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A Passivity Enforcement Scheme for Delay-Based Transmission Line Macromodels

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***Abstract*—This letter presents an algorithm for the enforcement of passivity in transmission-line macromodels based on the generalized Method of Characteristics. The algorithm is based on first-order perturbations of the solutions to a frequency-dependent eigenvalue problem. The theoretical formulation provides an extension of techniques that were available only for lumped models to the more general class of delay-based models.**

***Index Terms*—Eigenvalues, Hamiltonian matrices, passivity, perturbation, transmission lines.**

I. Introduction

THE automated design of highly interconnected systems requires accurate and efficient models for all components. This applies even to the simplest structures such as uniform transmission lines, since all spurious effects that have an influence on the signals must be considered in the models. Most of these effects are natively described in frequency domain, leading to frequency-dependent parameters. The conversion from frequency to time domain descriptions for transient analysis using standard circuit solvers such as SPICE has been a subject of intense research over the last few decades. Nonetheless, several open problems remain.

This letter deals with one of these problems. Namely, the preservation of passivity during the derivation of a SPICE-compatible transmission line model. This is a fundamental physical property, requiring that no energy can be generated from any passive structure. However, this property may be lost during the model manipulation and approximation steps required for the conversion. It is well-known that nonpassive models are unreliable, since they may lead to exponential instability in a transient simulation, depending on their terminations. We concentrate here on models based on the so-called Method of Characteristics (MoC), since it has been demonstrated that such models are the most efficient for lines characterized by a significant propagation delay. Preservation of passivity for such models is still an open issue.

Significant advancements have been recently achieved in [1] and [2]. In these papers, the authors present a systematic procedure for checking the passivity of MoC-based transmission line models. Here, we start from their formulation and we present a perturbation approach that is able to enforce model passivity once some passivity violations have been detected. The basics of MoC formulation are first reviewed in Section II, and the passivity check of [1], [2] is outlined in Section III in order to set the notations. The proposed perturbation scheme is presented in Section IV together with an application example.

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II. MoC Macromodels

We consider a multiconductor transmission line of length $\mathcal{L}$ governed by the telegrapher equations, here, stated in the Laplace-domain

$$\begin{aligned} -\frac{\mathrm{d}}{\mathrm{dz}}\boldsymbol{V}(z,s) &= \boldsymbol{Z}(s)\boldsymbol{I}(z,s) \\ -\frac{\mathrm{d}}{\mathrm{dz}}\boldsymbol{I}(z,s) &= \boldsymbol{Y}(s)\boldsymbol{V}(z,s) \end{aligned} \tag{1}$$

where z represents the longitudinal coordinate along which signals propagate. The matrices $\boldsymbol{Z}(s)$ and $\boldsymbol{Y}(s)$ denote the f-PUL (frequency dependent per-unit-length) impedance and admittance parameters, respectively. Following the MoC approach, the solution of telegrapher equations is obtained as [3]

$$\begin{aligned} \boldsymbol{I}_1(s) &= \boldsymbol{Y}_c(s)\boldsymbol{V}_1(s) - \boldsymbol{Q}(s)\left[\boldsymbol{Y}_c(s)\boldsymbol{V}_2(s) + \boldsymbol{I}_2(s)\right] \\ \boldsymbol{I}_2(s) &= \boldsymbol{Y}_c(s)\boldsymbol{V}_2(s) - \boldsymbol{Q}(s)\left[\boldsymbol{Y}_c(s)\boldsymbol{V}_1(s) + \boldsymbol{I}_1(s)\right] \end{aligned} \tag{2}$$

where $\boldsymbol{V}_{1,2}(s)$ and $\boldsymbol{I}_{1,2}(s)$ represent the terminal voltages and currents of the line and where $\boldsymbol{Q}(s) = \exp\{-\boldsymbol{\Gamma}(s)\mathcal{L}\}$, $\boldsymbol{\Gamma}^2(s) = \boldsymbol{Y}(s)\boldsymbol{Z}(s)$, and $\boldsymbol{Y}_c(s) = \boldsymbol{\Gamma}^{-1}(s)\boldsymbol{Y}(s)$. A SPICE-compatible stamp is derived from (2) by extracting the asymptotic modal delays $\boldsymbol{T} = \mathrm{diag}\{T_k\}$ from the propagation operator $\boldsymbol{Q}(s)$

$$\boldsymbol{P}(s) = e^{-s\boldsymbol{T}}\boldsymbol{M}^{-1}\boldsymbol{Q}(s)\boldsymbol{M} \tag{3}$$

using the asymptotic modal decomposition matrix $\boldsymbol{M}$, and by approximating the remaining matrix operators $\boldsymbol{Y}_c(s)$, $\boldsymbol{P}(s)$ with low-order rational functions $\tilde{\boldsymbol{Y}}_c(s)$, $\tilde{\boldsymbol{P}}(s)$, respectively. The well-known Vector Fitting algorithm [4] can be used for this task, leading to a state-space realization for $\tilde{\boldsymbol{Y}}_c(s)$

$$\tilde{\boldsymbol{Y}}_c(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D} \tag{4}$$

and similarly for $\tilde{\boldsymbol{P}}(s)$. Note that the model poles are the eigenvalues of $\boldsymbol{A}$, whereas the corresponding residues are stored in matrix $\boldsymbol{C}$.

III. Passivity Characterization

Following [1], [2], the passivity of the line MoC macromodel is here characterized using the short-circuit admittance matrix $\mathcal{Y}(s)$. The latter is readily obtained from (2) and (3) and reads

$$\mathcal{Y}(s) = \begin{bmatrix} \boldsymbol{W}_0^{-1}(s)\boldsymbol{W}_1(s)\tilde{\boldsymbol{Y}}_c(s) & \boldsymbol{W}_0^{-1}(s)\boldsymbol{W}_2(s)\tilde{\boldsymbol{Y}}_c(s) \\ \boldsymbol{W}_0^{-1}(s)\boldsymbol{W}_2(s)\tilde{\boldsymbol{Y}}_c(s) & \boldsymbol{W}_0^{-1}(s)\boldsymbol{W}_1(s)\tilde{\boldsymbol{Y}}_c(s) \end{bmatrix} \tag{5}$$

where

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$$\begin{aligned}\boldsymbol{W}_0 &= \boldsymbol{I} - \boldsymbol{M}\mathrm{e}^{-s\boldsymbol{T}}\tilde{\boldsymbol{P}}(s)\mathrm{e}^{-s\boldsymbol{T}}\tilde{\boldsymbol{P}}(s)\boldsymbol{M}^{-1}\\ \boldsymbol{W}_1 &= \boldsymbol{I} + \boldsymbol{M}\mathrm{e}^{-s\boldsymbol{T}}\tilde{\boldsymbol{P}}(s)\mathrm{e}^{-s\boldsymbol{T}}\tilde{\boldsymbol{P}}(s)\boldsymbol{M}^{-1}\\ \boldsymbol{W}_2 &= -2\boldsymbol{M}\mathrm{e}^{-s\boldsymbol{T}}\tilde{\boldsymbol{P}}(s)\boldsymbol{M}^{-1}\end{aligned} \tag{6}$$

with $\boldsymbol{I}$ denoting the identity matrix. As in [1], [2], we concentrate on the scalar case (two conductors transmission line) only, for which

$$\mathcal{Y}(s) = \begin{bmatrix} \frac{1+e^{-2sT}\tilde{P}^2(s)}{1-e^{-2sT}\tilde{P}^2(s)}\tilde{Y}_c(s) & \frac{-2e^{-sT}\tilde{P}(s)}{1-e^{-2sT}\tilde{P}^2(s)}\tilde{Y}_c(s) \\ \frac{-2e^{-sT}\tilde{P}(s)}{1-e^{-2sT}\tilde{P}^2(s)}\tilde{Y}_c(s) & \frac{1+e^{-2sT}\tilde{P}^2(s)}{1-e^{-2sT}\tilde{P}^2(s)}\tilde{Y}_c(s) \end{bmatrix}. \tag{7}$$

A multiport described by the admittance matrix $\mathcal{Y}(s)$ is passive if and only if $\mathcal{Y}(s)$ is positive real [5]. If $\tilde{Y}_c(s)$, $\tilde{P}(s)$, and $\mathcal{Y}(s)$ are asymptotically stable (do not have poles in the right-half plane of the s-domain) with $\tilde{P}(s) \to 0$ for $s \to \infty$, the positive realness of $\mathcal{Y}(s)$ is equivalent to the condition

$$2\mathcal{G}(s) = \mathcal{Y}^T(-s) + \mathcal{Y}(s) \geq 0, \quad \forall s = j\omega \tag{8}$$

which can be verified by checking that all eigenvalues of $\mathcal{G}(j\omega)$ are nonnegative throughout the frequency axis. It turns out that these eigenvalues correspond to the real parts (for $s = j\omega$) of the eigenvalues of $\mathcal{Y}(s)$, which can be easily calculated as

$$\lambda_{1,2}(s) = \frac{1 \pm e^{-sT}\tilde{P}(s)}{1 \mp e^{-sT}\tilde{P}(s)}\tilde{Y}_c(s). \tag{9}$$

The elegant theory in [1], [2] provides purely algebraic criteria for checking (8) that do not require any sampling the frequency axis, which is intrinsically problematic. This formulation involves restating $\lambda_{1,2}(s)$ in time-domain as algebraic delay-differential equations (ADDE), for which a delayed state-space realization is readily obtained from (9) using inverse Laplace transform [2]. Then, the passivity of the MoC macromodel is guaranteed when there are no purely imaginary values for s that satisfy the following frequency-dependent eigenvalue problems (FD–EP)

$$s\boldsymbol{\xi} = \boldsymbol{H}_{1,2}(s)\boldsymbol{\xi} \tag{10}$$

where

$$\boldsymbol{H}_{1,2}(s) = \boldsymbol{\mathcal{V}}_{1,2} + \boldsymbol{\mathcal{W}}^-_{1,2}e^{-sT} + \boldsymbol{\mathcal{W}}^+_{1,2}e^{sT}. \tag{11}$$

The constant matrices $\boldsymbol{\mathcal{V}}_{1,2}$ and $\boldsymbol{\mathcal{W}}^{\pm}_{1,2}$ are easily constructed following a tedious but simple algebraic manipulation, using as building blocks the state-space matrices defining the ADDEs above. We remark that, as a result, both matrices $\boldsymbol{H}_{1,2}(s)$ are a quadratic function of the state-space matrix $\boldsymbol{C}$ of (4) for any fixed frequency s. Note that the frequency-dependent matrices $\boldsymbol{H}_{1,2}(s)$ provide a generalization of the concept of Hamiltonian matrices [6] to the ADDE case. Hence, the purely imaginary eigenvalues of FD-EPs above, if any, correspond to those frequencies at which the eigenvalues of $\mathcal{G}(j\omega)$ change sign (Fig. 1). If no such solutions are found, the eigenvalues remain positive at all frequencies and the model is passive.

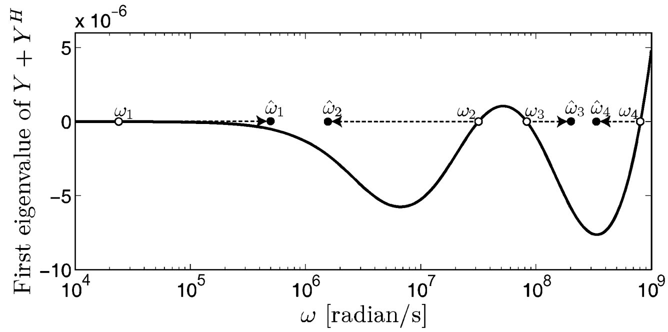

Fig. 1. Modeling a $\mathcal{L} = 10$ cm microstrip ($w = 0.007''$ and $t = 0.0014''$) over a $h = 1/16''$ FR4 substrate with permittivity $\varepsilon_r = 4.7$. Solutions ω_k of the FD–EP and their perturbation $\hat{\omega}_k$. The solid line depicts $\lambda_1(j\omega)$ as in (9).

IV. Passivity Enforcement

Once the above procedure for passivity characterization is applied to a given MoC model, and the model is found to be nonpassive, some correction must be applied in order to enforce its passivity. Let us consider the example in Fig. 1. In this case, four frequencies ω_k are found as solutions of the FD–EP in (10), leading to two separate frequency bands where passivity violations occur. In the following sections, we describe a procedure that allows to perturb matrix $\boldsymbol{C}$ in (4) (i.e., the residues of a partial fraction expansion of $\tilde{\boldsymbol{Y}}_c$) so that the nonpassive bands are eliminated. This is accomplished by a first-order perturbation of the frequencies ω_k. In fact, this is the main contribution of this work, namely an extension of the perturbation scheme of [6], which is applicable to lumped models only, to the delayed transmission-line case. Section IV-A provides a general result on the perturbation of nonlinear eigenvalue problems such as the FD–EP in (10). Section IV-B applies this result for the MoC passivity enforcement.

A. Perturbation of Eigenvalues

Let us consider the FD–EP

$$\boldsymbol{H}(s,\varepsilon)\boldsymbol{\xi}(\varepsilon) = s\boldsymbol{\xi}(\varepsilon) \tag{12}$$

where the system matrix depends on an additional parameter $\varepsilon \simeq 0$. We denote a generic eigensolution of (12) as $\{s(\varepsilon), \boldsymbol{\xi}(\varepsilon), \boldsymbol{\zeta}(\varepsilon)\}$, where $\boldsymbol{\xi}$ and $\boldsymbol{\zeta}$ are the right and left eigenvectors associated to the eigenvalue s, in order to highlight its dependence on ε. Also, we denote the reference eigensolution for $\varepsilon = 0$ as

$$\boldsymbol{H}(s_0,0)\boldsymbol{\xi}_0 = s_0\boldsymbol{\xi}_0, \quad \boldsymbol{\zeta}_0^H\boldsymbol{H}(s_0,0) = s_0\boldsymbol{\zeta}_0^H \tag{13}$$

where ^{H} denotes the conjugate transpose. Differentiating (12) with respect to ε and setting $\varepsilon = 0$ in the result, we obtain

$$\left(\boldsymbol{I} - \boldsymbol{H}_0^{(s)}\right)s_0'\boldsymbol{\xi}_0 = \boldsymbol{H}_0^{(\varepsilon)}\boldsymbol{\xi}_0 + \left(\boldsymbol{H}(s_0,0) - s_0\boldsymbol{I}\right)\boldsymbol{\xi}_0' \tag{14}$$

where

$$\boldsymbol{H}_0^{(s)} = \left.\frac{\partial\boldsymbol{H}}{\partial s}\right|_{\substack{s=s_0\\ \varepsilon=0}} \qquad \boldsymbol{H}_0^{(\varepsilon)} = \left.\frac{\partial\boldsymbol{H}}{\partial \varepsilon}\right|_{\substack{s=s_0\\ \varepsilon=0}} \tag{15}$$

and where s_0' and $\boldsymbol{\xi}_0'$ are the first-order perturbation coefficients of eigenvalue and eigenvector, respectively. Premultiplying now by the left eigenvector $\boldsymbol{\zeta}_0^H$ and using (13), we have

$$s_0' = \frac{\boldsymbol{\zeta}_0^H \boldsymbol{H}_0^{(\varepsilon)} \boldsymbol{\xi}_0}{\boldsymbol{\zeta}_0^H \left(\boldsymbol{I} - \boldsymbol{H}_0^{(s)}\right) \boldsymbol{\xi}_0} \tag{16}$$

which establishes a linear relation between the first-order perturbation coefficients of system matrix $\boldsymbol{H}(s,\varepsilon)$ and the corresponding eigenvalue $s(\varepsilon) \simeq s_0 + s_0'\varepsilon$. Of course, in case of a regular frequency-independent eigenvalue problem, we have $\boldsymbol{H}_0^{(s)} = 0$ and standard perturbation results are obtained [6], [7].

B. MoC Passivity Enforcement

Equation (16) enables the derivation of a passivity enforcement scheme similar to [6]. Each imaginary eigensolution $s_k = j\omega_k$ is displaced to a target location $\hat{s}_k = j\hat{\omega}_k$ inwards into the violation bandwidth (see Fig. 1). This is obtained by computing a new state-space matrix

$$\widehat{\boldsymbol{C}} = \boldsymbol{C} + \boldsymbol{\Delta} \tag{17}$$

such that the first-order perturbation induced in the system matrix $\boldsymbol{H}(s,\boldsymbol{\Delta})$ has the desired perturbed eigensolution. After some straightforward manipulation of (16) we obtain

$$2\Re\left\{\boldsymbol{z}_k^H \boldsymbol{\Delta} \boldsymbol{x}_k\right\} = (\hat{\omega}_k - \omega_k)\Im\left\{\boldsymbol{\zeta}_k^H \left(\boldsymbol{I} - \boldsymbol{H}_k^{(s)}\right)\boldsymbol{\xi}_k\right\} \tag{18}$$

to be enforced $\forall k$, where the complex vectors $\boldsymbol{z}_k$, $\boldsymbol{x}_k$ are easily derived using a first-order expansion of $\boldsymbol{H}(s,\boldsymbol{\Delta})$ in terms of $\boldsymbol{\Delta}$. The final (linear) system to be solved for eigenvalue displacement is obtained via the following equivalence

$$2\Re\left\{\boldsymbol{z}_k^H \boldsymbol{\Delta} \boldsymbol{x}_k\right\} = 2\Re\left\{\boldsymbol{x}_k^T \otimes \boldsymbol{z}_k^H\right\}\mathrm{vec}(\boldsymbol{\Delta}) \quad \forall k \tag{19}$$

where $\otimes$ is the Kronecker product [8] and the operator $\mathrm{vec}(\cdot)$ stacks the columns of its matrix argument. Passivity of the MoC model is enforced by iterative solution of (18) and (19).

C. Example

We apply the proposed methodology for the generation of a passive macromodel of the microstrip line of Fig. 1, which was characterized by four imaginary eigenvalues and two frequency bands with passivity violations. The passivity compensation algorithm of Section IV-B was applied in order to eliminate these violations. Fig. 1 provides a schematic view of the compensation process, by highlighting the perturbation that is applied in order to displace the imaginary eigenvalues. The final result after only one iteration of (18) and (19) is a passive macromodel, as depicted in Fig. 2, without any imaginary eigenvalues left. Fig. 3 shows a comparison between the Y_{11} elements of the original nonpassive and perturbed passive macromodels. As expected, the difference is almost unnoticeable, confirming that the accuracy is preserved during the passivity enforcement process.

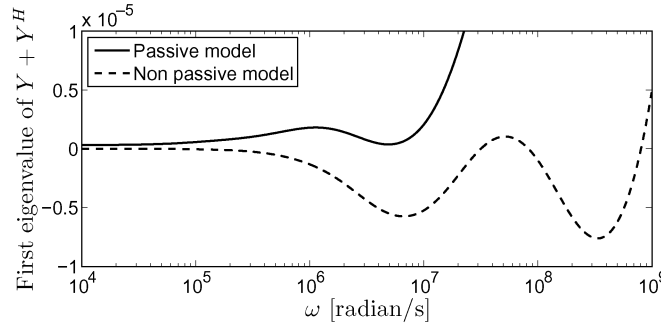

Fig. 2. Eigenvalue $\lambda_1(j\omega)$ of original and perturbed model. The perturbed model is passive because the eigenvalue is positive for all values of ω.

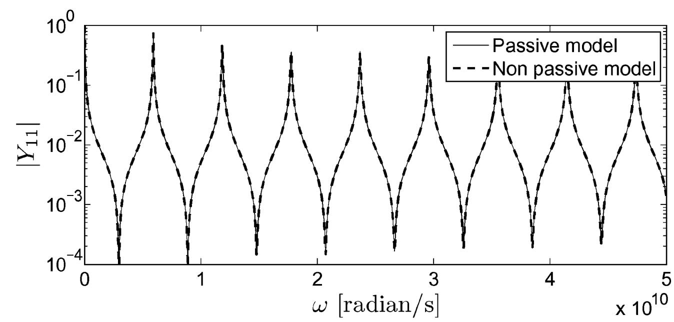

Fig. 3. Comparison of input short-circuit admittances of original and perturbed model.

V. Conclusion

We have presented a new algorithm for the passivity enforcement in transmission-line macromodels based on the generalized MoC. The proposed technique is able to perturb the model coefficients until passivity is achieved, with control over the model accuracy. The main algorithm extends to delay-extraction based macromodels existing methodologies that were available only for lumped macromodels. Only preliminary results on single transmission lines were presented here. Extension to multiconductor lines will be the subject of a forthcoming report.

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