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Qualitative Ordinal Scales: The Concept of Ordinal Range

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ABSTRACT

Many practical problems of quality control involve the use of ordinal scales. Questionnaires planned to collect judgments on qualitative or linguistic scales, whose levels are terms such as "good," "bad," "medium," etc., are extensively used both in evaluating service quality and in visual controls for manufacturing industry. In an ordinal environment, the concept of distance between two generic levels of the same scale is not defined. Therefore, a population (universe) of judgments cannot be described using "traditional" statistical distributions since they are based on the notion of distance. The concept of "distribution shape" cannot be defined as well. In this article, we introduce a new statistical entity, the so-called ordinal distribution, to describe a population of judgments expressed on an ordinal scale. We also discuss which of the traditional location and dispersion measures can be used in this context and we briefly analyze some of their properties. A new dispersion measure, the ordinal range, as an extension of the cardinal range to ordinal scales, is then proposed. A practical application in the field of quality is developed throughout the article.

Key Words: Quality; Quality measurements; Ordinal scales; Linguistic scales; OWA.

INTRODUCTION

Many practical problems involve the use of a linguistic or qualitative scale in assessing the attributes of products or services. This is the case, for example, when performing visual controls on manufactured products or when assessing the expected or perceived quality of a service.

Typical levels of a linguistic scale are terms such as "good," "bad," or "medium." (Agresti, 1984,

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Table 1. Results of the visual control of a sample of 30 corks.

"Reject"	"Poor quality"	"Medium quality"	"Good quality"	"Excellent quality"
1	2	10	9	8

2002). An example is reported in Franceschini and Romano (1999) for a production line of fine liqueurs. Operators in charge of visual control of the corking and closing process have the following assessment possibilities:

- "reject" if the cork does not work;
- "poor quality" if the cork must not be rejected but has some defects;
- "medium quality" if the cork has relevant aesthetic flaws but no other defects;
- "good quality" if the cork only has small aesthetic flaws;
- "excellent quality" if the cork is perfect.

An example of results of the visual control for a sample of 30 corks is reported in Table 1.

How can we analyze these data? A simple answer to this question is the numerical conversion of verbal information; i.e., the assignment of a numerical value to each level of the ordinal scale. However, this operation introduces into the scale the property of distance between the levels of the scale itself (Franceschini and Rossetto, 1995).

Let us assume, for example, the following codification:

- "reject" = 1;
- "poor quality" = 2;
- "medium quality" = 3;
- "good quality" = 4;
- "excellent quality" = 5.

This codification allows us to calculate all location and dispersion measures of the sample; for example, its arithmetic mean is: $\bar{x} = 3.7$.

This result seems to suggest that the mean of the sample is between "medium quality" and "good quality" and that it is nearer to the latter than to the former.

The numerical conversion we have adopted is based on the implicit assumption that, in the evaluator's mind, all scale levels are equispaced. However, we are not sure that the evaluator perceives the subsequent levels of the scale as equispaced, nor even if he or she has been preliminarily trained. For example, the evaluator might perceive the upper levels as more distinguished from the others. The suitable codification of the levels of the scale for this inspector might be the following (Roberts, 1979):

- "reject" = 1;
- "poor quality" = 3;
- "medium quality" = 9;
- "good quality" = 27;
- "excellent quality" = 81.

In case this codification were adopted, we would obtain an arithmetic mean equal to $\bar{x} = 32.9$, that is to say that the sample mean is near to "good quality," but between "good quality" and "excellent quality," not between "medium quality" and "good quality."

Which is the right value of the mean of the sample at hand? We cannot answer this question because an "exact" codification does not exist.

A more correct approach, alternative to a numerical conversion of the levels of an ordinal scale, is based on usage of the only properties of ordinal scales themselves. In practice, we do not convert the ordinal scale into a numerical one, but we focus our attention only on the order of levels. That is to say that if an evaluator asserts that the cork is of "good quality," he or she simply says that cork quality is better than "medium quality" but worse than "excellent quality."

In the next sections we analyze the consequences of this approach. Particularly, we point out the "traditional" statistical properties and measures that are still valid on ordinal scales. We also introduce new ones that are specific to ordinal scales.

THE CONCEPT OF ORDINAL DISTRIBUTION

In a framework where the distance between the levels of a scale is not defined, the use of traditional statistical distributions is not correct. A distribution requires the concept of distance to be defined, since its argument is a number on the real axis. Denoting by X a discrete random variable whose possible values belong to the set $\{x_1, x_2, \ldots, x_n\} \subset \mathbb{R}$, its probability distribution $f_X(x)$ can be defined as

(Montgomery and Runger, 1999; Vicario and Levi, 2001):

$$f_X(x) = \begin{cases} \mathbb{P}[X = x_j] & \text{if } x = x_j, \text{with } j = 1, 2, \ldots, n \\ 0 & \text{if } x \neq x_j. \end{cases}$$

In an ordinal environment, instead, the argument of a hypothetical probability distribution is an element of a set of ordered levels. Denoting by S an ordinal random variable whose values belong to the set $\{S_1, S_2, \ldots, S_t\}$, where S_i is the ith level of the ordinal scale and t is the number of levels of the scale, the equivalent of the probability distribution in an ordinal environment can be defined as

$$f_S(S) = \begin{cases} \mathbb{P}[S = S_j] & \text{if } S = S_j, \text{with } j = 1, 2, \ldots, t \\ 0 & \text{if } S \neq S_j. \end{cases}$$

The empirical frequency distribution in an ordinal environment can be obtained in the same way as in a cardinal space. Denoting by n the sample size and by n_i, $i = 1, 2, \ldots, t$, the number of judgments of the sample at level S_i, the relative frequency of S_i can be calculated as

$$p_i = \frac{n_i}{n}.$$

The abscissas of these values; i.e., the levels of the ordinal scale, cannot be fixed on a real axis, since their relative distance is undefined. Therefore, a probability distribution in an ordinal environment, hereinafter called ordinal distribution, is made up by points whose abscissas are "free to move" along their axis provided that they keep their order.

Figure 1 points out the difference between a traditional probability distribution and an ordinal distribution. Figure 1a represents the frequency distribution of data reported in Table 1 with the first numerical codification. Figure 1b shows the frequency distribution obtained with the second numerical codification. Figure 1c illustrates the ordinal distribution of the same data: vertical bars are not fixed at a precise point of the horizontal axis, since their relative distances are undefined. To represent this characteristic, a "skate" symbol is considered.

Figure 1c shows an ordinal distribution with equispaced bars. However, we must remember that the position of these bars on the real axis is undefined: the only information we have is about their order. The same ordinal distribution can be represented in various equivalent forms. For example, Figs. 1c and 2 are equivalent representations of the same ordinal distribution.

A direct consequence of the absence of the concept of distance among the levels of an ordinal scale is the lack of another important concept: the distribution shape. It is not correct to refer to the shape of an ordinal distribution, but only to analyze the heights of its vertical bars (probabilities or relative frequencies). For example, it is not correct to say that the distribution of judgments follows a binomial distribution, because the assumption of a specific distribution requires the fixing of its shape, which in turn requires the introduction of the concept of distance between the levels of the ordinal scale. Nevertheless, it is correct to assert that the heights of the vertical bars of an ordinal distribution are the same as those of a binomial distribution whose variable can assume values corresponding (one to one) to the ordinal distribution levels, since this statement does not require the introduction of the concept of distance.

LOCATION AND DISPERSION MEASURES IN AN ORDINAL ENVIRONMENT

All statistical measures that can be used in an ordinal environment cannot make use of the concept of distance between the levels of the scale.

Location Measures

A location measure for an ordinal environment is the median. Denoting by n the number of sample elements, a_i the ith element of the sample and b_i the ith element of the ordered sample, the sample median $\tilde{x}$ can be defined as (n odd)

$$\tilde{x} = b_k, \quad \text{where } k = \frac{n+1}{2}.$$

If n is even, the median is a couple of values $\tilde{x} = (b_i, b_j)$, with $i = n/2$ and $j = n/2 + 1$.

In the example at hand (Table 1), $n = 30$. Since $b_{15} =$ "good quality" and $b_{16} =$ "good quality," we have

$$\tilde{x} = (b_{15}, b_{16}) = \text{"good quality."}$$

The median is the 50th centile. Of course, all other centiles can be defined in the same way.

Another "traditional" location measure usable in an ordinal environment is the mode, which is the value of the scale with the maximum probability.

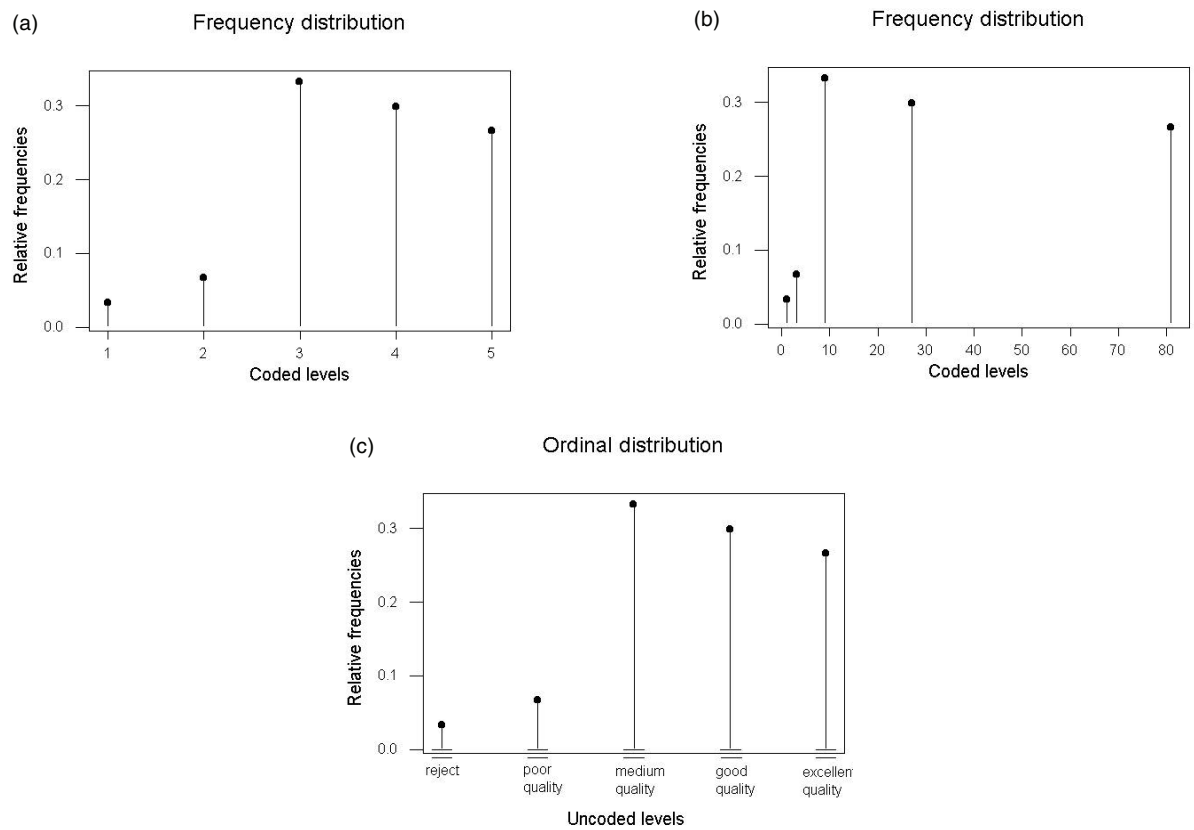

Figure 1. Frequency distributions (a), (b) and ordinal distribution (c) of data reported in Table 1. The "skate" symbol is used in the ordinal distribution to point out that only the relative position of the bars is known.

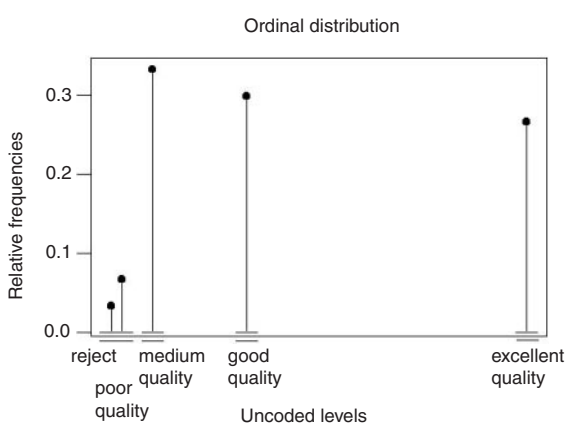

Figure 2. An alternative representation of the ordinal distribution of data reported in Table 1. The concept of distance is not defined.

Obviously, different values with the same maximum probability are possible, that is to say different modes. In the example of the corking process (Table 1), the modal level is "medium quality." A survey of the main properties of the median and the mode can be found in Kendall and Stuart (1977).

A more specific ordinal location measure is the OWA (ordered weighted average) emulator of arithmetic mean, described by Yager and Filev (Yager, 1993; Yager and Filev, 1994). This operator is typically used with linguistic scales. It is defined as

$$\mathrm{OWA} = \operatorname*{Max}_{k=1}^{n} [\mathrm{Min}\{Q(k), b_k\}],$$

where $Q(k) = S_{g(k)}$, $k = 1, 2, \ldots, n$, with:

- $Q(k)$ the average linguistic quantifier (the weights of the OWA operator);
- $g(k) = \mathrm{Int}\{1 + [k((t-1)/n)]\}$;
- $\mathrm{Int}(a)$ a function that gives the integer closest to a;
- b_k the kth element of the sample previously ordered in a decreasing order.

This OWA operator is said to be an emulator of arithmetic mean since it operates, in an ordinal environment, in the same way as the arithmetic mean

works in a cardinal one. It can take value only in the set of levels of the ordinal scale, while a numerical codification of these levels could lead to some intermediate mean values.

As an example, let us suppose we have a scale with $t = 5$ levels, namely, S_1, S_2, S_3, S_4, and S_5, and a sample of size $n = 10$, whose elements, previously ordered in a decreasing order, are [S_5, S_5, S_5, S_4, S_4, S_3, S_3, S_3, S_2, S_1].

The "weights" of the OWA operator are

- $Q(1) = S_1$;
- $Q(2) = Q(3) = S_2$;
- $Q(4) = Q(5) = Q(6) = S_3$;
- $Q(7) = Q(8) = S_4$;
- $Q(9) = Q(10) = S_5$.

Therefore, we have:

$$\begin{aligned}\text{OWA} = \text{Max}[&\text{Min}\{S_1, S_5\}, \text{Min}\{S_2, S_5\},\\ &\text{Min}\{S_2, S_5\}, \text{Min}\{S_3, S_4\}, \text{Min}\{S_3, S_4\},\\ &\text{Min}\{S_3, S_3\}, \text{Min}\{S_4, S_3\}, \text{Min}\{S_4, S_3\},\\ &\text{Min}\{S_5, S_2\}, \text{Min}\{S_5, S_1\}] = S_3.\end{aligned}$$

Figure 3 shows a graphical representation of the OWA calculation (Franceschini and Rossetto, 1999). The value of the OWA emulator of arithmetic mean is given by the intersection of the "ascending stair" (OWA weights) and the "descending stair" (ordered sample elements).

Referring to the example of the corking process (Table 1), we have

- $t = 5$; $n = 30$;
- $S_1 =$ "reject"; $S_2 =$ "poor quality"; $S_3 =$ "medium quality"; $S_4 =$ "good quality"; $S_5 =$ "excellent quality."

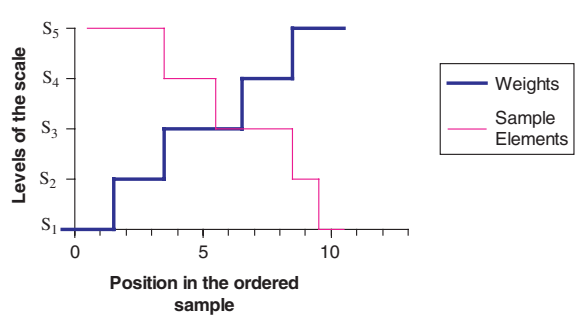

Figure 3. Graphical representation of the OWA calculation. The value of the OWA emulator of arithmetic mean is given by the intersection of the "ascending stair" (OWA weights) and the "descending stair" (ordered sample elements). *(View this art in color at www.dekker.com.)*

The weights of the OWA operator are

- $Q(1) = Q(2) = Q(3) =$ "reject";
- $Q(4) = Q(5) = \cdots = Q(11) =$ "poor quality";
- $Q(12) = Q(13) = \cdots = Q(18) =$ "medium quality";
- $Q(19) = Q(20) = \cdots = Q(26) =$ "good quality";
- $Q(27) = Q(28) = Q(29) = Q(30) =$ "excellent quality."

Therefore, we have

$$\begin{aligned}\text{OWA}\\ &= \text{Max}[\text{Min}\{S_1, S_5\}, \text{Min}\{S_1, S_5\}, \text{Min}\{S_1, S_5\},\\ &\quad \text{Min}\{S_2, S_5\}, \text{Min}\{S_2, S_5\}, \text{Min}\{S_2, S_5\},\\ &\quad \text{Min}\{S_2, S_5\}, \text{Min}\{S_2, S_5\}, \text{Min}\{S_2, S_4\},\\ &\quad \text{Min}\{S_2, S_4\}, \text{Min}\{S_2, S_4\}, \text{Min}\{S_3, S_4\},\\ &\quad \text{Min}\{S_3, S_4\}, \text{Min}\{S_3, S_4\}, \text{Min}\{S_3, S_4\},\\ &\quad \text{Min}\{S_3, S_4\}, \text{Min}\{S_3, S_4\}, \text{Min}\{S_3, S_3\},\\ &\quad \text{Min}\{S_4, S_3\}, \text{Min}\{S_4, S_3\}, \text{Min}\{S_4, S_3\},\\ &\quad \text{Min}\{S_4, S_3\}, \text{Min}\{S_4, S_3\}, \text{Min}\{S_4, S_3\},\\ &\quad \text{Min}\{S_4, S_3\}, \text{Min}\{S_4, S_3\}, \text{Min}\{S_5, S_3\},\\ &\quad \text{Min}\{S_5, S_2\}, \text{Min}\{S_5, S_2\}, \text{Min}\{S_5, S_1\}]\\ &= S_3 = \text{"medium quality."}\end{aligned}$$

This result is different from that obtained by the codification of the scale levels.

Dispersion Measures

With regard to dispersion measures, none of the "traditional" ones can be used in an ordinal environment, since they all need a cardinal codification of levels of the scale. They all require the concept of distance to be defined.

A preliminary ordinal dispersion measure, first introduced by Franceschini and Romano (1999), is the range of ranks r_S, defined as the total number of levels contained between the maximum and the minimum value of a sample of evaluations (the rank is the sequential number of a level on a ordinal scale):

$$r_s = [r(q)_{\max} - r(q)_{\min}],$$

where $r(q)$ is the rank of a generic ordinal level.

In the example of the corking process (Table 1), we would have

$$r_S = r(\text{Excellent quality}) - r(\text{Reject}) = r(S_5) - r(S_1) = 5 - 1 = 4.$$

This dispersion measure assumes that the scale ranks do not depend on the position of levels of the ordinal variable. Table 2 shows two different samples with the same value of r_S.

The actual dispersion of samples in Table 2 is the same if and only if the distance between "good quality" and "excellent quality" is equal to the distance between "reject" and "poor quality." However, this cannot be asserted since the concept of distance is not defined.

To overcome this difficulty, we introduce a new dispersion measure, the so-called ordinal range, that considers not only the number of levels between the maximum and the minimum value of the sample, but also their positioning on the scale. The ordinal range is based on the concept of "dangerousness" that in turn depends on the "meaning" of the particular scale at hand. It is defined on a scale with $t(t+1)/2$ levels, where t is the number of levels of the ordinal scale considered. Each level is ordered according to an increasing "dangerousness" of dispersion. With the same difference of scale levels, the dispersion is more "dangerous" if the sample is centered on a more "dangerous" level of the scale (that is a lower level in the example at hand, since lower levels are associated with a more negative judgment on product quality).

Table 3 shows, for the example of the corking process ($t = 5$), the 15 ($t(t+1)/2 = 15$) levels of the corresponding scale of ordinal range.

Table 4 reports some different samples of judgments from the example of the corking process and the related values of the ordinal range. Samples 1 and 2 have the same difference of levels between the maximum and the minimum rank value. However, their dispersion is not the same. Dispersion of sample 2 is more "dangerous" than dispersion of sample 1 because values of sample 2 are nearer to the lower values of the scale of judgments. As a consequence, the value of ordinal range of sample 2 is greater than the value of ordinal range of sample 1. A similar analysis can be developed regarding samples 3 and 4.

Table 2. Two different samples with the same sample size and the same range of ranks.

(a)				
"Reject"	"Poor quality"	"Medium quality"	"Good quality"	"Excellent quality"
0	3	10	9	8
(b)				
"Reject"	"Poor quality"	"Medium quality"	"Good quality"	"Excellent quality"
3	10	9	8	0

Table 3. Levels of the scale of ordinal range for the example of the corking process.

Minimum sample value	Maximum sample value	Level of the scale of ordinal range (in increasing "dangerousness")
Excellent quality	Excellent quality	R_1
Good quality	Good quality	R_2
Medium quality	Medium quality	R_3
Poor quality	Poor quality	R_4
Reject	Reject	R_5
Good quality	Excellent quality	R_6
Medium quality	Good quality	R_7
Poor quality	Medium quality	R_8
Reject	Poor quality	R_9
Medium quality	Excellent quality	R_{10}
Poor quality	Good quality	R_{11}
Reject	Medium quality	R_{12}
Poor quality	Excellent quality	R_{13}
Reject	Good quality	R_{14}
Reject	Excellent quality	R_{15}

Distribution of Location and Dispersion Measures in an Ordinal Environment

In "traditional" cardinal statistics, the introduction of location and dispersion measures is followed by the analysis of their statistical properties. The knowledge of their distributions is necessary to develop statistical techniques such as hypothesis testing.

The ordinal distribution of location and dispersion measures can be easily obtained from the ordinal distribution of the population (universe) of judgments. This can be done through the following procedure, based on the exploration of the entire sample space:

1. Initialize to zero all probabilities of location or dispersion measure at hand.

Table 4. Different samples of judgments and related values of the ordinal range for the example of the corking process.

Sample number	"Reject"	"Poor quality"	"Medium quality"	"Good quality"	"Excellent quality"	Ordinal range
1	0	0	0	2	28	R_6
2	0	0	10	20	0	R_7
3	0	0	10	15	5	R_{10}
4	0	3	8	19	0	R_{11}
5	0	0	0	0	30	R_1

Table 5. Ordinal distributions of the OWA emulator of arithmetic mean for different sample sizes (n) and different numbers of levels of the ordinal scale (t).

		Ordinal distribution of the population (universe) of judgments								
	t	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
	5	0.2	0.2	0.2	0.2	0.2				
	7	0.14	0.14	0.14	0.14	0.14	0.14	0.14		
	9	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
		Ordinal distribution of the OWA emulator of arithmetic mean								
t	n	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
5	5	0	0.09	0.83	0.09	0				
	10	0	0.05	0.89	0.05	0				
	20	0	0.02	0.96	0.02	0				
7	5	0	0.03	0.34	0.26	0.34	0.03	0		
	10	0	0	0.22	0.56	0.22	0	0		
	20	0	0	0.09	0.81	0.09	0	0		
7	5	0	0	0.04	0.35	0.21	0.35	0.04	0	0
	10	0	0	0.02	0.23	0.50	0.23	0.02	0	0
	20	0	0	0	0.12	0.76	0.12	0	0	0

2. Select a sample of size n from the population of judgments.
3. Calculate its probability (assuming that the sample elements are independent from each other).
4. Calculate the corresponding value of location or dispersion measure.
5. Add the probability at point (3) to probability of value at point (4).
6. Go to (2) until all possible samples have been analyzed.

The complexity of this procedure can be reduced by observing that all samples corresponding to the same ordered sample have the same probability.

By means of the described procedure, we can determine, for example, the ordinal distribution of the OWA emulator of arithmetic mean. Let us assume a uniform ordinal distribution of the population of judgments (i.e., all levels of the ordinal scale have the same probability to be selected by evaluators). Table 5 reports the ordinal distributions of the OWA emulator of arithmetic mean for different sample sizes and different numbers of levels of the ordinal scale. Results are obtained by a software program implemented in The MATLAB 5.2 environment.

The obtained ordinal distributions are symmetric (the concept of symmetry in an ordinal environment will be discussed in the next section). The probabil-

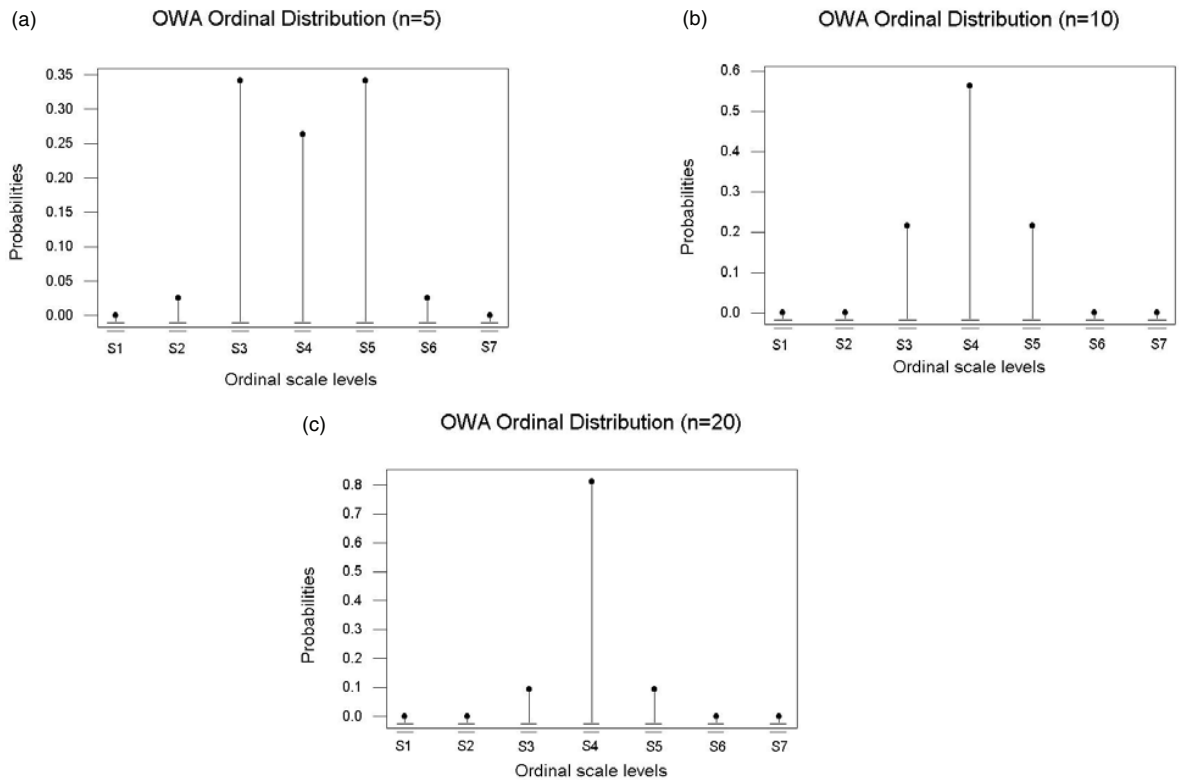

Figure 4. Ordinal distributions of OWA emulator of arithmetic mean for different values of sample size n. The initial ordinal distribution of the population (universe) of judgments is assumed to be uniform (see Table 5) on a scale with $t = 7$ levels.

ities concentrate on the central value of the scale as n increases and t decreases. Figure 4 shows the effect of n on the ordinal distribution of the OWA emulator of arithmetic mean.

This process of probability concentration (with increasing n and decreasing t) heavily depends on the distribution of judgment population (universe).

Table 5 and Fig. 4 show a process of probability concentration (with increasing n and decreasing t). These results suggest the existence of a theorem "similar" to the central limit theorem for an ordinal environment.

The "discovery" of an asymptotic ordinal distribution for each location and dispersion measure, independent of the distribution of the population (universe) of judgments, would represent an important step toward the development of techniques like hypothesis testing or statistical process control tools for an ordinal environment. Such a discovery would overcome the use of the described procedure to calculate the ordinal distribution of each location and dispersion measures, which requires the knowledge of the ordinal distribution of the population (universe) of judgments. However, as matters stand, this requirement is still inevitable, because we do not have an equivalent of the central limit theorem for an ordinal environment yet. With regard to the OWA emulator of arithmetic mean, an asymptotic ordinal distribution seems very unlikely to exist. In Table 5 and Fig. 4, the OWA ordinal distribution concentrates on the central level of the scale when judgments are expressed according to a uniform distribution (i.e., all levels of the ordinal scale have the same probability to be selected by evaluators).

As a counter example, Fig. 5 shows the ordinal distribution of the OWA emulator of arithmetic mean for a particular ordinal distribution of the population of judgments on a scale with $t = 9$ levels. In this case, the OWA ordinal distribution concentrates on two different levels of the scale instead of only one. This seems to suggest that an asymptotic ordinal distribution of OWA emulator of arithmetic mean does not exist.

The Concept of Symmetry in an Ordinal Environment

Denoting by t the number of levels of an ordinal scale, we can say that the ordinal distribution is

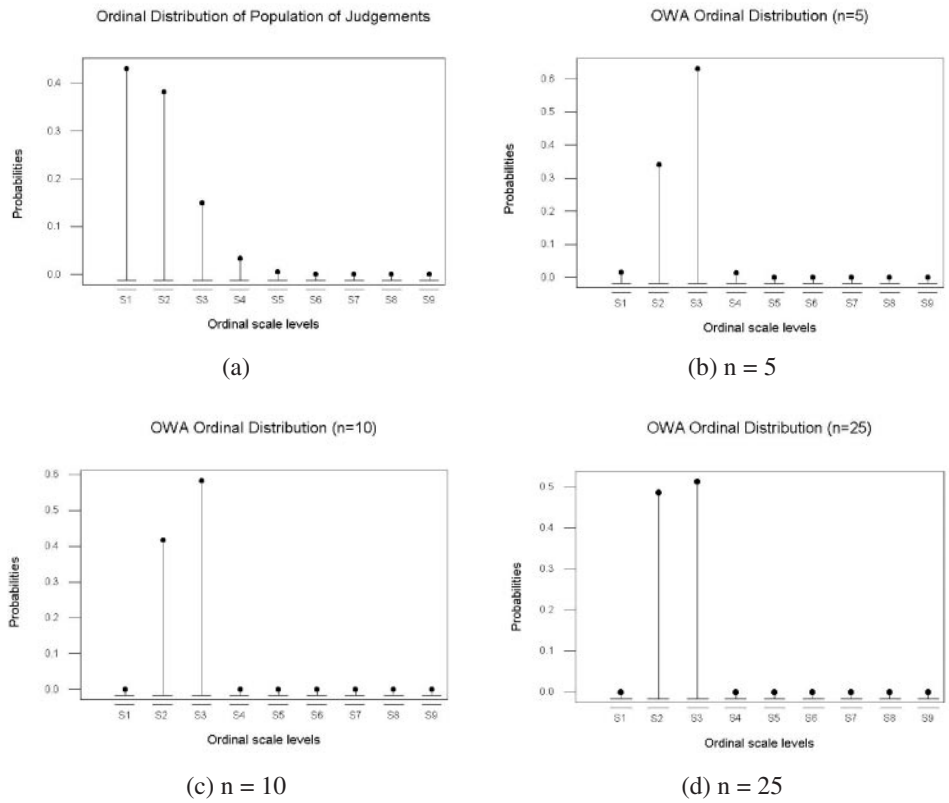

Figure 5. Graphical representation of ordinal distributions reported in Table 5. Ordinal distribution of population of judgments (a) and ordinal distribution of the OWA emulator of arithmetic mean for different sample sizes (b), (c), (d), for a scale with $t = 9$ levels.

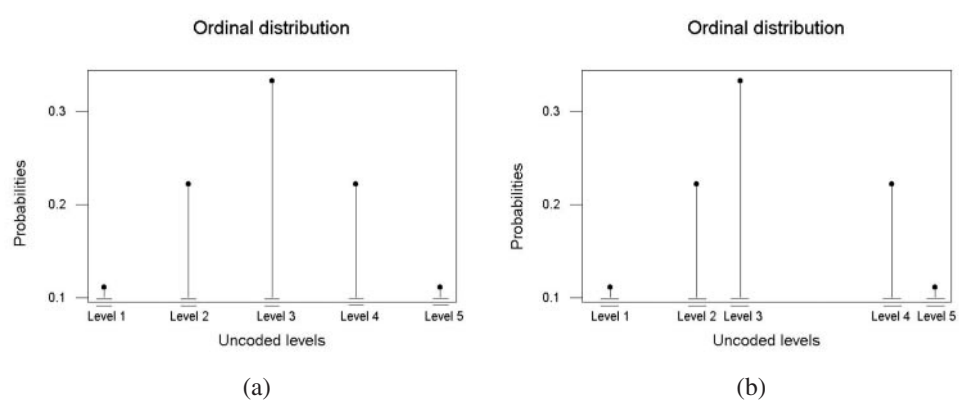

Figure 6. Two equivalent representations of the same symmetric ordinal distribution. The concept of distance is not defined.

symmetric if and only if the probability of level S_i, $f(S_i)$, equals the probability of level S_j, $f(S_j)$; i.e., $f(S_i) = f(S_j)$, where $i + j = t + 1$, $\forall i, j = 1, 2, \ldots, t$.

Figure 6 shows two equivalent representations of the same symmetric ordinal distribution on a scale with an odd number of levels. Since each ordinal distribution has infinite equivalent possible representations, the concept of symmetry cannot be defined on the basis of the concept of shape. This is a direct consequence of the most important feature of ordinal

scales: the lack of the concept of distance between the levels of the scale.

CONCLUSIONS

In this article, we pointed out some problems that arise when dealing with evaluations or measurements expressed on an ordinal scale. We analyzed the properties of these scales and we extended the concept of probability distribution to an ordinal environment, by the introduction of the so-called ordinal distribution. The main location and dispersion measures that can be used in an ordinal environment are discussed, and a methodology to calculate their ordinal distribution from an ordinal distribution of population (universe) of judgments is presented.

These studies on properties of location and dispersion measures can lead to the development of new tools able to manage processes monitored by ordinal scales only.

Future research will aim at finding an asymptotic ($n \to \infty$) ordinal distribution for each location and dispersion measure of interest, provided that it actually does exist. A parallel theme of research will be the analysis of statistical properties of location and dispersion measures, such as correctness, consistency, and efficiency.

ABOUT THE AUTHORS

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