Composition laws for learning curves of industrial manufacturingprocesses

Composition laws for learning curves of industrial manufacturing processes

F. FRANCESCHINI†* and M. GALETTO†

The theory of learning curves is widely investigated in many fields related to production planning, quality improvement and cost analysis. Many different approaches to describe the learning mechanism of a process are reported in the academic literature. The aim is to analyse the behaviour of complex systems composed of a network of elementary processes whose learning curve is known. Composition laws of two basic aggregation structures, series and parallel, are discussed and analysed. The effects of these composition laws are shown in a series of practical examples.

1. Introduction

The theory of learning curves has been widely investigated by many authors (Yelle 1979, Venezia 1985, Muth 1986, Cherrington *et al*. 1987, Zangwill and Kantor 1998, Dar-El Ezey 2000, Franceschini 2002). Its main applications are in the areas of production planning, quality improvement and cost analysis.

Learning curves are employed to describe the evolution over time of many business processes, such as labour costs, time to market, outgoing defect levels, time delivery, lead time, manufacturing cycle time, process defect level, yield, etc. Organizations use these curves to promote the elimination of problems, to enhance the rate of improvement and to provide process behaviour forecasts. They are also employed to estimate the labour costs related to a production process of new products.

Many different learning models based on theoretical or empirical approaches have been proposed (Levy 1965, Sahal 1979, Roberts 1983, Venezia 1985, Muth 1986, Cherrington *et al*. 1987, Schneiderman 1988, Zangwill and Kantor 1998). The most common ones are the exponential and the power models, which are applied in a wide variety of fields from process analysis to quality and cost analysis (Cherrington *et al*. 1987, Schneiderman 1988, Franceschini 2002).

The main characteristics of learning curves, as described in a wide part of literature, are as follows (figure 1).

- Upward concavity (Muth 1986, Zangwill and Kantor 1998).
- Asymptotic ‘plateau effect’ (due to the eventual lack of any improvement with additional output) (Conway and Shultz 1959, Baloff 1971).
- In some cases, an initial downward concavity (Garg and Milliman 1961, Muth 1986).

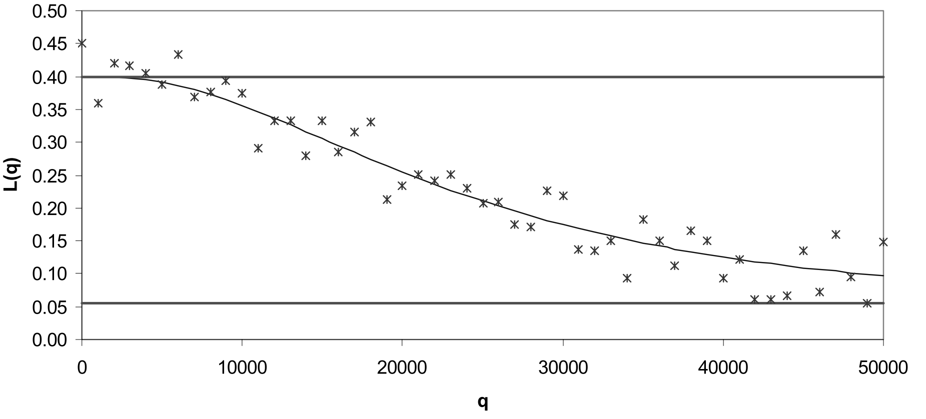

Figure 1. Typical example of an average learning curve model (continuous line) and the corresponding experimental results (stars). $L(q)$ is the fraction of non-conforming units of a manufacturing process, where q is the production cycle (cumulative input) after the start-up.

Further, some authors observe that, in certain circumstances, an unexpected discontinuity happens (e.g. in production cycles learning, a sudden reduction in labour hours may occur after significant periods of no improvement at all (Abernathy and Wayne 1974).

As a first approximation, a manufacturing process can be described as a framework able to convert raw materials into finished products. Complex production systems can be thought of as a composition of single processes connected to each other in different ways. The aim of this paper is to present a method for the study of learning, interpreted as a Quality Improvement, in complex manufacturing systems, composed of a network of processes.

2. Learning curves in production analysis

Theories attempting to describe the learning curve for a manufacturing process have been developed by Crossman (1959), Levy (1965), Sahal (1979), Roberts (1983), Venezia (1985), Muth (1986) and Zangwill and Kantor (1998). Experimental analyses have been proposed by Cherrington *et al.* (1987), Schneiderman (1988) and Bailey and McIntyre (1997).

The earliest model attempting to explain productivity improvements and technological change was formulated by March and Simon (1958). Their theory relies on a ‘performance gap’, which is the difference between actual and desired (aspired) performance. Search activity intensifies as the performance gap widens and wanes as it narrows. The theory asserts that search activity never fully stops, not even in the limit, so the theory is inconsistent with a level plateau (Muth 1986).

Crossman (1959) assumed that an individual facing a new task tries out various methods, retaining the more successful ones and rejecting the less successful (’trial-and-error learning’).

Levy’s (1965) theoretical approach, at least under certain conditions, predicts the change in concavity. The model developed by Levy is based on the assumption that the rate of increase in the rate of production as the firm gains experience is proportional to the amount that the process can improve. The family of curves that comes out has an initial downward concavity and approaches a plateau gradually. An important limitation of Levy's model is that a differential equation describes little of the process by which innovation takes place (Muth 1986).

Venezia (1985) indicates that if the firm learns about the parameters of the production function from previous observations of allocations and their outputs, then a learning curve phenomenon will emerge. The model assumes the existence of an objective function $f(x)$ with a maximum at $x = x^*$. The maximizing value x^* is not known, but is approximated with decreasing error as additional information is received. Venezia's model implies that better estimates from past operating data lead to improved allocation of resources.

Another approach is presented by Muth (1986), who analysed the construction of learning curves on a theory based upon random search within a fixed population of technological possibilities. The theory is consistent with the power function relation between unit costs and cumulative output. It is also consistent with initial rates of improvement smaller than those predicted later by the power function relation.

From an empirical point of view, Cherrington *et al.* (1987) show that learning curves can be described by two basic families of models: the hyperbolic and the exponential.

Schneiderman (1988) argued that for the analysis of Quality Improvement Process the most practical model is the exponential model:

$$L(t) - L_{\min} = (L_0 - L_{\min}) \cdot \mathrm{e}^{\left(-a \cdot \frac{(t-t_0)}{t_{1/2}}\right)} \tag{1}$$

where $L(t)$ is the defect level over time (errors, rework, yield loss, unnecessary reports, and, in general, any measurable quantity that is in need of improvement), $L_{\min}$ equals the minimum achievable defect level, L_0 is the initial defect level, t equals time, t_0 equals initial time, a equals $ln(2)$ and $t_{1/2}$ equals 'defect half-life' (the nature of this parameter is that for each increment in time that is equal to the half-life, the defect level drops, on average, by 50%). Many experimental investigations confirm this law (Schneiderman 1988).

In a recent work, Bailey and McIntyre (1997) analyse the relation between fit of experimental data and prediction. Their analysis is based on power models that take into account learning and relearning effects.

Recently, Zangwill and Kantor (1998) showed that a great deal of basic models to describe evolution curves may all be related to a unique differential equation based on the Volterra–Lotka form (usually called 'prey–predator equation') (Murray 1989). This represents an important contribution to the unification of the entire learning curves theory. The relevant part of the Zangwill and Kantor model is the so-called 'Postulate Five'. It states that the total metric value (which can be, for example, the fraction of non-conforming units of a manufacturing process) for the entire process is the sum of the metric values on the components of the process. In this way, the basic hypothesis of Zangwill and Kantor's model can be extended to the analysis of networks of elementary processes.

3. Proposed method

The present paper proposes a new approach for describing the learning mechanism of a complex system. The basic idea is to interpret a complex manufacturing system as a 'network' of processes ('elementary blocks'), referring to one variable factor of production (Venezia 1985). This approach is reminiscent of network use in

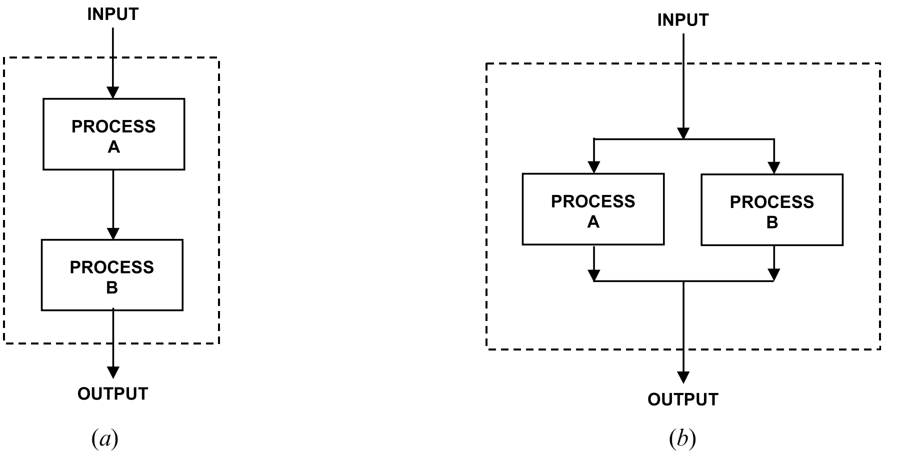

Figure 2. (*a*) Example of a 'learning block diagram' for a series structure. (*b*) Example of a 'learning block diagram' for a parallel structure.

calculating the failure rate of a complex system, in accordance with reliability theory. We assume as a factor of production the rate of non-conforming units of an industrial manufacturing plant. We define as an 'elementary block' a process whose learning curve can be described by means of one of the models described in the previous section. A 'series structure' is a combination of cascade blocks (figure 2a). It is assumed that each elementary block operates independently (without mutual influence) of other blocks. A 'parallel structure' consists of n elementary blocks sharing the production load (figure 2b). We define a 'network of processes' as a complex structure composed of series and/or parallel processes.

Let us consider, for example, a manufacturing plant for the production of coachworks in an automotive firm. The system can be subdivided into the following processes (figure 3): cutting of metal sheet, pressing and assembling. Each manufacturing phase can be represented as an elementary block. The learning curve of the whole plant is conditioned by the learning curve of each single process and by their specific connections. Monitoring Quality Improvement means detecting the fraction of non-conforming products over production cycles of the entire manufacturing plant, i.e. its learning curve (Schneiderman 1988).

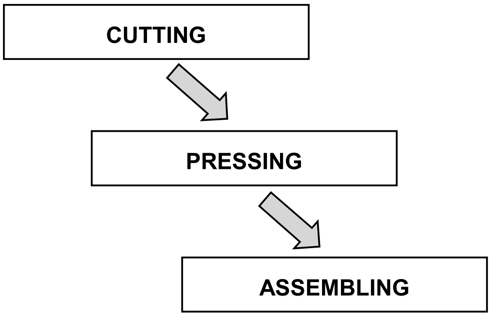

Figure 3. Scheme of a manufacturing system for the production of automotive coachworks.

4. Quality improvement learning curve

Let us consider a generic manufacturing process. We define as q the production cycle (cumulative input) of the entire process after the start-up, and $D(q)$ the cumulative number of scrapped components. Let us also define as $F(q)$ the fraction of cumulative scrapped components over cumulative input (figure 4):

$$F(q) = \frac{D(q)}{q} \tag{2}$$

In accordance with the cited learning models, the theoretical learning curve of the whole system can be expressed as follows:

$$L(q) = \frac{dD(q)}{dq}. \tag{3a}$$

The learning curve for a discrete manufacturing process can be expressed as:

$$L(q) = \frac{\Delta D(q)}{\Delta q} = \frac{D(q+N) - D(q)}{(q+N) - q}, \tag{3b}$$

where N is an established number of production cycles (as, for example, a daily production).

According to equations (2) and (3) the relationship between $F(q)$ and $L(q)$ is:

$$L(q) = F(q) + q \cdot \frac{\mathrm{d}F(q)}{\mathrm{d}q}, \tag{4a}$$

or, equivalently:

$$F(q) = \frac{\int_0^q L(x) \cdot \mathrm{d}x}{q}. \tag{4b}$$

Defining $D(q)$ as a positive non-decreasing quantity, $L(q)$ is also a positive function describable by the mathematical models of learning curve literature (Zangwill and Kantor 1998). Figure 4 shows a comparison between $D(q)$, $F(q)$ and $L(q)$, when $L(q)$ has an exponential behaviour.

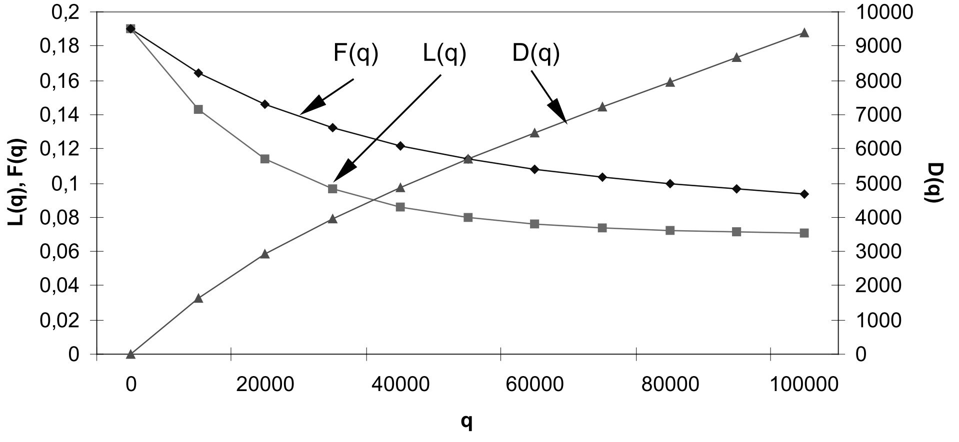

Figure 4. Comparison between $D(q)$, $F(q)$ and $L(q)$ for a generic manufacturing process.

5. Series structures

Let us consider the series manufacturing process reported in figure 5. Let us define as q the production cycle (cumulative input) of the entire process after the start-up, $D_A(q_A)$ the cumulative number of scrapped components after the processing of q_Ath cycle in the block A, $D_B(q_B)$ the cumulative number of scrapped components after the processing of q_Bth cycle in the block B, $D(q) = D_A(q_A) + D_B(q_B)$ the cumulative number of scrapped components after the processing of qth cycle in the series structure $A \xrightarrow{S} B$.

Let us also define the following quantities.

- $F_A(q_A) = \dfrac{D_A(q_A)}{q_A}$ the fraction of scrapped components at q_Ath cycle by process A.
- $F_B(q_B) = \dfrac{D_B(q_B)}{q_B}$ the fraction of scrapped components at q_Bth cycle by process B.
- $F_{eq}(q) = \dfrac{D(q)}{q}$ the fraction of scrapped components at qth cycle by the 'equivalent' process (process $A \xrightarrow{S} B$).
- $L_A(q_A) = \dfrac{dD_A(q_A)}{dq_A}$ process A learning curve at q_Ath cycle.
- $L_B(q_B) = \dfrac{dD_B(q_B)}{dq_B}$ process B learning curve at q_Bth cycle.
- $L_{eq}(q) = \dfrac{dD(q)}{dq}$ process A $\xrightarrow{S}$ B learning curve at qth cycle.

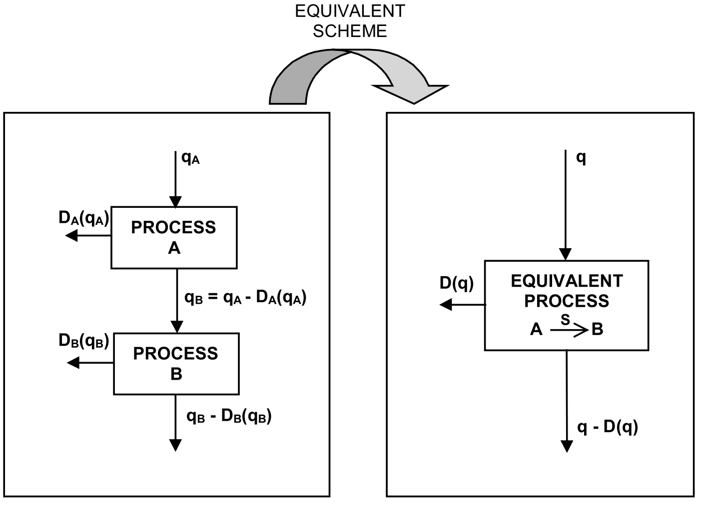

Figure 5. Scheme of a series structure, where $D(q)$ is the cumulative number of scrapped components after the processing of qth cycle in the series structure, $D_A(q_A)$ is the cumulative number of scrapped components of process A at the q_Ath cycle, $D_B(q_B)$ is the cumulative number of scrapped components of process B at the q_Bth cycle.

Under the condition of a complete independence among processes (i.e. without 'knowledge' exchange between processes), it can be shown that (see Appendix):

$$L_{\mathrm{eq}}(q) = L_{\mathrm{A}}(q) + L_{\mathrm{B}}\{q \cdot [1 - F_{\mathrm{A}}(q)]\} - L_{\mathrm{A}}(q) \cdot L_{\mathrm{B}}\{q \cdot [1 - F_{\mathrm{A}}(q)]\} \tag{5}$$

Knowing processes A and B learning curves, we can determine $L_{\mathrm{eq}}(q)$.

5.1. *Example of series structures*

Let us consider again the manufacturing plant for the production of automotive coachworks (figure 3).

(1) A is a process (cutting of metal sheet) with an average learning curve described by the following exponential form:

$$L_{\mathrm{A}}(q_{\mathrm{A}}) = 0.07 + (0.19 - 0.07) \cdot e^{-\frac{q_{\mathrm{A}}}{30000}},$$

where q_{A} is the cumulative number of cycles performed by process A.

From a practical point of view, the process presents at the start up phase a percentage of non-conforming equal to 19% and an asymptotic non-conforming value of 7%. Furthermore, 30 000 is the process 'time constant'.

(2) B is a process (pressing) with an average learning curve described by the following exponential form:

$$L_{\mathrm{B}}(q_{\mathrm{B}}) = 0.08 + (0.21 - 0.08) \cdot e^{-\frac{q_{\mathrm{B}}}{20000}},$$

where q_B is the cumulative number of cycles performed by process B.

This process, at the start up, presents a percentage of non-conforming equal to 21% and an asymptotic value of 8%. 20 000 is the B process 'time constant'.

Figure 6 shows, respectively, the learning curves for the processes A and B, and the equivalent process A $\xrightarrow{S}$ B.

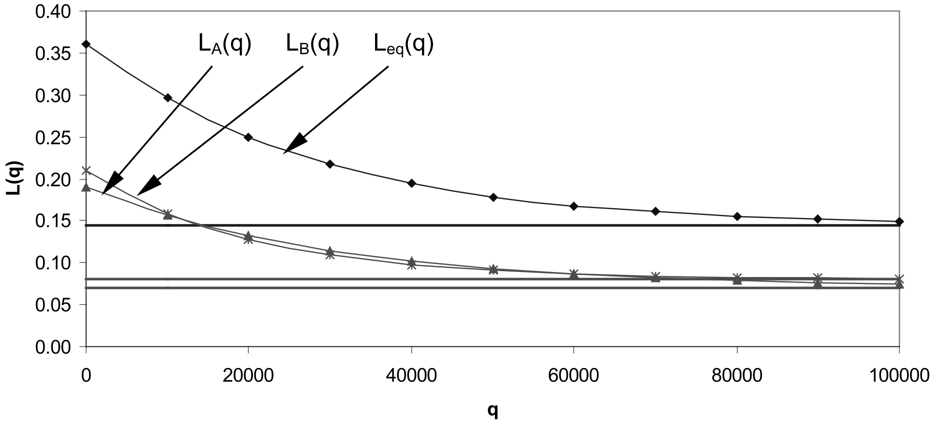

Figure 6. Average learning curve (fraction of non-conforming) for a series structure: Process A learning curve (cutting of metal sheet) (triangles); Process B learning curve (pressing) (stars); Equivalent series process learning curve $A \xrightarrow{S} B$ (cutting of metal sheet $\xrightarrow{S}$ pressing) (squares).

In accordance with equation (5), the equivalent learning curve for the whole manufacturing process is:

$$L_{\mathrm{eq}}(q) = 0.1444 + 0.1104 \cdot e^{-\frac{q}{30000}} + 0.1209 \cdot e^{-\frac{q \cdot \left[1-\left(0.07+\frac{(0.19-0.07)\cdot 30000}{q} \cdot \left(1-e^{-\frac{q}{30000}}\right)\right)\right]}{20000}} +$$

$$- 0.0156 \cdot e^{-\frac{q}{30000}} \cdot e^{-\frac{q \cdot \left[1-\left(0.07+\frac{(0.19-0.07)\cdot 30000}{q} \cdot \left(1-e^{-\frac{q}{30000}}\right)\right)\right]}{20000}}.$$

The start up value is $L_{\mathrm{eq}}(0) = 36.01\%$, and the asymptotic value is $L_{\mathrm{eq}}(\infty) = 14.44\%$.

As intuitively expected, these results show that the series structure has a $L_{\mathrm{eq}}(q)$ constantly higher than the two single process learning curves. From the learning point of view, a series connection manifests a 'learning delay'. This is due to the fact that the number of working cycles of block B depends on the output of block A. As a consequence, the learning process of block B is 'lowered' by that of block A.

6. Parallel structures

Let us consider the parallel structure reported in figure 7. We define as q the production cycle (cumulative input) of the entire process after the start-up, $D_{\mathrm{A}}(q_{\mathrm{A}})$ the cumulative number of scrapped components after the processing of q_{A}th cycle in block A, $D_{\mathrm{B}}(q_{\mathrm{B}})$ the cumulative number of scrapped components after the processing of q_{B}th cycle in block B, and $D(q) = D_{\mathrm{A}}(q_{\mathrm{A}}) + D_{\mathrm{B}}(q_{\mathrm{B}})$ the cumulative number of scrapped components by the parallel structure A//B after the processing of qth cycle.

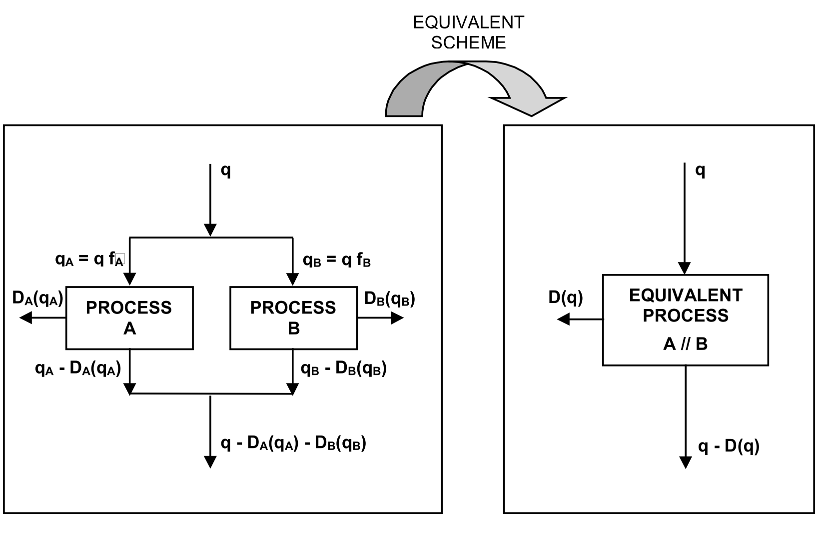

Figure 7. Scheme of a parallel structure, where D(q) is the cumulative number of scrapped components after the processing of the qth cycle in the parallel structure, $D_{\mathrm{A}}(q_{\mathrm{A}})$ is the cumulative number of scrapped components of process A at the q_{A}th cycle, $D_{\mathrm{B}}(q_{\mathrm{B}})$ is the cumulative number of scrapped components of process B at the q_{B}th cycle, and f_{A} and f_{B} are, respectively, the fraction of components worked by process A and B.

Let us also define:

- $f_A = \dfrac{q_A}{q}$ the fraction of components worked by process A, when the whole system has worked q components; and
- $f_B = \dfrac{q_B}{q}$ the fraction of components worked by process B, when the whole system has worked q components.

Under the condition of a complete independence among processes (i.e. without 'knowledge' exchange between processes), it can be shown that (see Appendix):

$$L_{eq}(q) = f_A \cdot L_A(q \cdot f_A) + f_B \cdot L_B(q \cdot f_B). \tag{6}$$

Introducing the 'capacity parameter' $f = f_A/f_B$, equation (3) becomes:

$$L_{eq}(q) = \frac{f}{1+f} \cdot L_A\left(q \cdot \frac{f}{1+f}\right) + \frac{1}{1+f} \cdot L_B\left(q \cdot \frac{1}{1+f}\right). \tag{6'}$$

Knowing processes A and B learning curves we can determine $L_{eq}(q)$.

6.1. *Example of parallel structure*

Let us consider again the manufacturing plant of automotive coachworks. The learning curves of two similar assembling processes, A and B, are respectively:

$$L_A(q_A) = 0.04 + (0.12 - 0.04) \cdot e^{-\frac{q_A}{30000}} \quad \text{and} \quad L_B(q_B) = 0.03 + (0.10 - 0.03) \cdot e^{-\frac{q_B}{20000}}.$$

Processes A and B have different asymptotic values. Furthermore, process B is quicker in learning than process A, so the plateau level is reached earlier by process B and later by process A.

Process A and B capacity parameter is $f = \frac{3}{2}$.

In accordance with equation (6′) the equivalent learning curve becomes (figure 8):

$$L_{eq}(q) = 0.036 + 0.028 \cdot e^{-\frac{q}{50000}} + 0.048 \cdot e^{-\frac{q}{50000}}.$$

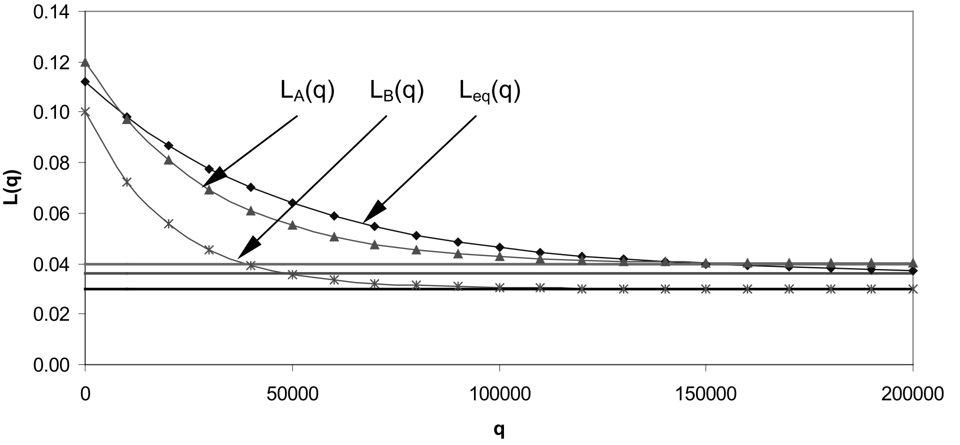

Figure 8. Average learning curve for a parallel structure: Process A learning curve (assembling 1) (triangles); Process B learning curve (assembling 2) (stars); Equivalent process learning curve $A//B$ (assembling 1//assembling 2) (squares).

The start up value for the equivalent learning curve is $L_{eq}(0) = 11.2\%$, and the asymptotic value $L_{eq}(\infty) = 3.6\%$. As intuitively expected, the asymptotic value of the parallel structure is contained between the asymptotic values of each single process. However, the equivalent learning curve can be higher than the two single learning curves of the two composing blocks (figure 8).

It is interesting to note that, if $f = 1$:

$$L_{eq}(q) = \frac{L_A\left(\frac{q}{2}\right) + L_B\left(\frac{q}{2}\right)}{2}. \tag{7}$$

Furthermore, if $L_A(q) = L_B(q)$:

$$L_{eq}(q) = L_A\left(\frac{q}{2}\right) = L_B\left(\frac{q}{2}\right). \tag{8}$$

From the learning point of view, equation (8) shows that a system composed of two equal parallel blocks is slower than each single block. However, as production rate increases over time the equivalent process learning increases too.

7. Process network

A complex manufacturing plant can be interpreted as a network of processes connected to each other by series or parallel structures. Figure 9 shows a manufacturing scheme of a coachwork in an automotive firm (see section 3). A and B are two parallel processes for the cutting of metal sheet, and C is a pressing process.

The learning curves for A, B and C are respectively (figure 10):

$$L_A(q_A) = L_B(q_B) = 0.065 + (0.21 - 0.065) \cdot e^{-\frac{q_A}{35000}}$$

and

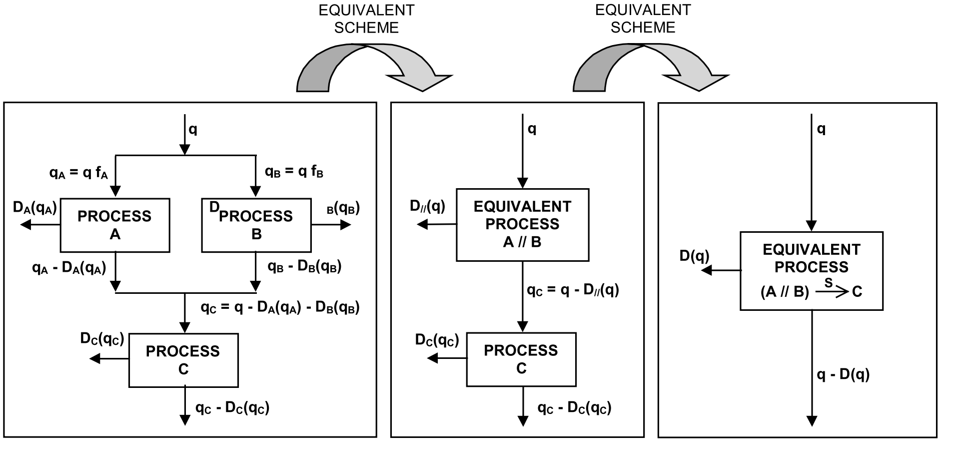

Figure 9. Scheme of a network of process, where $D(q)$ is the cumulative number of scrapped components after the processing of the qth cycle in the series structure, $D_A(q_A)$, $D_B(q_B)$ and $D_C(q_C)$ are the cumulative numbers of scrapped components by process A, B and C, respectively, f_A is the fraction of components worked by process A, and f_B is the fraction of components worked by process B.

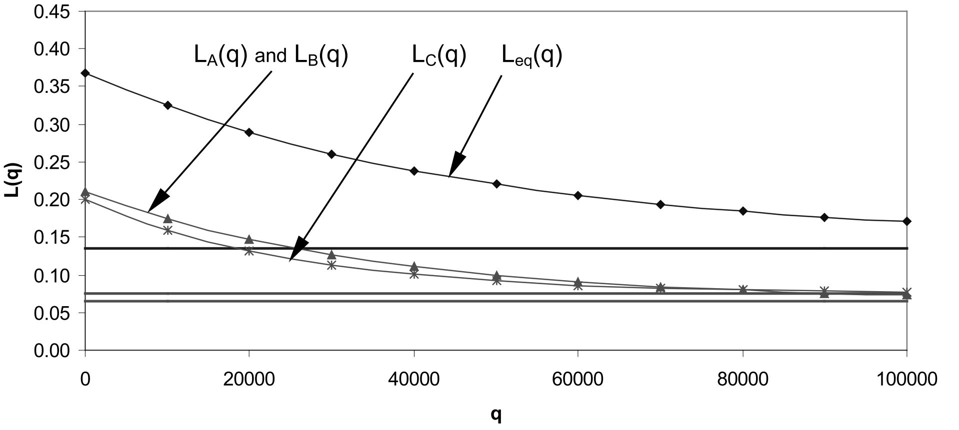

Figure 10. Average learning curve for the process network reported in figure 9. Processes A and B learning curve (cutting of metal sheet) (triangles); Process C learning curve (pressing) (stars). Total equivalent process network learning curve $(A//B) \xrightarrow{S} C$ (squares).

$$L_C(q_C) = 0.075 + (0.2 - 0.075) \cdot e^{-\frac{q_C}{25000}}.$$

Processes A and B capacity parameter is $f = \frac{3}{2}$.

By sequentially applying the composition laws, we can define the equivalent curve for A//B and the total equivalent curve $L_{eq}(q)$ for $(A//B) \xrightarrow{S} C$ (figure 10).

The start up learning value for this network is $L_{eq}(0) = 36.8\%$, and its asymptotic value is $L_{eq}(\infty) = 13.5\%$.

These values are bigger than each single elementary process. This is due to the combined effect of series and parallel structures which produces a delay in the learning of the composed system.

In accordance with the proposed model, only similar processes can be connected in parallel. We define 'similar' two processes that produce the same part or component (e.g. two processes producing the same kind of a car door). If the two processes are not similar, the situation becomes more complicated. Referring to the case of two 'not-similar' processes that produce complementary components (e.g. respectively producing right- and left-side car doors), the effective production of the whole system is really the minimum of their two productions and not a combination of them. In this case, the two different parallel processes must be considered as one unique process with a learning curve equal to the minimum of the two.

Furthermore, it is possible to highlight certain special behaviours for the output of particular networks. Let us consider, for instance, a series structure composed by the following processes:

$$L_A(q_A) = 0.06 + (0.9 - 0.06) \cdot e^{-\frac{q_A}{40000}} \quad \text{and} \quad L_B(q_B) = 0.04 + (0.8 - 0.04) \cdot e^{-\frac{q_B}{14000}}.$$

Figure 11 shows the learning curves of processes A, B and their composition. We observe a change in concavity of the equivalent learning curve. The curve shows an initial downward concavity, followed by an upward concavity which presents a 'plateau effect'. This behaviour is probably due to a lack of any improvement with additional input. The shape can be explained by considering a sort of inertial effect due to the contemporary activation of the two systems (Muth 1986).

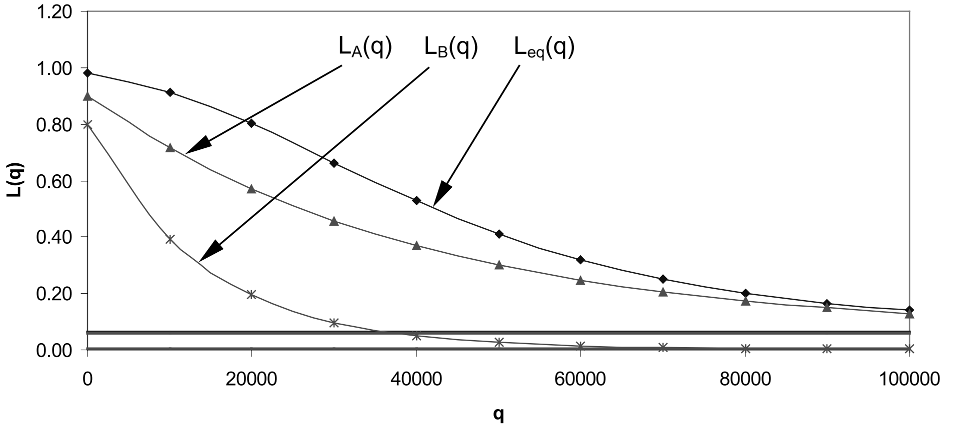

Figure 11. Example of an average learning curve for a particular series structure: Process A learning curve (triangles); Process B learning curve (stars); Series process learning curve $A \xrightarrow{S} B$ (squares).

It must be noted that this phenomenon is due to a very high percentage of non-conforming units at the start up (90% process A and 80% process B) and a very fast decreasing of the learning curves till the condition of plateau (very small, in comparison to the start-up value).

The series start up value is $L_S(0) = 98\%$, and the asymptotic value is $L_{eq}(\infty) = 6.38\%$. As in the previous series cases, the composed learning curve is constantly higher than the original two.

8. Organization of the method

The proposed method can be used as a planning tool to forecast the learning behaviour of new complex systems or plants over cumulative production.

From a practical point of view, it can be organized as follows.

- Definition of one or more possible plant configurations (design alternatives) in order to satisfy specific production requirements.
- Determination of the learning curves for each simple process; management can study the performance of similar processes or may determine them by means of experimentation or simulation analysis.
- Reduction of the complex network to a single equivalent block (equivalent learning curve L_{eq}).
- Comparison and analysis of the equivalent learning curves; evaluation of 'time constant' and asymptotic behaviours.
- Choice of the best solution from the learning point of view (under the same productivity conditions) (figure 12).

The main characteristic of the method is related to its capability to provide a forecast of non-conforming units of a complex plant. This information is very helpful during the preliminary design phases, allowing a rationalization of process schemes.

If we consider two different design solutions to satisfy a given production, the method helps to evaluate which has to be preferred from the asymptotic non-

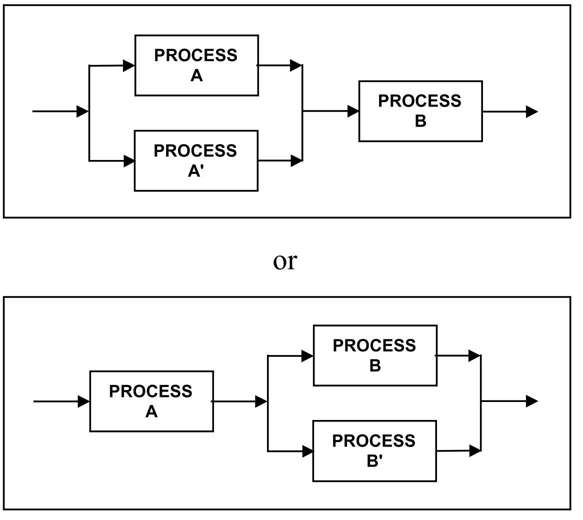

Figure 12. Comparison between two different design alternatives from the learning point of view (under the same productivity conditions).

conforming (learning) point of view. This analysis can be conducted both on existing production systems and on new plants.

Furthermore, this approach is superior to the trivial 'practitioner approach' which consists in defining for each of the system components its own learning curve equation, calculating the number of defective units for every production cycle, summing them, and generating a new learning curve based on a regression line of the summation points. First, as we have shown, the new learning curve cannot be obtained by a mere summation of the learning curves of each single block. Second, the proposed method can be used as a planning tool to make an 'a priori' comparison of different plant configurations without building them operatively.

The method can be easily automated by implementing a software package able to analyse complex networks composed by the two basic series and parallel structures.

Furthermore, suppose management wants to speed up the learning in a production process. To begin doing this, it wants to invest some money to improve the learning rate of one operation (elementary block). Suppose the cost to obtain a given percent improvement in the learning rate is the same for all operations. With the support of the proposed method, it is possible to evaluate all possible configurations, and select which operation the management should try to improve first.

9. Further considerations

As it is possible to see by previous examples the learning expression for series and parallel structures confirm the intuitive hypothesis that the equivalent learning process is decreasing with the increasing of system complexity. Up to this point, we only analysed processes that interact in a complex structure, without influencing each other. All conclusions achieved in the first part of the paper are obtained under the condition of a complete independence among processes. However, in real manufacturing systems it is easy to find configurations with a 'mutual influence' among processes. Let us consider, for example, a parallel structure composed by two communicating processes. The exchange of 'knowledge' between the two processes can influence the respective learning curves. As a first approximation, this event can be modelled by introducing a new parameter describing the learning exchange.

For two parallel processes A and B with $f = 1$ and $L_A(q) = L_B(q)$, we can model the synergic effect defining a learning coefficient α which takes into account their mutual influence:

$$L_{\alpha-\mathrm{eq}}(q) = (1-\alpha) \cdot L_A\left(\frac{q}{2}\right) \qquad 0 \leq \alpha < 1, \tag{9}$$

where α is the parameter that describes the synergy under the hypothesis of a 'constructive' influence. If $\alpha = 0$ there is no synergy. Some preliminary experimentation shows that α can assume values close to 0 (typical values are 0/0.05).

Let us consider the configuration reported in figure 13. The equivalent learning curve with no mutual influence, with $L_B(q) = L_A(q)$ and $f = 1$, is:

$$L_{\mathrm{eq}}(q) = L_A\left(\frac{q}{2}\right) + L_C\left[q \cdot \left(1 - F_A\left(\frac{q}{2}\right)\right)\right] - L_A\left(\frac{q}{2}\right) \cdot L_C\left[q \cdot \left(1 - F_A\left(\frac{q}{2}\right)\right)\right]. \tag{10}$$

Considering a mutual influence between the two parallel blocks A and B, we obtain the following expression:

$$\begin{aligned} L_{\mathrm{eq}}(q) = (1-\alpha) \cdot L_A\left(\frac{q}{2}\right) + L_C\left[q \cdot \left(1 - (1-\alpha) \cdot F_A\left(\frac{q}{2}\right)\right)\right] + \\ - (1-\alpha) \cdot L_A\left(\frac{q}{2}\right) \cdot L_C\left[q \cdot \left(1 - (1-\alpha) \cdot F_A\left(\frac{q}{2}\right)\right)\right]. \end{aligned} \tag{11}$$

This last relationship produces a sensible reduction of the equivalent learning curve values. Figure 14 shows the two equivalent learning curves with, respectively, $\alpha = 0$ and 0.05.

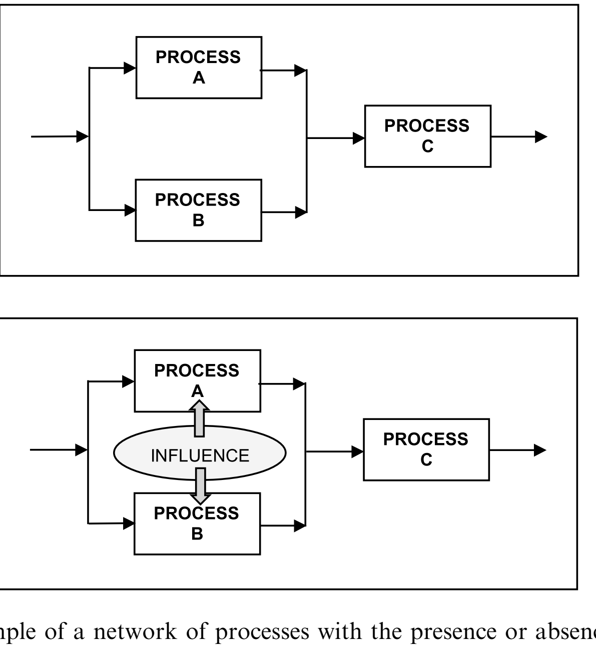

Figure 13. Example of a network of processes with the presence or absence of a mutual influence between blocks.

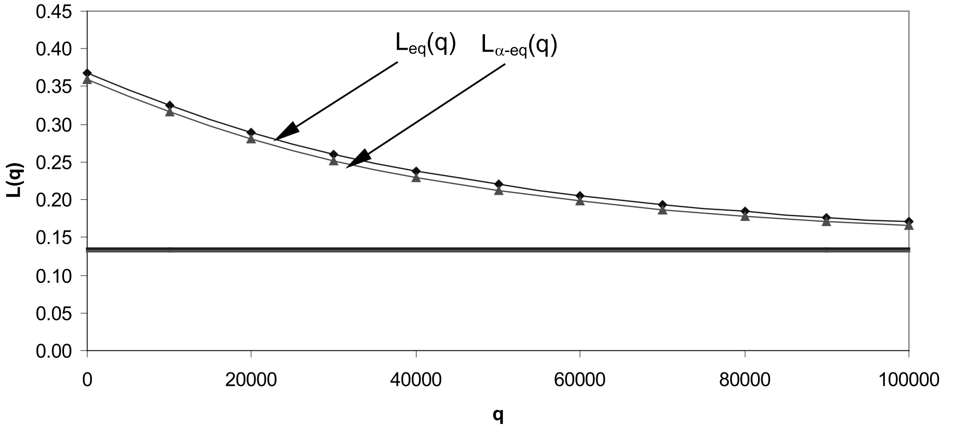

Figure 14. Comparison between the two equivalent average learning curves for the network of processes in figure 13. No influence between processes A and B (squares), mutual influence (triangles) (with $\alpha = 0.05$).

According to Landry and Oral's classification, five types of validation can be defined: conceptual, logical, experimental, operational and data validation (Landry *et al.* 1983, Landry and Oral 1993). In this paper, we mainly focus our attention towards conceptual and experimental validation. In order to verify the behaviour of a complex network composed by a set of elementary blocks (processes), theoretical results have been tested by a MATLAB simulation program. Tests have been carried out with the assumption of a complete independence among processes. Currently, an experimental campaign to test composition laws validity, conducted on automotive exhaust-systems assembling process, is in progress.

In particular, we are analysing the launching phase of a new production regarding the complete exhaust-systems of a new model of automobile. Two production lines, related to different powered car (motorization A and B), are investigated. The two plants can be schematized using a series structure composed by six functional macro-phases including one or more working station. After each phase, a quality control is performed in order to individuate and scrape the defective units. For every macro-phase and for the whole systems the characteristic learning curves are calculated and compared.

First results appear in agreement, at least from the asymptotic point of view, with model predictions.

10. Conclusion

The paper introduces a new approach to determine learning curves of complex manufacturing systems or plants. The method is based on the composition of simple structures: series and parallel, which constitute the basic elementary blocks (processes) of a generic complex manufacturing system.

The main novelty of the method is its ability to provide a preliminary forecast of the learning performances of complex manufacturing plants. Some examples related to Quality Improvement have been analysed. Moreover, the method allows one to highlight the synergic learning effect due to the exchange of 'knowledge' (mutual 'knowledge') between processes.

Future work will be dedicated to the development of a thorough model for overall 'mutual influence' analysis, as well as differentiated experimental investigations for on-field validation of the method.

Appendix

For a series structure (where $q_A = q$ and $q_B = q \cdot [1 - F_A(q)]$), the following equation holds:

$$
\begin{aligned}
F_{eq}(q) &= \frac{D_A(q_A) + D_B(q_B)}{q} = \frac{q_A \cdot F_A(q_A) + q_B \cdot F_B(q_B)}{q} \\
&= \frac{q \cdot F_A(q) + q \cdot [1 - F_A(q)] \cdot F_B(q \cdot [1 - F_A(q)])}{q} \\
&= F_A(q) + F_B\{q \cdot [1 - F_A(q)]\} - F_A(q) \cdot F_B\{q \cdot [1 - F_A(q)]\}
\end{aligned} \tag{12}
$$

Substituting equation (4) into equation (12), we obtain:

$$L_{eq}(q) = L_A(q) + L_B\{q \cdot [1 - F_A(q)]\} - L_A(q) \cdot L_B\{q \cdot [1 - F_A(q)]\}, \tag{13}$$

which is the same as equation (5).

For a parallel structure (where $q_A = q \cdot f_A$, $q_B = q \cdot f_B$ and $f_B = 1 - f_A$), the following equation holds:

$$
\begin{aligned}
F_{eq}(q) &= \frac{D_A(q_A) + D_B(q_B)}{q} = \frac{q_A \cdot F_A(q_A) + q_B \cdot F_B(q_B)}{q} \\
&= \frac{q \cdot f_A \cdot F_A(q \cdot f_A) + q \cdot f_B \cdot F_B(q \cdot f_B)}{q} \\
&= f_A \cdot F_A(q \cdot f_A) + f_B \cdot F_B(q \cdot f_B).
\end{aligned} \tag{14}
$$

Substituting equation (4) into equation (14), we obtain:

$$L_{eq}(q) = f_A \cdot L_A(q \cdot f_A) + f_B \cdot L_B(q \cdot f_B), \tag{15}$$

which is the same as equation (6).

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