

# The Time-Domain Vector Fitting Algorithm for Linear Macromodeling

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**Abstract** The Time-Domain Vector Fitting (TD-VF) algorithm for macromodeling of linear multiport systems is presented. A rational approximation for the system matrix transfer function is easily derived from transient input/output port responses. Several validations illustrate the high accuracy of the method.

**Keywords** Time-Domain Vector Fitting, Macromodeling, Rational Approximations.

## 1. Introduction

This paper presents an extension to the so-called Vector-Fitting (VF) algorithm recently proposed in [1]. This algorithm provides very accurate rational approximations to matrix transfer functions of linear systems starting from frequency-domain samples. This method was originally proposed for the analysis of lossy transmission lines, but the technique is general and applicable in principle to the macromodeling of any multiport linear system. The main advantage of VF is use of a two-stage linear least squares approximation for the estimation of the system poles and residues matrices.

The extension herewith presented is denoted Time-Domain Vector Fitting (TD-VF). This indicates that the proposed algorithm works entirely in the time domain. A rational approximation to the system matrix transfer function is derived from a set of input-output transient port responses using a combination of digital filtering and linear least squares solutions. We will show that the resulting TD-VF algorithm is simple, accurate, and efficient.

The material is here presented with focus on the methodology and not on specific applications. However, an application to the macromodeling of an electronic package structure characterized via full-wave electromagnetic analysis will be presented in the numerical results section. This example will show that very accurate macromodels can be obtained via TD-VF even for large structures with a large number of ports.

## 2. Frequency-Domain Vector Fitting

Here we briefly review the basics of the standard Vector-Fitting algorithm. More details can be found in [1]. Let us consider a scalar transfer function  $H(s)$ , where  $s$  is the Laplace variable, to be approximated by a rational function

$$H(s) \simeq H_\infty + \sum_{n=1}^N \frac{R_n}{s - p_n} \quad (1)$$

with unknown poles  $\{p_n\}$  and residues  $\{R_n\}$ . The dataset characterizing the Device Under Modeling (DUM) is a set of frequency samples  $H(j\omega_k)$ .

Standard VF avoids solving the nonlinear fit condition (1) by introducing the following weight function

$$\sigma(s) = 1 + \sum_{n=1}^N \frac{k_n}{s - q_n} = \frac{\prod_{n=1}^N (s - z_n)}{\prod_{n=1}^N (s - q_n)} \quad (2)$$

with fixed (a priori) poles  $\{q_n\}$  and unknown residues  $\{k_n\}$ . For best performance, the poles should be chosen to span uniformly the frequency bandwidth over which the approximation is sought for. Then, the following condition

$$\sigma(s)H(s) \simeq c_\infty + \sum_{n=1}^N \frac{c_n}{s - q_n} \quad (3)$$

is enforced in least-squares sense at the given frequency points, using the available data  $H(j\omega_k)$ . The solution provides the coefficients  $\{c_n\}$ , which are not used for further processing, and the residues  $\{k_n\}$  of  $\sigma(s)$ . The VF condition (3) implies that the poles of the best rational approximation to  $H(s)$  must cancel with the zeros of the weight function, i.e.,  $\{p_n\} = \{z_n\}$ . The latter can be easily derived from  $\{k_n\}$  as the eigenvalues of matrix  $A - b c^T$ , where  $A = \text{diag}\{q_n\}$ ,  $b = (1, \dots, 1)^T$ , and  $c^T = (k_1, \dots, k_n)$ . See [1] for details. This procedure is called “poles relocation”. Once the poles  $\{p_n\}$  are known, a second least-squares solution to Eq. (1) provides the residues. As a result, a rational approximation is computed using a two-stage linear least squares solution, thus avoiding use of possibly critical nonlinear optimization techniques for the direct solution of (1).

## 3. Time-Domain Vector Fitting

The TD-VF algorithm restates the rational approximation (1) in time-domain, using as raw data for the DUM characterization a pair of transient excitation  $x(t)$  and response  $y(t)$  waveforms known at discrete times  $t_k$ . These waveforms are related in the Laplace domain through  $Y(s) = H(s)X(s)$ .

The main VF condition (3) can be restated in input-output form as

$$\sigma(s)Y(s) \simeq \left\{ c_\infty + \sum_{n=1}^N \frac{c_n}{s - q_n} \right\} X(s). \quad (4)$$

Applying inverse Laplace transform and using (2), this conditions translates into

$$y(t) + \sum_{n=1}^N k_n y_n(t) \simeq c_\infty x(t) + \sum_{n=1}^N c_n x_n(t), \quad (5)$$

where the sequences  $x_n(t)$  are defined via the convolution integrals

$$x_n(t) = \int_0^t e^{q_n(t-\tau)} x(\tau) d\tau, \quad \forall n, \quad (6)$$

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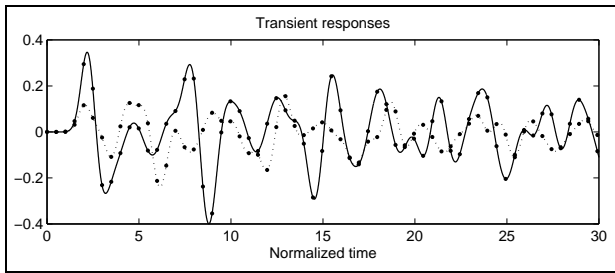


Fig. 1. Macromodeling of a four-port test structure. Transient responses  $y_{11}(t)$  (continuous line) and  $y_{34}(t)$  (dotted line) and their approximations (dots) obtained via TD-VF.

and similarly for  $y_n(t)$ . Due to the exponential nature of the convolution kernels in (6), the actual implementation can be performed via recursive convolutions or, equivalently, with first-order digital filters.

Equation (5) is the main TD-VF condition. Enforcing this condition at discrete times  $t_k$ , using the available data for excitation and responses, and solving in least-squares sense provides the residues  $\{k_n\}$ , and thus the poles  $\{p_n\}$ . If necessary, the poles relocation via (5) can be iterated until convergence. Very few iterations are usually needed. The residues  $\{R_n\}$  and the direct coupling constant  $H_\infty$  are finally computed by solving a second linear least squares problem

$$y(t) \simeq H_\infty x(t) + \sum_{n=1}^N R_n \hat{x}_n(t), \quad (7)$$

equivalent to (1), where the filtered sequences  $\hat{x}_n(t)$  are obtained as in (6) by replacing the initial poles  $\{q_n\}$  with the actual poles  $\{p_n\}$ . The extension to the multi-input multi-output case of the above TD-VF procedure is straightforward, requiring a componentwise application of (5) and (7).

## 4. Numerical examples

In order to validate the TD-VF algorithm a large number of synthetic passive multiports with known poles and residues have been generated. We have performed tests with a number of ports ranging from one up to 16, and with a dynamic order ranging from two up to 41. For each case, a set of synthetic responses have been generated. The excitation signal was set to a Gaussian waveform with a 20dB bandwidth encompassing all poles (in order to excite all independent modes). These responses have been fed to the TD-VF algorithm, which in all cases led to poles estimates that are within machine precision from the original ones in up to three iterations. As a result, also the residues are very accurate, thus leading to undistinguishable approximate responses with respect to the original ones. As an example, we report two relevant waveforms for a 21-th order four-port low-loss structure in Figure 1. The same tests were repeated with different excitation signals, in particular with random multi-sine waveforms. The same accuracy was achieved in the results.

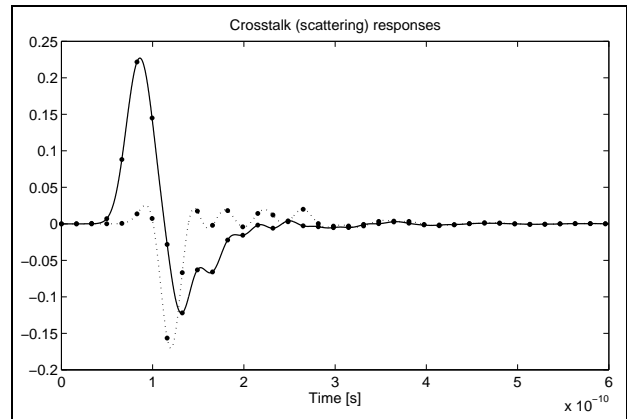


Fig. 2. Macromodeling of a commercial 14-pin package. Near-end (continuous line) and far-end (dotted line) crosstalk scattering waves and their 17-th order approximations (dots) obtained via TD-VF.

We present now the results for a more realistic application. A commercial 14-pin surface mount package has been meshed and analyzed through a full-wave electromagnetic solver based on the Finite-Difference Time-Domain method [2]. The structure has 28 ports, since each pin leads to one port on the die side and one port on the board side. As a result, a complete set of  $28 \times 28$  responses (transient scattering waves) have been obtained, using a unitary Gaussian pulse as excitation with a 20dB bandwidth of 30 GHz. The TD-VF algorithm was able to fit all responses with a 17-th order macromodel, with a maximum deviation between reference and approximate responses below  $4 \times 10^{-3}$ . Figure 2 shows the crosstalk scattering waves induced on a pin adjacent to the excited one. The approximation is excellent. We remark that this rational representation of the matrix transfer function can be synthesized into an equivalent multiport circuit for the entire structure via standard techniques. This equivalent circuit takes into account all relevant signal degradation effects due to the package like mutual couplings, radiation and conductor losses, etc. Therefore, such equivalent circuit can be used for accurate system-level simulations including the typical nonlinearities of the termination networks (drivers and receivers). Such task would be not feasible using a FDTD-based full-wave solver. Finally, we remark that, due to the geometrical complexity of the package, the FDTD simulation required more than three hours for a single run (28 runs were necessary to record the complete set of port responses), whereas the simulation of the generated equivalent circuit required only less than 5 minutes on the same machine.

## 5. Conclusions

In summary, the presented Time-Domain Vector Fitting algorithm appears to provide very accurate rational approximations for linear structures with a possibly large number of ports and dynamic order. This makes TD-VF an interesting technique for the characterization of linear structures in several application areas. We conclude

by noting that other techniques for linear macromodeling from time-domain port responses are available in the literature. Among these, the most relevant are the so-called Subspace-based State-Space System Identification (4SID) techniques [3] and the Generalized Pencil-Of-Function (GPOF) methods [4]. As a preliminary discussion, we remark that both these methods are based on the construction of Hankel matrices storing the time samples of the port responses. These matrices can grow very large in case of high-order structures with many ports. The proposed TD-VF method allows to circumvent this problem since it is based on less storage and computing power demanding steps. A detailed comparison is outside the scope of this short paper but will be the subject of a future investigation.

## References

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