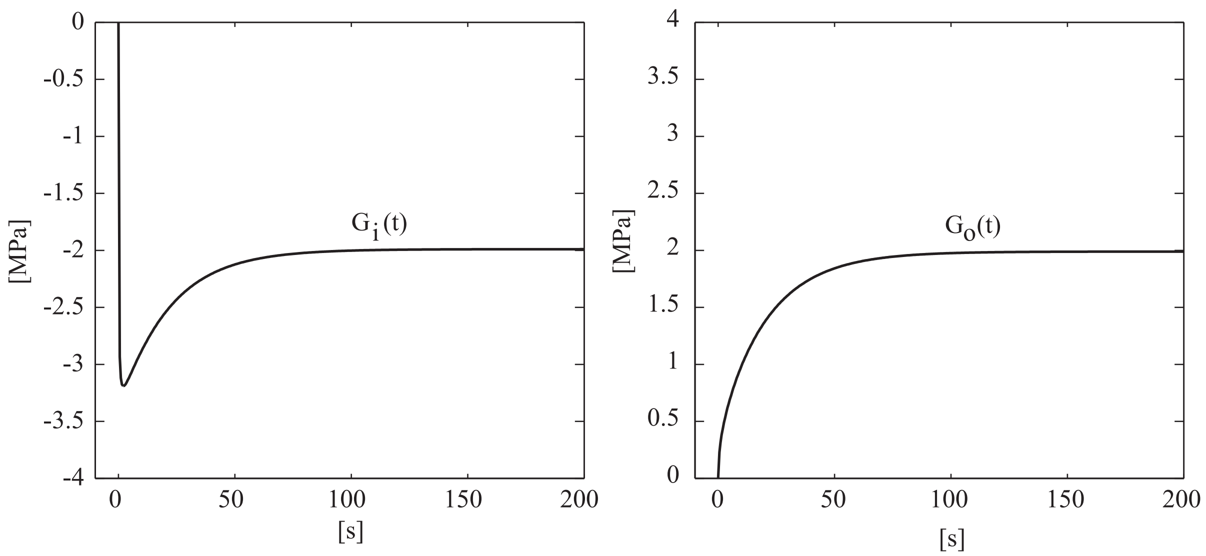


Fig. 7 - Study case 2: Green's functions due to metal temperatures at the inner and at the outer radius.

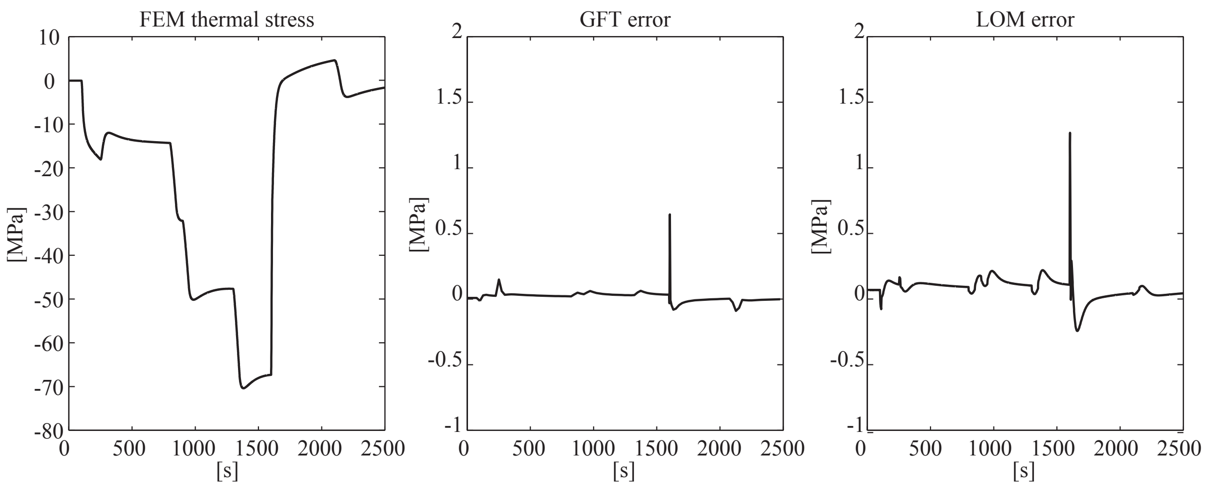


Fig. 8 - Study case 2: FEM results and accuracy of both the GFT and the LOM.

Tab. 1 - Study case 1: amplitudes and exponents of the exponential series fitting the Green's functions.

G_{fl1}			G_{fl2}		
Number of terms	Values of λ [Hz]	Values of η [MPa]	Number of terms	Values of λ [Hz]	Values of η [MPa]
6	-0.4581 -0.0065 -0.0683 -0.2851 -0.8798 -2.7393	-0.0002 0.5124 -0.5830 -0.2513 -0.1277 -0.0818	5	-0.1489 -0.7328 -0.0063 -0.0489 -0.1165	-0.1059 -0.0005 0.3193 0.2203 0.0985

Tab. 2 - Study case 2: convective coefficient variation.

time [s]	0	1600	1610	2500
h_i [W/m^2/K]	150	1200	150	150
h_o [W/m^2/K]	500	600	500	500

Tab. 3 - Study case 2: amplitudes and exponents of the exponential series fitting the Green's functions.

Green's function due to T_i			Green's function due to T_o		
Number of terms	Values of λ [Hz]	Values of η [MPa]	Number of terms	Values of λ [Hz]	Values of η [MPa]
4	-0.044 -1.36 -4.62 -76.3	1.360 -0.645 -0.605 -2.099	4	-0.049 -0.65 -3.38 -69.0	1.653 0.193 0.046 0.098