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STATIC SPLAY-STRIPES IN A HYBRID ALIGNED NEMATIC LAYER

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Abstract A usual aperiodic hybrid alignment can appear in a nematic layer with weak anchoring only if the cell thickness is greater than a critical value d_h, below which a static periodic pattern instead of the hybrid aperiodic structure could be preferred, if the energy cost for a three dimensional deformation, involving twist, is less than the cost for the two-dimensional deformation of splay-bend type. We have studied the occurrence of the mechanical instability leading to the static periodic splay-stripes, i. e. in the case of the tilt anchoring stronger at the one of the walls, in which the anchoring is planar, for several values of the twist anchoring strengths. Here the behavior of the threshold d_p for the periodic stripes is presented and discussed as a function of the anchoring energies and of the ratio of nematic bulk elastic constants, in the frame of the usual continuum theory.

INTRODUCTION

In the last decade, several authors dealt with hybrid aligned nematic (HAN) cells, i. e. nematic layers possessing homeotropic anchoring at the one of the walls and unidirectional planar at the other (see fig. 1). It was shown that a HAN cell can be very interesting both for a basic and for an applied point of view. In fact, a HAN structure can be useful for measuring the splay-bend elastic ratio K_{11}/K_{33} by means of dielectric[1] and optical methods[2], or for studying flexoelectricity[3], total internal reflectivity[4], and nonlinear optical reorientation[5], also in the presence of weak anchoring[6]. It must be pointed out, that a HAN cell with finite anchoring energy is stable only if the cell thickness d is bigger than a threshold value d_h,

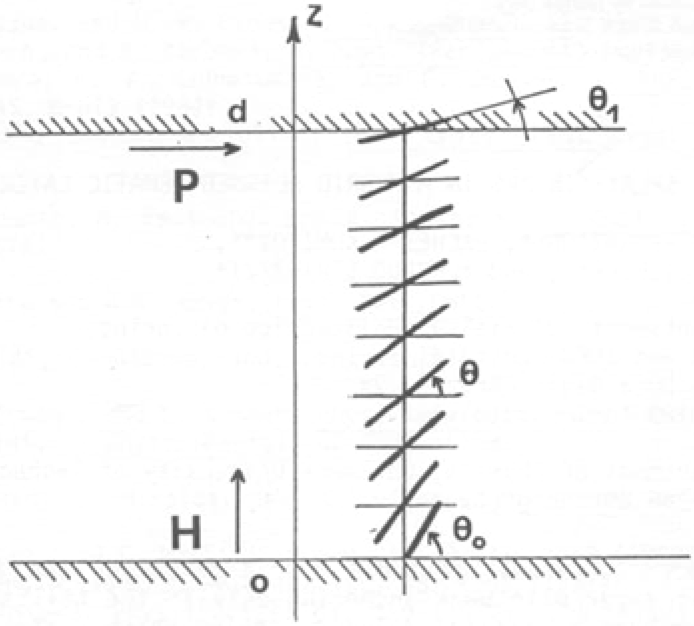

FIGURE 1. Aperiodic hybrid aligned nematic (HAN) cell with thickness d and weak anchoring at both walls. H and P are the easy directions at the lower and upper substrates, respectively. The tilt anchoring strength at the H-wall ($W_{\theta 0}$) is less than the one at the P-wall ($W_{\theta 1}$).

which depends on the tilt anchoring energy at both walls[7]. The recognizing of any threshold becomes essential indeed for all possible applications of HAN to electro-optical devices[8].

On the other hand, after Lonberg and Meyer[9] discovered the occurrence of periodic pattern in a unidirectional planar liquid crystal layer strongly anchored, due to a magnetic field with intensity smaller than the usual aperiodic Fréedericksz threshold, a considerable effort has been made by several authors in order to study the periodic stripes as dependent on the anchoring energy[10] with different external fields[11] in various geometries[12].

Recently, Lavrentovich and Pergamenshchik[13] and Chuvyrov[14] reported for the first time the experimental evidence of the periodic pattern in HAN layers weakly anchored with conveniently small thickness in the absence of external fields. In the ref. /13/ the authors considered theoretically just the ideal situation in which the twist anchoring at both surfaces was degenerated, involving no energy cost at all, whereas the tilt anchoring energy was finite; then they found by means of numerical calculation the condition for the appearing of the periodic pattern.

In the present work we are dealing with a HAN cell with weak tilt and twist anchoring at both walls without degeneration, where the planar alignment appears to be the preferred one at smaller layer thickness than the threshold value d_p. The occurrence of splay-stripes is considered, when the layer thickness is in the interval $d_p < d < d_h$. Our goal is to show the dependence of d_p on the splay-twist elastic ratio and on the anchoring conditions.

THEORY

With the aim of studying the splay-periodic pattern which could arise in a HAN cell at certain thickness d, let us consider a nematic layer limited by two surfaces, $z = 0$ and $z = d$, z being normal to the walls themselves.

The easy axes are homeotropic (H) at $z = 0$ and unidirectional planar (P) at $z = d$; the surface treatment provides weak anchoring both for tilt (θ) and for twist (φ) (see fig. 2). The out-plane angle θ and the in-plane angle φ are assumed to be zero if the director $\mathbf{n}$ is parallel to the x-axis. As usual, we are looking for the transverse periodicity (along the y-axis): hence $\theta = \theta(y,z)$ and $\varphi = \varphi(y,z)$. At the walls, $\theta(y,0) = \theta_0(y)$, $\varphi(y,0) = \varphi_0(y)$ and $\theta(y,d) = \theta_1(y)$, $\varphi(y,d) = \varphi_1(y)$. Since we are interested in splay-stripes, thus the tilt anchoring strength[15] at the upper wall ($W_{\theta 1}$) is bigger than the one at the lower wall ($W_{\theta o}$). No restrictions are requested for the twist anchoring strengths $W_{\varphi o}$ and $W_{\varphi 1}$.

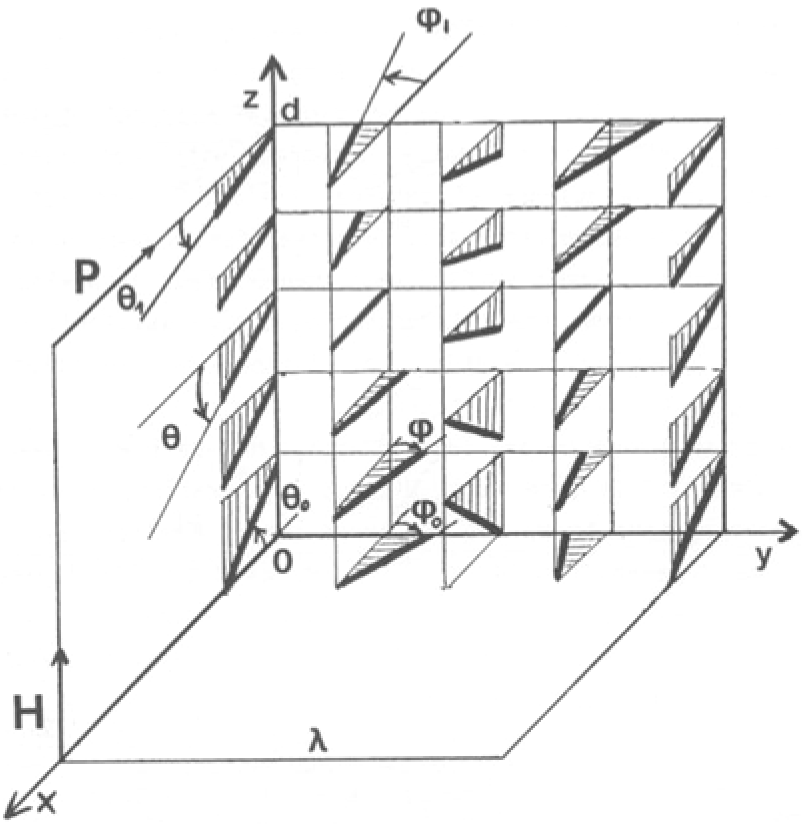

FIGURE 2. Periodic hybrid aligned nematic (PHAN) cell of thickness d with weak anchoring for tilt (θ) and for twist (φ) at both walls.

This means that if the usual hybrid alignment is not allowed[7], the cell thickness d being smaller than a convenient threshold thickness d_h, then in the absence of periodic stripes the layer alignment is transformed into a uniform unidirectional planar one. On the contrary, if there exist a threshold thickness d_p for periodic stripes, and if $d_p < d < d_h$, thus a periodic hybrid alignment (PHAN) takes place, which for d close to d_p involves essentially splay and twist.

In order to find the threshold thickness d_p, let us consider the reduced free energy $G = 2\,F/\,K_{22}\,d_x$ of the cell portion having arbitrary length d_x along the x-axis, and length λ along the y-axis, corresponding to the periodic pattern wavelength. F is the free energy of the same cell portion, and K_{22} is the nematic twist elastic constant. G reads

$$G = \int_0^{\lambda} dy \int_0^{d} g_b \, dz + G_s \tag{1}$$

where g_b is the bulk kernel and G_s is the surface contribution.

By linearizing close to the threshold thickness d_p we have, in the frame of the usual continuum elasticity[16]:

$$g_b = \varkappa_1 \left(\dot{\varphi}_y + \dot{\theta}_z \right)^2 + (\dot{\theta}_y - \dot{\varphi}_z)^2 \tag{2}$$

where $\varkappa_1 = K_{11}/K_{22}$ is the elastic ratio between the splay- (K_{11}) and the twist elastic constant, and the dot means the derivative with respect to y or to z respectively, as defined by the relevant subscript.

For the sake of simplicity, let us suppose that the surface-like saddle-splay elastic constant is given by $K_{24} = -\,K_{22}$: hence the surface contribution, just due to the anchoring, writes

$$G_s = \int_0^{\lambda} \Big[L_{\varphi o}^{\ \ -1} \varphi_o^{\ 2} - \varkappa_1 L_{\theta o}^{\ \ -1} \theta_o^{\ 2} + \\ + L_{\varphi 1}^{\ \ -1} \varphi_1^{\ 2} + \varkappa_1 L_{\theta 1}^{\ \ -1} \theta_1^{\ 2} \Big]\, dy \tag{3}$$

here L_{ij} - $i = \varphi, \theta$; $j = o, 1$ - are de Gennes-Kléman extrapolation lengths[17], given by

$$\begin{cases} L_{\varphi j} = K_{22}/\,W_{\varphi j} \\ L_{\theta j} = K_{11}/W_{\theta j} \end{cases} \tag{4}$$

According to the usual procedures, the Euler-Lagrange equations read[10]

$$\begin{cases} \ddot{\theta}_{yy} + \varkappa_1 \ddot{\theta}_{zz} + (\varkappa_1 - 1)\ddot{\varphi}_{yz} = 0 \\ \varkappa_1 \ddot{\varphi}_{yy} + \ddot{\varphi}_{zz} + (\varkappa_1 - 1)\ddot{\theta}_{yz} = 0 \end{cases} \tag{5}$$

which, if $\varkappa_1 > 1$ as in the present case, are coupled homogeneous second order equations, with the boundary conditions:

$$\begin{cases} \dot{\theta}_{yo} + L_{\varphi o}^{-1} \varphi_o - \dot{\varphi}_{zo} = 0 \\ \dot{\varphi}_{yo} + L_{\theta o}^{-1} \theta_o + \dot{\theta}_{zo} = 0 \\ \dot{\theta}_{y1} - L_{\varphi 1}^{-1} \varphi_1 - \dot{\varphi}_{z1} = 0 \\ \dot{\varphi}_{y1} + L_{\theta 1}^{-1} \theta_1 + \dot{\theta}_{z1} = 0 \end{cases} \tag{6}$$

which are explicity independent of $\varkappa_1$.

The solutions of system (5) have to be searched in the complex range

$$\begin{cases} \theta = \Theta \exp\left[i\,(\alpha z + \beta y)\right] \\ \varphi = \Phi \exp\left[i\,(\alpha z + \beta y)\right] \end{cases} \tag{7}$$

β being the real wave number of the in-plane twist periodic pattern, and α being the possible splay-wave number along z.

By substituting (7) in (5), the characteristic equation is obtained as:

$$(\alpha^2 + \beta^2)^2 = 0 \tag{8}$$

which has four solutions, given by

$$\alpha_1 = \alpha_2 = - i \beta \quad , \quad \alpha_3 = \alpha_4 = i \beta.$$

Thus system (7) reads:

$$\begin{cases} \theta = (C_1 \text{ ch } \beta z + C_2 \, z \text{ ch } \beta z + C_3 \text{ sh } \beta z + C_4 \, z \text{ sh } \beta z) \cos \beta y \\ \varphi = (C'_1 \text{ ch } \beta z + C'_2 \, z \text{ ch } \beta z + C'_3 \text{ sh } \beta z + C'_4 \, z \text{ sh } \beta z) \sin \beta y \end{cases} \qquad (9)$$

with only four independent integration constants:

$$\begin{cases} C_1 = A \, C'_2 - C'_3 \\ C_2 = - C'_4 \\ C_3 = A \, C'_4 - C'_1 \\ C_4 = - C'_2 \end{cases} \qquad (10)$$

where $A = (\varkappa_1 + 1) / [\beta (\varkappa_1 - 1)]$ keeps the information relevant to the bulk elasticity.

By substituting the solutions (9) with the constraint (10) into the boundary conditions (6), we recognize that the 4 x 4 coefficient determinant D, which must be zero in order to avoid trivial case, has the following elements:

1st line: $a_{11} = 1$; $a_{12} = - L_{\varphi o} (\beta A + 1)$; $a_{13} = 0$; $a_{14} = 0$

2nd line: $a_{21} = 0$; $a_{22} = A$; $a_{23} = - 1$; $a_{24} = L_{\theta o} (\beta A - 1)$

3rd line: $a_{31} = \text{ch } \beta d$; $a_{32} = [L_{\varphi 1} (\beta A + 1) + d] \text{ ch } \beta d$;
$a_{33} = \text{sh } \beta d$; $a_{34} = [L_{\varphi 1} (\beta A + 1) + d] \text{ sh } \beta d$

4th line: $a_{41} = -\,\mathrm{sh}\,\beta d$; $a_{42} = A\,\mathrm{ch}\,\beta d + [L_{\theta 1}(\beta A - 1) - d]\,\mathrm{sh}\,\beta d$;
$a_{43} = -\,\mathrm{ch}\,\beta d$; $a_{44} = A\,\mathrm{sh}\,\beta d + [L_{\theta 1}(\beta A - 1) - d]\,\mathrm{ch}\,\beta d$

where the condition $D = 0$ has to be interpreted as an implicit function $d(\beta)$: the minimum of this function gives both the threshold thickness d_p for PHAN splay-stripes and the wave number β_p at this thickness.

DISCUSSION

The aperiodic HAN threshold thickness obtained by Barbero and Barberi[7] was

$$d_h = L_{\theta o} - L_{\theta 1} \tag{11}$$

we point out that just if $d_p = d_h$ the alignment in a layer with $d < d_h$ is unidirectional planar.

Let us now treat some particular cases.

No torsional anchoring

In ref. /13/ the authors dealt with theoretical considerations concerning a special HAN cell, having no anchoring for twist at both walls: the problem was solved without linearizing, but just numerically, by imposing directly the minimum condition to the second variation of the free energy.

Here we found simply that if $L_{\varphi o}^{-1} = L_{\varphi 1}^{-1} = 0$, thus the critical condition $D = 0$ reads $\mathrm{sh}\,\beta d = 0$, i.e. $d_p = 0$ with β indeterminate. Physically, it means that if the torsional anchoring at both walls is very weak, the PHAN is expected to arise for $d_p = 0$, with exceptionally small critical wavelength of the order of the molecular interaction: for every $d < d_h$ the undeformed planar state is unstable, no matter how are the values of the tilt anchoring energies at both walls. We conclude that the experimental disappearance of the stripe domains observed at very small thicknesses of PHAN cells by Lavrentovich et alt.[13] and by Chuvyrov[14] implies that the actual finite anchoring for twist cannot be theoretically disregarded.

Strong twist anchoring

Now the opposite situation has to be considered: strong twist anchoring at both walls. In such a case, $L_{\varphi o} = L_{\varphi 1} = 0$; as a consequence, we always found a region in which PHAN is stable, provided the elastic ratio $r = 1/\varkappa_1$ is sufficiently small. The critical value is $r_c = 0.32$, close to the one reported for the periodic stripes occurring in the planar geometry, due to magnetic field[10]; in fig. 3, it is shown the behavior of both d_p and β_p vs. r for $L_{\theta 1} = 0$, $L_{\theta o} = 2\,\mu m$. Note that in this case $d_h = L_{\theta o}$.

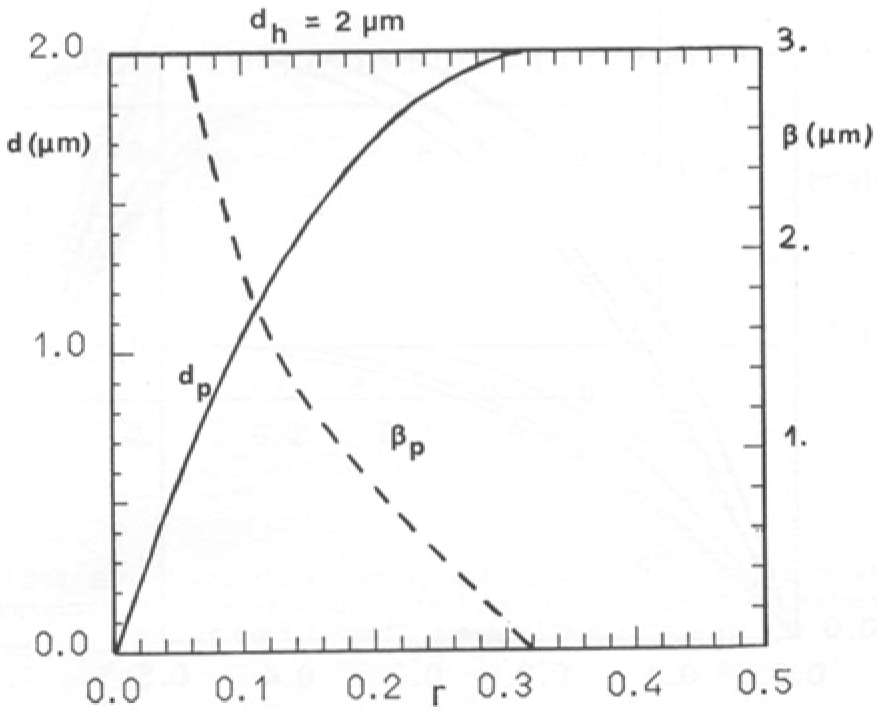

FIGURE 3. Threshold values d_p and d_h vs. $r = K_{22}/K_{11}$ for twist strong anchoring, with $L_{\theta 1} = 0$, and $L_{\theta o} = 1\,\mu m$. The PHAN configuration is stable for $d_p < d < d_h$. The broken line represents the behavior of the wave number β_p at the threshold thickness d_p.

Weak twist anchoring only at the P-wall

By relaxing the twist surface energy at the one of the walls, two different situations arise. Let us consider first, only the twist anchoring at the P-wall to be weak.

The condition $L_{\varphi o} = 0$, by fixing the tilt plane at the side where the favored alignment is homeotropic, enables the PHAN to appear just for small values of r, even though the decrease of the twist anchoring strength at the P-wall yields a small enhancement of the critical value: if $L_{\varphi 1}$ goes from 0 to 5 μm, thus r_c goes from ~0.32 to ~0.42, provided the tilt anchoring at the P-wall is also strong ($L_{\theta 1} = 0$) - see fig. 4; whereas the weakening of the tilt anchoring at the

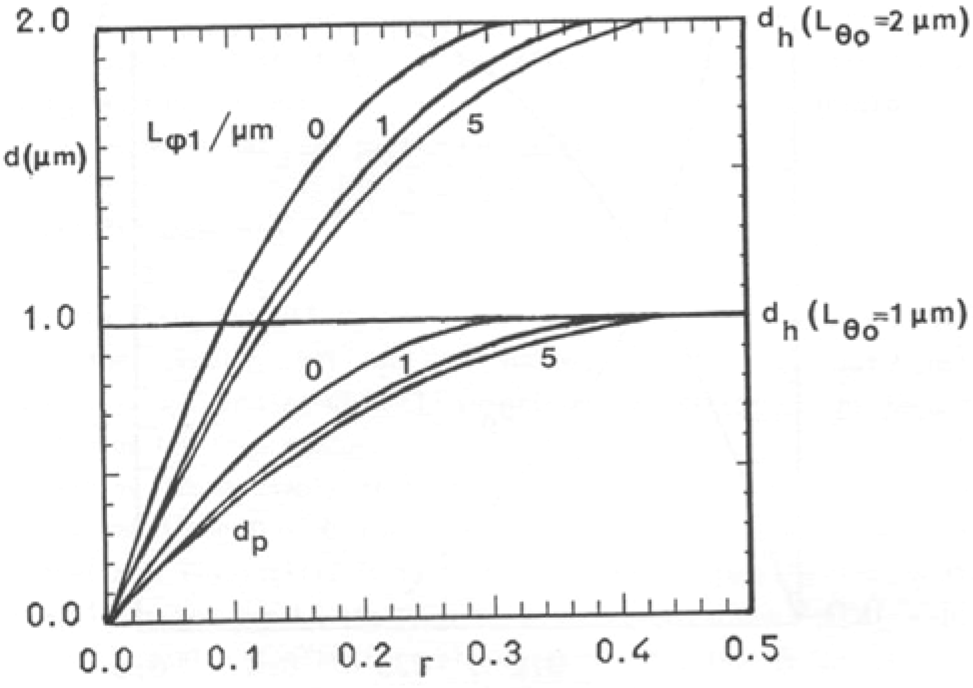

FIGURE 4. Threshold values d_p and d_h vs. r if the twist anchoring at the H-wall is strong, for $L_{\theta 1} = 0$ and for two different values $L_{\theta o} = 1$, 2 μm. In both cases, $L_{\varphi 1}$ ranges from 0 to 5 μm.

P-wall hinders in a certain sense the formation of PHAN, by diminishing the bulk splay energy for the same elastic ratio r, whose critical value is ranging from ~0.15 to ~0.20 and from ~0.20 to ~0.26 when $L_{\theta 1}$ = 1 μm and either $L_{\theta o}$ = 2 μm or $L_{\theta o}$ = 3 μm, respectively - see fig. 5.

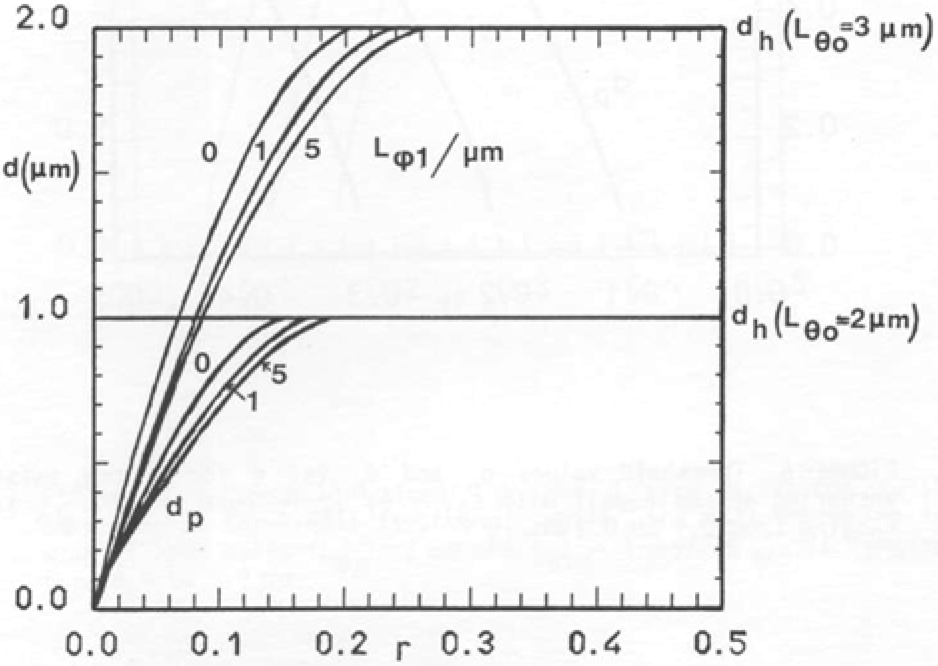

FIGURE 5. Threshold values d_p and d_h vs. r for strong twist anchoring at the H-wall and $L_{\theta 1}$ = 1 μm with two different values $L_{\theta o}$ = 2, 3 μm. In both cases $L_{\varphi 1}$ ranges from 0 to 5 μm.

Weak twist anchoring only at the H-wall

The most interesting result is relevant to the case $L_{\varphi o} \neq 0$. Figs. 6, 7 show the behavior of d_p as a function of r when $L_{\varphi o}$ = 0.1, 0.2, 0.3 μm, i.e. when the anchoring strength for the twist at the H-wall is high again, even if the anchoring could not be considered strong.

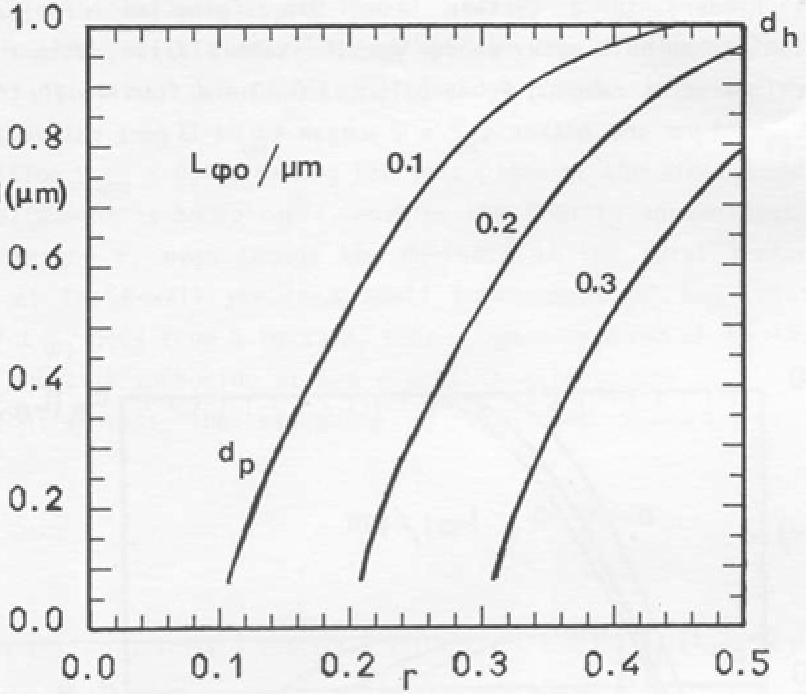

FIGURE 6. Threshold values d_p and d_h vs. r for strong twist anchoring at the P-wall with $L_{\theta 1} = 0$, $L_{\theta o} = 1\ \mu m$ while $L_{\varphi o}$ is ranging from 0.1 to 0.3 μm.

In fig. 6 the case of $L_{\theta o} = 1\ \mu m$, $L_{\theta 1} = 0$ is reported, while in fig. 7 one has $L_{\theta o} = 2\ \mu m$, $L_{\theta 1} = 1\ \mu m$. There it has demonstrated that d_p is strongly dependent on $L_{\varphi o}$; as the twist anchoring energy decreases at the H-wall, d_p deeply diminishes as well, and such an effect is much more greater if $L_{\theta 1} = 0$, the splay energy cost becoming higher.

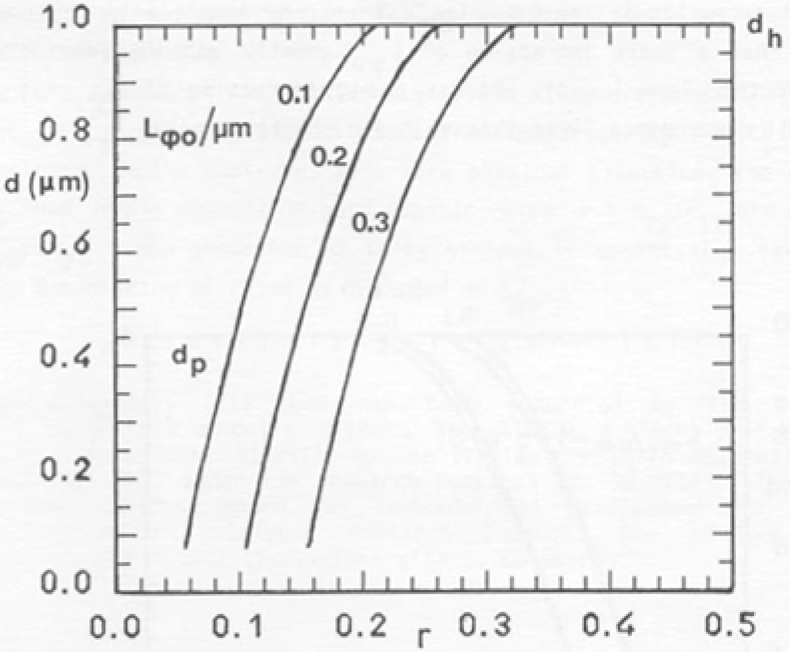

FIGURE 7. Threshold values d_p and d_h vs. r if the twist anchoring at the P-wall is strong, while the tilt anchoring is weak at both walls ($L_{\theta o}$ = 2 μm and $L_{\theta 1}$ = 1 μm). $L_{\varphi o}$ is ranging from 0.1 to 0.3 μm.

General case

When all the anchoring on both surfaces is weak, the effect of the twist surface energy at the H-wall is expected to play the major role, particularly if the tilt anchoring involves high bulk energy cost for splay. In fact, in fig. 8 the effect of different twist anchoring strengths on the PHAN threshold thickness d_p is reported, for a given weak tilt anchoring ($L_{\theta o}$ = 2 μm, $L_{\theta 1}$ = 1 μm). By simple inspection we may see that a small increasing of $L_{\varphi o}$ greatly enhances the PHAN region in the plane [r, d], whereas a big increasing of $L_{\varphi 1}$ has a very small consequence (less than ~2 order of magnitude).

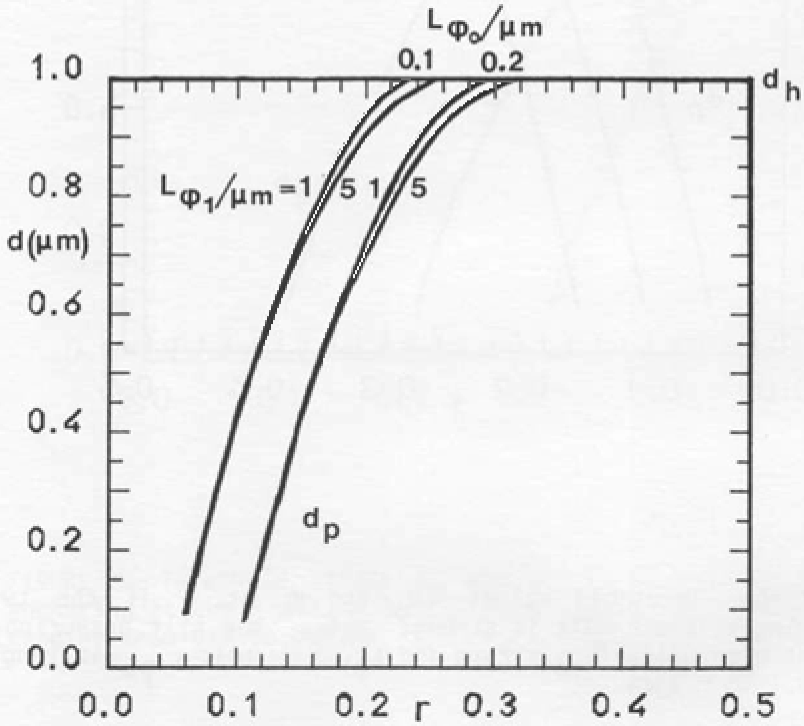

FIGURE 8. Threshold value d_p and d_h vs. r in the general case of weak anchoring both for twist and for tilt ($L_{\varphi o}$ = 0.1, 0.2 μm; $L_{\varphi 1}$ =1, 5 μm; $L_{\theta o}$ = 2 μm; $L_{\theta 1}$ = 1 μm)

CONCLUSIONS

The occurence of a mechanical instability in a HAN layer weakly anchored has been studied. The HAN instability leads to a splay-twist periodic deformation (PHAN), due to the fact that a decreasing of the cell thickness produces a higher splay energy cost, as a result of competitive boundary conditions: such a competition plays the role of an external field.

We have found that the PHAN configuration always is present for $d < d_h = L_{\theta 0} - L_{\theta 1}$, if the twist extrapolation length at the H-wall ($L_{\varphi 0}$) is infinite. On the contrary, in a more physical situation, the presence of PHAN deeply depends on both elastic ratio $r = K_{22}/K_{11}$ and on $L_{\varphi 0} = K_{22}/W_{\varphi 0}$: the occurence of splay stripes is essentially favored by the diminishing of r and by the rise of $L_{\varphi 0}$.

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REFERENCES

1. G. Barbero, and A. Strigazzi, Fizika, **13**, 85 (1981)

2. F. Scudieri, Opt. Commun., **37**, 37 (1981)

3. I Dozov, Ph. Martinot-Lagarde, and G. Durand, J. Phys. (Paris) Lett., **43**, L-365 (1982); **44**, L-817 (1983);
G. Durand, Mol. Cryst. Liq. Cryst., **113**, 237 (1984)
G. Barbero, and A. Strigazzi, J. Appl. Phys., **58**, 2544 (1985);
G. Barbero, and M. Meuti, J. Phys. (Paris), **47**, 311 (1986)
B. Valenti, C. Bertoni, G. Barbero, P. Taverna, and R. Bartolino, Mol. Cryst. Liq. Cryst., **146**, 307 (1987).

4. F. Bloisi, L.R.M. Vicari, and F. Simoni, Mol. Cryst. Liq. Cryst., **179**, 45 (1990)

5. G. Barbero, F. Simoni, and P. Aiello, J. Appl. Phys., **55**, 304 (1984);
F. Simoni, R. Bartolino, G. Barbero, and P. Aiello, Mol. Cryst. Liq. Cryst., **143**, 303 (1984);
F. Simoni, and C. Umeton, Mol. Cryst. Liq. Cryst., **143**, 39 (1987) and references therein

6. In the case of finite boundary energy (weak anchoring), the average directions of the nematic molecules at the boundary are determined both by the anchoring conditions and by the bulk distortion. On the contrary, the strong anchoring implies that the average molecular directions at the substrate are prefixed by the surface treatment.

7. A. Hochbaum, and M. M. Labes, J. Appl. Phys., 53, 2998 (1982)
G. Barbero, and R. Barberi, J. Phys. (Paris), 44, 609 (1983)
G. Barbero, N. V. Madhusudana, and G. Durand, J. Phys. (Paris) Lett., 45, L-613 (1984)
G. Barbero and R. Bartolino, Mol. Cryst. Liq. Cryst., 99, 89 (1983)

8. S. Matsumoto, M. Kawamoto, and K. Mizunaya, J. Appl. Phys., 47, 3842 (1976)

9. F. Lonberg and R.B. Meyer, Phys. Rev. Lett., 55, 718 (1985)

10. E. Miraldi, C. Oldano, and A. Strigazzi, Phys. Rev. A, 34, 4348 (1986)
W. Zimmermann, and L. Kramer, Phys. Rev. Lett., 56, 2655 (1986)
U. D. Kini, J. Phys. (Paris), 47 1829, (1986)

11. G. Barbero, E. Miraldi, and C. Oldano, Phys. Rev. A, 38, 519 (1988)
G. Barbero, and E. Miraldi, Nuovo Cim. D, 11, 1265 (1989)
H. M. Zenginoglou, J. Phys.(Paris), 48, 1599 (1987)
U. D. Kini, J. Phys.(Paris), 51, 529 (1990); Liq. Cryst., 8, 745 (1990)
B. J. Frisken, and P. Palffy-Muhoray, Phys. Rev. A, 40, 6099 (1989).
D. W. Allender, B. J. Frisken, and P. Palffy-Muhoray, Liq. Cryst., 5, 735 (1989)

12. B. J. Frisken, and P. Palffy-Muhoray, Phys. Rev. A, 39, 1513 (1989);
D. W. Allender, R. M. Hornreich, and D. L. Johnson, Phys. Rev. Lett., 59, 2654 (1987)
U. D. Kini, J. Phys. (Paris), 48, 1187 (1987); 49, 527 (1988); Liq. Cryst., 7, 185 (1990)

13. O. D. Lavrentovich, V.M. Pergamenshchik, Mol. Cryst. Liq. Cryst., 179, 125 (1990)

14. A. N. Chuvyrov, private communication

15. G. Barbero, and G. Durand, J. Phys. (Paris), 47, 2129 (1986);
H. Yokoyama, Mol. Cryst. Liq. Cryst., 165, 265 (1988) and references therein;

16. G. Barbero, A. Sparavigna, and A. Strigazzi, Nuovo Cim. D, 12, 1259 (1990)

17. P.G. de Gennes, The Physics of Liquid Crystals (Clarendon Press, Oxford 1974);
M. Kléman, Points, Lignes, Parois (Ed. Physique, Paris 1977)