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NARX APPROACH TO BLACK-BOX MODELING OF CIRCUIT ELEMENTS

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ABSTRACT

This paper deals with the identification of NARX (Nonlinear AutoRegression with eXtra input) models for the numerical simulation of circuit containing nonlinear dynamic elements. NARX identification, based on a sequence of input/output samples, is useful for black-box modeling and for the refinement of models of nonlinear circuit elements. In order to assess the suitability of such an approach, we apply it to a CMOS inverter gate and experiment with the main elements controlling the identification process. We obtain accurate models with relatively simple structure and observe reliable operation of the identification process, as well as a good insensitivity to the noise content of the output samples. Such results confirm that NARX identification could be a useful tool for circuit simulations.

1. INTRODUCTION

The development of efficient and accurate numerical models to describe the behavior of circuit elements is of paramount importance in the area of circuit simulation. In particular, the availability of techniques for the black-box modeling of nonlinear dynamic circuit elements and for the simplification (*i.e.,* order reduction) of existing models would be very useful. Black-box modeling is the most general approach to the description of poorly known devices, whereas model simplification allows the balancing of model accuracy and efficiency.

The latter point is going to be a dominant issue of future simulation problems. In fact, as circuit applications become more complex and their operation more critical, conventional circuit simulation and functional simulation become ineffective, since they are, respectively, too expensive and/or too idealized. Fast digital circuits are important examples of this evolution. Their short switching times activate many parasitic effects and require the use of non ideal models, whereas their size limits the complexity of models that can be used. However, for logic gates, the trade-off between model accuracy and complexity is not trivial. The transistor level description of gates leads to complex models, not affordable in the simulation of realistic problems, whereas simpler nonideal models are hardly devised.

In this scenario, the NARX models could provide a useful modeling approach. Such models are the extension of the widely used ARX models to nonlinear systems and are general enough to describe a wide class of them (*e.g.,* see [1]), possibly including many nonlinear electric and electronic components. The identification of NARX models from input/output signals could be exploited for both the black-box modeling and the refinement of models of nonlinear circuit elements. Besides, NARX identification could be applied to the input/output signals of existing accurate models as a method for their simplification. Although such a method has a brute-force nature, its use is justified by lack of systematic methods for the direct simplification of nonlinear models [2].

NARX models have been widely studied in the area of *control systems,* where suitable identification algorithms have been developed and successfully applied to moderately nonlinear dynamic systems [3]. Also, the direct derivation of NARX models from nonlinear differential models and a discussion of their effectiveness in the modeling of physical systems have been carried out [4].

In this paper, we try to assess the performances of NARX models and of their identification in the modeling of highly nonlinear fast dynamic circuit elements. We carry out the study by applying the NARX identification algorithm proposed in [5] to a CMOS inverter and by experimenting with the relevant identification parameters. Since the inverter is the basic element of logic gates, the results obtained in this study should give a first indication of the possibilities of the considered approach in the modeling of digital devices.

2. IDENTIFICATION ALGORITHM

NARX models are discrete-time linear-in-parameter models defined by Kolmogorov-Gabor polynomials [5]

$$y(k) = \bar{y} + \sum_{i=1}^{\gamma} a_i x_i + \sum_{i=1}^{\gamma}\sum_{j=1}^{\gamma} a_{ij} x_i x_j + \dots + \underbrace{\sum_{i=1}^{\gamma}\sum_{j=1}^{\gamma}\dots\sum_{p=1}^{\gamma}}_{q \text{ terms}} a_{ij\dots p} x_i x_j \dots x_p + e(k) \quad (1)$$

$$X = \{u(k), u(k-1), \dots, u(k-r), y(k-1), \dots, y(k-r)\} \quad (2)$$

where $u(k)$, $e(k)$ and $y(k)$ are the samples at the k-th time point of the input, disturbance and output signals, respectively, x_p is the generic element of X, *i.e.,* the present sample of u or the past samples of u and y up to the time $k-r$, and $\gamma = 2r+1$. The integer variables r and q are the *dynamic order* and the *nonlinear degree* of the model, respectively. Every possible product of up to q elements of X is a *potential component* of the model and appears in (1). The potential components with nonzero coefficients are the model *actual components* (or *components* in short) and their coefficients are the *model parameters.*

In order to identify a NARX model from a sequence of input/output samples (*i.e.,* to select the components of the model and compute their parameters), we implement and use the algorithm of Pottman et Al. [5]. Since the number of potential components that can compose a NARX model grows rapidly with r and q, making the identification computationally expensive, we base our implementation on a step forward approach. In such an approach, the model is built by starting from a minimal *guess model* (possibly with no components) and by adding at each step the potential component that mostly reduces the model mean-square error.

In detail, the implemented identification algorithm is organized as follows. A model dynamic order r and a nonlinear degree q are chosen. The model dynamic order is estimated a priori from the input/output sequence by the algorithm of [6], whereas the nonlinear degree is estimated empirically. Such a choice defines the set of potential components. Then a guess model is decided and the following three steps are repeated.

1. The reduction of the model mean-square error produced by each potential component not in the *current model* is estimated off-line by orthogonalization of the time sequences [5]. A *new model* is generated by adding to the current one the potential component that minimizes the mean-square error.
2. The stability of the new model is verified and, if necessary, the added component is discarded.
3. For the new model, the values of suitable statistical indexes are computed [5].

Each execution of the above steps generates a new model, whose statistical significance is assessed by the index values computed in step (3). When a new model has values of the statistical indexes not better than the previous one, the process is terminated and the model with the best values of the statistical indexes is retained as the final one. Eventually, the final model is validated by checking its ability to reproduce the system output for input signals different from those used in the identification process.

As an example, we apply the above procedure to the input/output sequences of a NARX test system and verify its ability to correctly retrieve such system. The test system is defined by the following particular Kolmogorov-Gabor polynomial with $r = 1$ and $q = 4$

$$\begin{aligned} y(k) &= 0.2025 + 0.405y(k-1) + 0.09u(k-1) \\ &+ 0.0008y^4(k-1) - 0.0056u(k-1)y^2(k-1) \\ &+ 0.177y^2(k-1) + 0.09u(k-1)y(k-1) \\ &- 0.0253y^3(k-1) + 0.01u^2(k-1) \end{aligned} \quad (3)$$

The input *identification sequence* used is obtained by sampling a random multilevel signal with a small white noise superimposed. The identification sequence has 3600 samples, and its level variations are wide enough to highlight the nonlinear nature of (3). Table 1 shows the main figures of some of the models generated by the identification procedure for this example. Each row of such a Table describes a different model and lists, from left to right, the number of components of the model (n), the maximum value ($e_{\max}$) and the variance (σ_e^2) of the error between the model and the reference outputs, and the value of the OVF index [5]. The OVF index is a decreasing function of σ_e^2 and of n, so that its maximum should indicate the most significant model of the sequence. For this example, the OVF index is maximum for the model with $n = 9$ (see the bold row of Tab. 1), which indeed coincides with the original system. The detailed structure of models with $n = 8, 9, 10$ is reported in Tab. 2. Models with $n < 9$ are composed of a subset of the component of the original system and their parameters approximate the corresponding parameters of the original system. On the other hand, models with $n > 9$ have the same components and parameters of the original systems plus spurious components with negligibly small coefficients.

n	e_{max}	σ_e^2	OVF
6	0.2386	2.014×10^{-3}	7.477×10^{5}
7	0.1703	2.014×10^{-3}	8.916×10^{5}
8	0.0712	1.312×10^{-13}	1.381×10^{7}
9	$\mathbf{1.249 \times 10^{-6}}$	$\mathbf{1.314 \times 10^{-13}}$	$\mathbf{7.249 \times 10^{15}}$
10	1.229×10^{-6}	1.306×10^{-13}	6.43×10^{15}
11	1.256×10^{-6}	1.3×10^{-13}	5.8×10^{15}

Table 1: *Main figures for models of the example of Sec. 2 .*

n	8	9	10
$\bar{y}$	0.1809	0.2025	0.2025
$y(k-1)$	0.3938	0.405	0.405
$u(k-1)$	0.1245	0.09	0.09
$y^4(k-1)$	0.0009	0.0008	0.0008
$u(k-1)y^2(k-1)$	−0.0052	−0.0056	−0.0056
$y^2(k-1)$	0.1824	0.177	0.177
$u(k-1)y(k-1)$	0.0901	0.09	0.09
$y^3(k-1)$	−0.0267	−0.0253	−0.0253
$u^2(k-1)$	—	0.01	0.01
$u^2(k)y^2(k-1)$	—	—	0.0

Table 2: *Structure of models of the example of Sec. 2 .*

3. NARX INVERTER MODELS

In this Section, we address the NARX modeling of logic gates by applying the identification algorithm of Sec. 2 to an inverter gate of CMOS technology. We generate the output sequences for the identification process via Spice simulations based on the CMOS *level 2 model*, which is a detailed transistor model including several parasitics. The simulated output sequences are used either directly, to identify NARX models as simple as possible (model simplification), or corrupted by noise, to reproduce identification from measured data (black-box modeling).

We start by considering a Single Input Single Output (SISO) configuration of the inverter system, obtained by loading the inverter with an identical one and by using the voltages at its input and output ports as the input ($u(t)$) and the output ($y(t)$) signals, respectively. For this system, the estimated order is $r = 1$ and we start with $q = 3$ and a guess model with no components. The identification process yields the sequence of models described in Tab. 3. Since the modeled system is not of NARX type, the figures of Tab. 3 do not show the net threshold phenomenon shown in Tab. 1. However, among the identified models, the one with the maximum value of the OVF index ($n = 8$) still reveals the most faithful static and dynamic behavior and can be considered the final model. Furthermore, as in the example of Sec. 2, the parameters of the models with $n = 6$ and 7 still approximate the corresponding parameters of the model with $n = 8$. The accuracy of the model with $n = 8$ is good and can be appreciated in Fig. 1, where its response to a *validation input* (*i.e.,* a signal different from the identification input) and the reference response of the original system are compared. Finally, the complete identification process

n	e_{max}	σ_e^2	OVF
4	2.427	0.7444	1223
5	2.902	0.8784	916.6
6	1.572	0.1919	2969
7	0.7939	0.01882	27230
8	**0.3862**	**0.01266**	**34640**
9	0.3506	0.01253	30420
10	0.3346	0.01048	32140
11	0.3574	0.0108	28000
12	0.3651	0.01122	24450
13	0.3497	0.01139	22190

Table 3: *Main figures of NARX models obtained for a CMOS inverter in SISO configuration by using $r = 1$, $q = 3$ and a guess model with no components*

to obtain this model requires about 20 s on a 60 MHz Pentium PC.

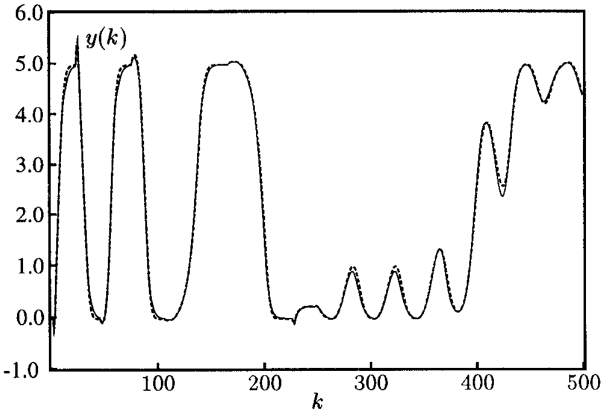

Figure 1: *Reference response to a validation signal (solid line) compared with the response of the $n = 8$ model of Tab. 3 (dashed line)*

In order to further assess the performance of NARX identification on the inverter device, we carry out a complete set of identification experiments for the simple SISO configuration.

The first point considered in such experiments is the influence of the guess model. In fact, the sensitivity of the NARX identification to the order in which the model components are selected is a known weakness [4], and the guess model affects such order. We verify that guess models defined by a subset of the components of the $n = 8$ model of Tab. 3 lead to the same final model. In contrast, guess models with components not in the $n = 8$ model may lead to different final models and show that the pure forward approach does not ensure the neutralization of inappropriate components.

The next element considered is the identification signal. The selection of suitable identification signals is a critical point in the identification of nonlinear system, because of the lack of theoretical guidelines. We try different types of identification signals and different lengths of the identification sequences. The best results are obtained with the random multilevel signals defined in Sec. 2 when the constant levels last enough in comparison with the system "local time constants". In this case we observe good identification properties and a weak sensitivity of the variance error to the length of the identification sequence.

Then we consider the ability of the identification process to obtain models from corrupted output sequences, which is the key property required to use NARX identification for black-box modeling. To check such an ability, we add white noise signals of different variance to the simulated output signal and use the resulting signal for the identification. In this way, we observe remarkably good insensitivity to the added noise. In fact, the identification process works also for the noisy output sequences and the final model is hardly affected (*i.e.,* has the same components and parameter values) for SNR values as low as 25 dB. Moreover, though lower values of the SNR of the output sequence lead to different final models, such models remain able to reproduce the qualitative behavior of the original system. As an example, Fig. 2 and Fig. 3 show, respectively, a part of a noisy output identification sequence and a response of the model identified from such a noisy sequence. In this example, the output identification sequence has SNR= 20 dB, and the response of the identified model to a validation signal, shown in Fig. 3, still tracks the reference output.

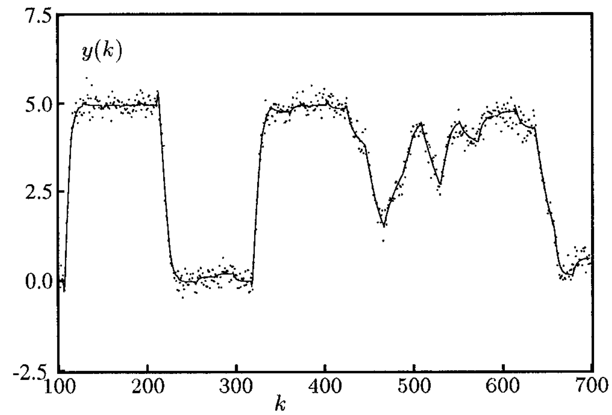

Figure 2: *Samples of an output identification sequence. Solid line: exact values; dots: values after the addition of a noise signal with SNR= 20 dB*

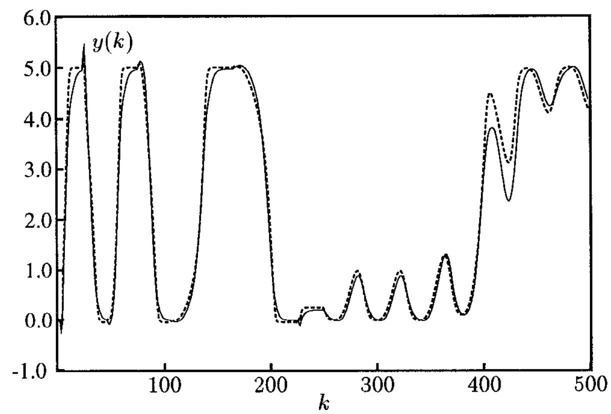

Figure 3: *Reference response to a validation signal (solid line) compared with the response of a model identified from the noisy sequence of Fig. 2 (dashed line)*

Finally, we check the sensitivity of the models to the non linear degree q by identifying NARX inverter models with different q

values. Such experiments highlight the ability of the NARX models to represent highly nonlinear systems even for moderate q values. In these comparisons, the static characteristic of the model is used as an additional index of its ability to reproduce the nonlinear behavior of the original system. For $q = 2$, the shape of the static characteristic cannot be obtained and the identification fails. For $q = 3$, instead, the characteristic is correctly reproduced and the accurate model with $n = 8$ of Tab. 3 is obtained. Moreover, the value $q = 3$ turns out to be an optimum choice for the problem at hand, as higher q values yield only minor improvements. This can be appreciated in Fig. 4, where the reference characteristic and the characteristics of two models with $q = 3$ and $q = 4$ are shown. An interesting method to improve the accuracy of NARX models without increasing q (and hence n) is the use of piecewise models [5]. We identify a piecewise model composed of two submodels with $r = 1$, $q = 3$ and $n = 7$, which works safely and is more accurate than the model with $n = 8$ of Tab. 3. The variance of its error, σ_e^2, is one order of magnitude smaller than the one reported in Tab. 3 and its response to the validation signal is better than the one shown in Fig. 1 (results are not presented for lack of space). In such a 2-piece model, the submodel domains are the two half planes $y < 2.5$ and $y > 2.5$, whereas the switching rule is hysteretic and takes into account the last two y samples.

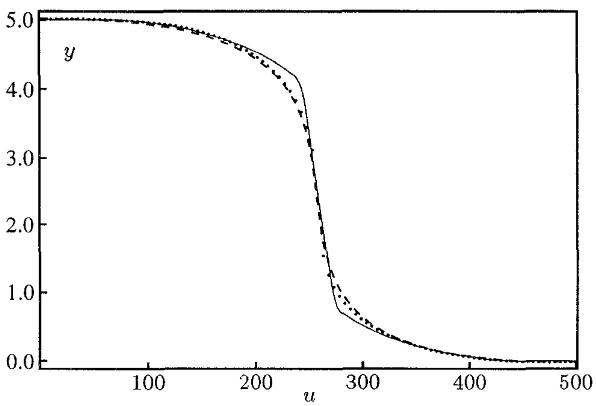

Figure 4: *Static characteristics of the CMOS inverter (solid line), of the model with $q = 3$ and $n = 8$ of Tab.3 (dashed line) and of a model with $q = 4$ and $n = 12$ (dotted line)*

SISO models are useful to assess the possibilities of the NARX approach, but the representation of multiport circuit elements requires Multiple Input Multiple Output (MIMO) models. In the inverter example, we deal with a 2-port element, where, in principle, two of its port variables are controlled by the other two variables. Since CMOS gates operate by forcing the voltages of their output ports, the natural input and output variables of the inverter circuit are v_1, i_2 and v_2, i_1, respectively, defined in the insert of Fig. 5. We identify a NARX MIMO inverter model by exciting the circuit through the input variables v_1 and i_2 and by using $r = 1$ and $q = 4$. The algorithm is applied to a set x defined as $X = \{v_1(k), v_1(k-1), i_2(k), i_2(k-1), v_2(k-1)\}$ and, in spite of the increase of the number of potential components, the identification remains affordable and produces accurate models. The best MIMO model obtained in this way has 16 components and its accuracy can be appreciated in Fig. 5. Such a Figure shows the waveform $v_2(k)$ produced by the MIMO model when it is loaded by a capacitor and is driven by a validation sequence $v_{in}(k)$. The waveform $v_2(k)$ obtained by the MIMO model tracks well the reference response of the simple test circuit.

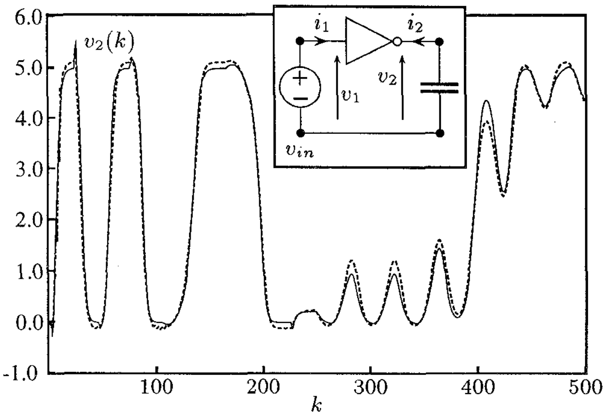

Figure 5: *Waveform $v_2(k)$ of the circuit of the insert for a validation input v_{in}. Solid curve: reference response; dashed curve: response produced by the NARX MIMO inverter model described in the text*

4. CONCLUSION

In this work, we investigate the performances of NARX identification applied to a dynamic highly nonlinear two-port element: the CMOS inverter. The numerical test carried out shows that such an approach has the potential to handle this type of nonlinear systems. The identification process yields accurate models with a moderate number of components, is robust and is practicable also for multiple inputs. Although many aspects should be further investigated before practical applications, the results obtained suggest that NARX identification could be a useful tool for circuit simulations.

5. REFERENCES

[1] A. Ponchet, J. L. Ponchet, and G. S. Moschytz, "On the input/output approximation of nonlinear systems," *Proceeding of ISCAS-95*, pp. 1500–1503, May 1995.

[2] L. Fortuna, G. Nunnari, and A. Gallo, *Model order reduction techiniques with applications in electrical engineering.* Springer-Verlag, 1992.

[3] R. Haber and H. Unbehauen, "Structure identification of nonlinear dynamic system-a survey on input/output approaches," *Automatica*, vol. 26, pp. 651–677, Apr. 1990.

[4] S. McCabe, P. Davies, and D. Seidel, "On the use of nonlinear autoregressive moving average models for simulation and system identification," *American Control Conference*, pp. 1758–1763, 1991.

[5] M. Pottman, H. Unbehauen, and D. Seborg, "Application of general multi-model approach for identification of highly nonlinear processes-a case study," *Int. J. Control*, vol. 57, pp. 97–120, Jan. 1993.

[6] M. Autin, M. Biey, and M. Hasler, "Order of discrete time nonlinear systems determined from input/output signals," *Proceedings of ISCAS-92*, pp. 296–299, May 1992.