Fast passivity enforcement for large and sparse macromodels

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Abstract: This paper presents a fast technique for testing and enforcing passivity of linear macromodels characterized by a large and sparse structure. An optimized algorithm is proposed for the computation of the imaginary eigenvalues of the associated Hamiltonian matrix, which are iteratively perturbed until passivity is enforced. Each iteration of the proposed scheme requires a small computational cost, that scales only linearly with the size of the problem.

1 Introduction

The designers of high-speed electronic systems continuously face the problem of insuring the integrity of the signals in their products. Any parasitic effect due to unwanted electromagnetic coupling, crosstalk, dispersion, losses, etc. must have no influence on the functional performance of the system. In order to reach this goal, an accurate characterization and modeling procedure must be applied in the early design stage to all the critical parts of the system having some influence on the signals. Examples of typical important structures are connectors, package structures, signal and power buses, via arrays on printed circuit boards, and discontinuities.

We consider here only macromodeling approaches based on native structure characterizations via measured or simulated sets of port responses in time or frequency domain, frequency-domain scattering parameters being the most common scenario. All available techniques for macromodel generation (Vector Fitting [8] being one of the most popular) share the common objective of generating a rational approximation of the transfer matrix of the structure. Most of these techniques lead to stable approximations having poles with negative real part. However, it is now widely recognized that this is not sufficient, since any macromodel should also be passive. In fact, stable but non-passive models cannot be reliably used for system-level analysis since they may lead to unstable behavior depending on the termination networks.

Several techniques for enforcement of macromodel passivity have been recently proposed. Some of these schemes may lead to overtreatment and to reduced accuracy [11]. Some other may leave small passivity violations in the resulting macromodels [9]. Finally, convex optimization methods are currently limited by problem size due to the large computational cost [2, 3]. Here we consider passivity compensation schemes based on the eigenstructure of the Hamiltonian matrix associated to the macromodel [6, 7]. We show that the particular structure of this matrix allows a fast computation of its eigenvalues when the macromodel is characterized by large and sparse matrices. This is indeed a situation which is typically encountered in practice. The main contribution of this paper is a passivity check and compensation algorithm requiring a number of operations which scales only linearly with the size of the problem. Compared to previous techniques, which need at least $O(n^3)$ operations, the proposed scheme leads to significant savings of CPU time and storage requirements.

2 Preliminaries

We consider a linear time-invariant macromodel characterized by a $p \times p$ rational transfer matrix with strictly stable poles

$$H(s) = D + \sum_k \frac{R_k}{s - q_k} = D + C(sI - A)^{-1}B\,, \qquad \text{with} \quad \left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t)\,, \\ y(t) & = & Cx(t) + Du(t)\,. \end{array} \right. \tag{1}$$

The state-space realization in (1) is assumed to be in real Gilbert's form [10]. The $n \times n$ matrix A is block-diagonal with blocks of size 1 for real poles and size 2 for complex poles. The number of its nonvanishing entries is thus at most $2n$. We will concentrate here on the realistic case $p \ll n$, i.e., the number of ports is assumed to be much less than the number of macromodel states. The corresponding state matrices B and C are generally full. However, for complex structures with a large number p of ports and dynamic order, the poles-residues terms in (1) are often computed separately for disjoint groups of transfer matrix entries. This procedure leads to poles that are not common to all transfer functions of the model. Consequently, the residues matrices R_k are possibly sparse with many vanishing entries. This leads naturally to state-space realizations with also B and C sparse. The limit case is when all entries of $H(s)$ are dealt with separately during the rational approximation stage. In this case, the number of nonvanishing entries in both B and C is at most n.

Passivity of the macromodel is guaranteed when the transfer matrix $H(s)$ is bounded real (case of scattering representations) or positive real (in case of impedance, admittance, and hybrid representations). We consider in this work only the scattering case, although the same procedure can be applied with obvious modifications to any other representation [7]. Bounded realness is guaranteed if all the singular values of the scattering matrix, computed at any frequency $s = j\omega$, are bounded by one. A purely algebraic test for passivity which avoids testing all possible frequencies is provided by simple conditions on the eigenspectrum of the Hamiltonian

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matrix associated to the macromodel. This matrix has size $2n$ and for the scattering case reads

$$\mathcal{M}_\gamma = \begin{pmatrix} A - BR^{-1}D^TC & -\gamma BR^{-1}B^T \\ \gamma C^TS^{-1}C & -A^T + C^TDR^{-1}B^T \end{pmatrix}, \tag{2}$$

with $R = (D^TD - \gamma^2 I)$ and $S = (DD^T - \gamma^2 I)$. The results in [7] show that the macromodel is passive when there are no imaginary eigenvalues of $\mathcal{M}_{\gamma=1}$. In fact, the scalar parameter γ acts as a threshold on the singular values of the scattering matrix. The location of the purely imaginary eigenvalues of the Hamiltonian matrix pinpoint precisely the frequencies at which at least one singular value of the scattering matrix assumes the value γ. When this result is complemented by a first-order perturbation analysis [7], it is possible to compute the complete set of frequency intervals (ω_{i-1}, ω_i) where passivity violations occur. This analysis requires knowledge of both imaginary eigenvalues of $\mathcal{M}_{\gamma=1}$ and their associated eigenvectors.

The rich information provided by the eigenspectrum of the Hamiltonian matrix can be exploited to setup an iterative scheme leading to compensation of the detected passivity violations. The main derivation of such scheme is presented in [6, 7], so we recall the main steps in Algorithm 1. We compute a correction matrix Δ for the iterative modification of state space matrix C. This correction is linearly related via a first-order perturbation analysis to the perturbed location of the Hamiltonian imaginary eigenvalues. Therefore, the computation of Δ (step 5) is achieved by solving small linear underdetermined least squares problems. The particular solution minimizing the impact on the macromodel accuracy is chosen (see [7] for details). We remark that all computations required by steps 1, 3, 4, 7 of the algorithm are based on eigenvalue computations.

Algorithm 1 Compute a passive macromodel via perturbation of Hamiltonian eigenspectrum.

Require: state-space matrices A, B, C, D

1: compute the set Λ of imaginary eigenvalues of $\mathcal{M}_{\gamma=1}$
2: **while** $\Lambda \neq \emptyset$ **do**
3: determine the violation bandwidths (ω_{i-1}, ω_i)
4: estimate maximum singular value $\gamma_{\max}$ in each violation bandwidth
5: perturb the imaginary eigenvalues and compute correction matrix Δ
6: $C \leftarrow C + \Delta$
7: compute the set Λ of imaginary eigenvalues of $\mathcal{M}_{\gamma=1}$
8: **end while**

3 Sparse passivity enforcement

The computation of purely imaginary eigenvalues of Hamiltonian matrices plays a crucial role in checking and enforcing passivity of the associated macromodels. The simplest approach for this computation is to use a full eigenvalue solver in order to find the complete eigenspectrum of the Hamiltonian matrix, and to extract a posteriori the imaginary eigenvalues and the associated eigenvectors. When the matrix size is very large, however, the computational cost becomes excessive since the number of operations required by a full eigensolver scales as the third power of the problem size.

We propose in this paper a fast technique for the computation of the few imaginary eigenvalues of interest. The number of required operations scales only linearly with the problem size, thus allowing passivity compensation for very large macromodels. The key factor for such a fast algorithm is the Shermann-Morrison-Woodbury Lemma [5], which applied to the Hamiltonian matrix (2) reads

$$(\mathcal{M}_\gamma - \sigma\mathcal{I})^{-1} = \begin{bmatrix} A_\sigma & \\ & -A_{-\sigma}^T \end{bmatrix} - \begin{bmatrix} A_\sigma & \\ & -A_{-\sigma}^T \end{bmatrix} \begin{bmatrix} B & \\ & -C^T \end{bmatrix} \begin{bmatrix} -H_\sigma & \gamma I \\ \gamma I & -H_{-\sigma}^T \end{bmatrix}^{-1} \begin{bmatrix} C & \\ & B^T \end{bmatrix} \begin{bmatrix} A_\sigma & \\ & -A_{-\sigma}^T \end{bmatrix}, \tag{3}$$

where σ is a given point (shift) in the complex plane, $A_{\pm\sigma} = (A \pm \sigma I)^{-1}$ and $H_{\pm\sigma} = D - CA_{\pm\sigma}B$. Since A is almost diagonal due to the adopted Gilbert state-space realization, matrices $A_{\pm\sigma}$ can be computed analytically and preserve the almost diagonal structure. All matrices in (3) are therefore sparse, except for the inner matrix to be inverted, which however has only size $2p$.

Equation (3) can be used to construct at a reduced computational cost the Krylov subspace associated to any given vector v,

$$Kr^d\left((\mathcal{M}_\gamma - \sigma\mathcal{I})^{-1}, v\right) = \text{span}\left\{v, (\mathcal{M}_\gamma - \sigma\mathcal{I})^{-1}v, \ldots, (\mathcal{M}_\gamma - \sigma\mathcal{I})^{-d+1}v\right\} \tag{4}$$

The well-known Arnoldi algorithm [1] can be used to construct an orthogonal basis for this subspace. This basis is employed to build a Galerkin projection of the Hamiltonian matrix onto the d-dimensional Krylov subspace. The eigenvalues of the $d \times d$ obtained matrix are approximations to the eigenvalues of $\mathcal{M}_\gamma$ close to the shift σ. We use here a more refined version of the Arnoldi scheme, including automatic restarts (selections of starting vector v in (4)) and deflation (in order to find converging eigenvalues more quickly). The developed algorithm is a modification of the basic version in [1]. It turns out that the number of operations needed to compute each vector in (4) is bounded by $8n + 2np + 4p^2$, whereas the cost of the orthogonalization process is about $4nd^2$ operations. Therefore, the total number of required operations is linear in the number of macromodel states.

The above procedure is embedded in an outer loop aimed at the optimal selection of multiple shifts σ_k in the complex plane, leading to the computation of the imaginary eigenvalues of the Hamiltonian matrix. Since this eigenspectrum is symmetric with

System	n	p	$\#\{A\}$	$\#\{B\}$	$\#\{C\}$	$\#\{D\}$
I	820	10	1400	530	820	100
II	1488	12	2688	888	1488	144

Table 1: Two test cases for illustration of the proposed sparse Hamiltonian eigenvalue solver. For each of the two systems the number of states n, the number of ports p, and the number of nonvanishing entries $\#\{\cdot\}$ of each state matrix is reported.

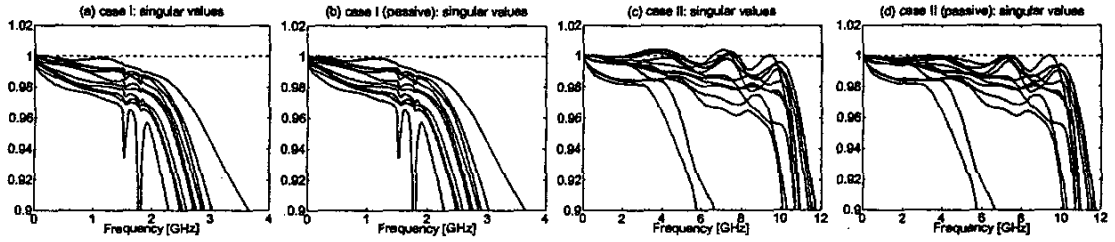

Figure 1: Singular values of the scattering matrix for test cases I and II plotted versus frequency.

respect to the imaginary axis, we choose purely imaginary shifts. A bisection process similar to the well-known Complex Frequency Hopping (CFH) algorithm [4] is used until the entire bandwidth of interest is covered by the convergence region (a circle) encompassing the eigenvalues detected by each shift (center). This procedure leads to the determination of all imaginary eigenvalues of the Hamiltonian matrix with any prescribed accuracy. We emphasize that the computational cost is weakly (linearly) dependent on the problem size. Since several types of iterations are employed (Arnoldi iterations, shift iterations, and passivity compensation iterations), the actual computational cost is problem-dependent and is expected to increase when the passivity violations are significant. The numerical results presented in next section show that the proposed technique leads to large CPU time savings for the passivity compensation when applied to structures of practical interest.

4 Examples

We illustrate the efficiency of the proposed technique by applying it to two large and sparse macromodels representing the scattering port behavior of two high-speed packaging structures (courtesy by Sigrity, Inc.). Table 1 summarizes the dynamic order, the number of ports, and the number of nonzero elements for each of the state space matrices. Note that the size of the Hamiltonian matrix for the two cases is $2n$ =1640 and 2976, respectively.

Panels (a) and (c) in Fig. 1 report the frequency behavior of the singular values of the scattering matrix of the two macromodels. The passivity violations in both cases are very small. As detailed in Sec. 2, the imaginary eigenvalues of the associated Hamiltonian matrices correspond to the frequencies of the intersection points of these curves with horizontal lines at a given level γ. In order to validate our proposed sparse eigenvalue solver, we selected a significant range $\gamma \in [0.9, 1.02]$ and we performed a sweep on γ over this range. For each fixed γ, we computed all the purely imaginary eigenvalues of the Hamiltonian matrix using (i) the proposed sparse technique; (ii) a full eigenvalue solver without computation of the eigenvectors; and (iii) a full eigenvalue solver with computation of the eigenvectors. MATLAB running on a 1.8 GHz Pentium IV PC with a 1 GByte of RAM was used for all numerical examples. The number of imaginary eigenvalues for the selected range of γ varies from 0 to 24 for the two cases. The relative difference between the eigenvalues computed by the sparse and full solver resulted in all cases below 10^{-12}, which was the actual threshold used to stop the Arnoldi iterations. The CPU time required for all computations is reported in Fig. 2. The advantages of the proposed sparse technique are evident. We remark that, since the proposed algorithm is based on subspace iterations, also eigenvectors are returned. Therefore, the CPU time of the sparse solver should be compared to the CPU time required by the full solver returning also the eigenvectors. For Case I, the average CPU times for the sparse, full, and full with eigenvectors solutions are 46, 131, and 434 seconds, respectively. For Case II, we obtained 53, 965, and 2611 seconds. As expected, the CPU time required for the sparse solver has a weak dependence on the actual dimension of the Hamiltonian matrix. Conversely, the CPU time required for the full solution scales as the third power of this dimension, leading to large execution times for large matrices.

We report now the results of the entire passivity compensation scheme using the proposed sparse Hamiltonian eigensolver. Both Case I and II were analyzed and successfully corrected for passivity in 9 and 13 iterations, respectively. The resulting singular values are plotted in panels (b) and (d) of Fig. 1. The relative perturbation of the state matrix C that was required to reach passivity was $||\Delta||_F/||C||_F = 0.0013$ for Case I and 0.0048 for Case II (subscript $_F$ denotes the Frobenius norm). These amounts correspond to the relative amount of perturbation, measured in the energy norm, on the cumulative set of impulse responses. The small perturbation amounts obtained indicate a minimal impact on the accuracy of the macromodels. Of course, these results were obtained without distinction both with the full and the sparse version of the passivity enforcement algorithm. However, the CPU time required for the passivity compensation clearly shows the advantages of the proposed sparse technique. Case I required only 6.7 minutes compared

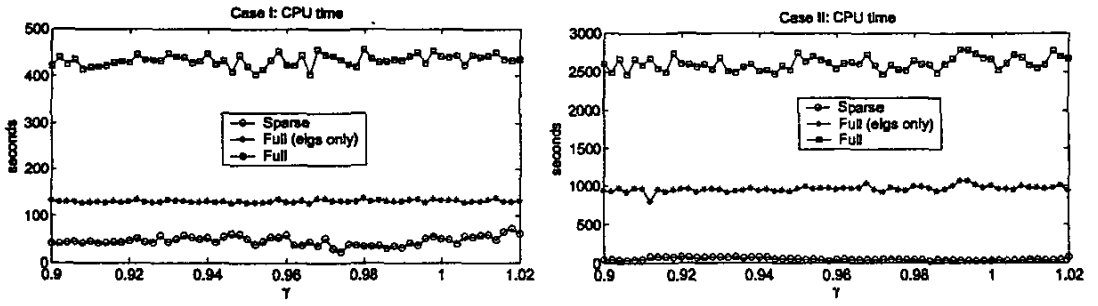

Figure 2: Comparison of sparse and full eigenvalue solvers for Case I (left) and Case II (right). The two panels report the CPU time that was required for the computation of all purely imaginary eigenpairs.

to 84 minutes needed by the full implementation. Case II required only 17 minutes, compared to 18 hours of the full version.

5 Conclusions

We have presented an algorithm for the passivity check and enforcement of large and sparse macromodels. The proposed technique is based on iterative computation and perturbation of purely imaginary eigenvalues of the Hamiltonian matrix associated to the macromodel. We perform this task using an optimized computation of these eigenvalues, which requires a number of operations that scales only linearly with the problem size. As a result, passivity can be enforced for very large macromodels representing high-speed interconnects with a possibly large number of ports over a broad frequency range. The numerical results reported in this paper show a significant increase in efficiency with respect to previous passivity enforcement schemes.

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References

[1] Z. Bai, J. Demmel, J. Dongarra, A. Ruhe, and H. van der Vorst, editors. *Templates for the Solution of Algebraic Eigenvalue Problems: A Practical Guide.* SIAM, Philadelphia, 2000.

[2] C.P.Coelho, J.Phillips, L.M.Silveira, "A Convex Programming Approach for GEnerating Guaranteed Passive Approximations to Tabulated Freqyency-Data", *IEEE Trans. Computed-Aided Design of Integrated Circuits and Systems*, Vol. 23, No. 2, February 2004.

[3] H. Chen, J. Fang, "Enforcing Bounded Realness of S parameter through trace parameterization", in *12th IEEE Topical Meeting on Electrical Performance of Electronic Packaging*, October 27–29, 2003, Princeton, NJ, pp. 291–294.

[4] E. Chiprout, M.S.Nakhla, "Analysis of Interconnects Networks using Complex Frequency Hopping (CFH)", *IEEE Trans. Computer-Aided Design*, vol. 14, pp. 186-200, Feb. 1995.

[5] G. H. Golub, C. F. van Loan, *Matrix computations*, 3rd ed., Baltimore: Johns Hopkins University Press, 1996

[6] S. Grivet-Talocia, "Enforcing Passivity of Macromodels via Spectral Perturbation of Hamiltonian Matrices," in *7th IEEE Workshop on Signal Propagation on Interconnects, Siena, Italy*, May 11–14, 2003, pp. 33–36.

[7] S. Grivet-Talocia, "Passivity enforcement via perturbation of Hamiltonian matrices", *IEEE Trans. CAS-I*, 2004, in press.

[8] B. Gustavsen, A. Semlyen, "Rational approximation of frequency responses by vector fitting", *IEEE Trans. Power Delivery*, Vol. 14, July 1999, pp. 1052–1061.

[9] B. Gustavsen, A. Semlyen, "Enforcing passivity for admittance matrices approximated by rational functions", *IEEE Trans. Power Systems*, Vol. 16, 2001, 97–104.

[10] T. Kailath, *Linear systems*, Prentice Hall, Englewood Cliffs, NJ, 1980.

[11] J. Morsey, A. C. Cangellaris, "PRIME: Passive Realization of Interconnects Models from Measures Data", in *Proc. IEEE* 10th *Topical Meeting on Electr. Perf. of Electron. Packag.*, 2001, pp. 47–50.