Fair Power Allocation Policies for Power-Domain Non-Orthogonal Multiple Access Transmission With Complete or Limited Successive Interference Cancellation

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RESEARCH ARTICLE

Fair Power Allocation Policies for Power-Domain Non-Orthogonal Multiple Access Transmission With Complete or Limited Successive Interference Cancellation

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ABSTRACT Power-Domain Non-Orthogonal Multiple Access (NOMA) transmission has been addressed in this paper with a proportional fairness optimization criterion (which includes MAX-MIN fairness as a special case) and an arbitrary number of users. The optimization of the power allocation coefficients required to achieve the optimum proportional fairness objective leads to a nonconvex optimization problem, which is generally hard to solve and may lead to multiple local optima. However, a simple optimality condition is characterized in the paper, leading to the solution of a nonlinear equation in a single variable. This equation reduces to polynomial form in the case of MAX-MIN fairness. Departing from the complete Successive Interference Cancellation (SIC) paradigm, typical of NOMA systems, a limited SIC technique is discussed and the relevant power allocation coefficients are obtained with the same optimization criterion. This approach eases the implementation of downlink NOMA when a large number of low-complexity hand-held terminals cannot sustain the computationally intensive task of complete SIC, at the cost of reduced their achievable rates. Numerical results are presented to illustrate the impact of complete and limited SIC, with power allocation optimization and two proportional fairness criteria. Among these results, the sum-rate loss due to proportional fairness and the impact of limited SIC on the system performance are illustrated.

INDEX TERMS Power-Domain Non-Orthogonal Multiple Access, Successive Interference Cancellation, Nonconvex Optimization, Proportional Fairness, MAX-MIN Fairness.

I. INTRODUCTION

Multiple-access techniques can be classified as orthogonal multiple access (OMA) and non-orthogonal multiple access (NOMA). OMA techniques allow complete separation of the useful signal from the interfering signals by using orthogonal signal sets for the different user transmissions. Typical examples of OMA schemes are time-division multiple access (TDMA), orthogonal frequency-division multiple access (OFDMA), and code-division multiple access (CDMA). In contrast, NOMA schemes allow different users to share a single communication channel, thereby achieving a number of advantages such as better spectral efficiency and throughput, lower latency, and simpler channel state information recovery. Because of these characteristics, NOMA techniques have been pointed out as a key enabling technology for the forthcoming 5G and 6G wireless communication networks [1]. From a general standpoint, NOMA techniques can be broadly classified as power-domain and code-domain NOMA, the former of which are the focus of this paper. More precisely, NOMA has been proposed in several forms in the literature, including power-domain NOMA [2], sparse code multiple access (SCMA) [3], pattern division multiple access (PDMA) [4], low density spreading (LDS) [5], lattice partition multiple access (LPMA) [6], and interleave division multiple access (IDMA) [7]. The common denominator of these techniques is the fact that more than one user is served in each orthogonal resource block (RB), which may

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be a space/time/code/frequency slot. This work focuses on Power-Domain NOMA (PD-NOMA), where many users are served by a single RB. Different users are allocated different power levels and coexist without any orthogonality requirements. It is worth noticing that the power allocation strategy derived in PD-NOMA is different from other, more conventional, power allocation schemes, like water-filling [8]. In fact, water-filling aims at maximizing the overall sum-rate of a set of independent channels, regardless of the individual requirements, in view of the fact that, typically, a single user can access all (or at least a subset of) the different channels. Water-filling tends to increase the power allocated to the channels operating according to good signal-to-noise ratio (SNR) conditions. On the contrary, NOMA tends to provide more transmission power to users experiencing weak channel conditions [9], because of the tight assignment of different channels to different users. Thus, a key objective with NOMA is trading off system throughput for user fairness, with the goal of reducing the unbalances between the different user achievable rates.

An additional discussion about NOMA techniques and their classification is reported in Section I-A.

User fairness plays a major role in the development of NOMA systems. In this framework, fairness, has received several definitions, though there is no common agreement about its interpretation. One way to characterize fairness is by using the Jain index [10]. This index is the ratio between the squared sample average of the user rates and the sample average of the squared user rates, namely,

$$J \triangleq \frac{(K^{-1}\sum_{k=1}^K R_k)^2}{K^{-1}\sum_{k=1}^K R_k^2} = \frac{(\sum_{k=1}^K R_k)^2}{K\sum_{k=1}^K R_k^2}. \tag{1}$$

This definition satisfies the inequalities $1/K \le J \le 1$, where the lower and upper limits correspond to minimum and maximum fairness, respectively (i.e., only one positive R_k in the former case and constant R_k's in the latter).

Another index has been proposed in [11]. The authors of [11] recognized that a fairness index should incorporate the power distribution of different user rates, so that they define a *fair rate* applicable to the case of constant channel gains considered therein. More precisely, [11] addresses an uplink NOMA system where all users have the same channel gains (which is a somewhat specific and unlikely assumption) to the base station and the total received SNR is $\rho = P/\sigma^2$. Here, P and σ^2 are the received power and the noise power, respectively, and the user powers $P_k, k = 1, \ldots, K$, are allocated by scaling P by the coefficients α_k (nonnegative and adding up to 1). Then, they define $R_{sum} \triangleq \log_2(1+\rho)$ and $R_{cum} \triangleq \sum_{k=1}^K \log_2(1+\alpha_k\rho)$ (so that, by Jensen's inequality, $R_{sum} \le R_{cum}$). Next, they define the *fair rates*

$$R_k^f \triangleq \log_2(1+\alpha_k\rho)\frac{R_{sum}}{R_{cum}}. \tag{2}$$

Finally, their fairness index is given by

$$F \triangleq 1 - \frac{K}{K-1}\frac{\sum_{k=1}^K (R_k^f - R_k)^2}{R_{sum}^2}, \tag{3}$$

where

$$R_k = \log_2\left(1 + \frac{\alpha_k\rho}{\rho\sum_{m=k+1}^K \alpha_m + 1}\right). \tag{4}$$

A simpler definition of fairness consists of maximizing the minimum user rate (MAX-MIN fairness). In this way, all users are enabled to achieve, at least, a certain common rate. The drawback of MAX-MIN fairness is that, when the channel conditions are much variable (and, as a consequence, the achievable user rate variance is large), the benefit to the users with bad channel conditions is limited while the penalty to the users with good channel conditions is very large. In order to overcome this drawback, an alternative approach has been proposed in [12], called *proportional fairness*. Proportional fairness is a generalization of the MAX-MIN fairness criterion. It still aims at the maximization of a minimum user performance metric but this metric is the ratio between the user achievable rate R_k and the user target rate T_k. Setting $T_k = 1$, proportional fairness becomes equivalent to MAX-MIN fairness. If the target rates take into account the channel conditions, users experiencing a good channel may expect to achieve higher target rates than users experiencing a bad channel. Thus, a simple assumption is setting the target rate equal to the Shannon capacity in the absence of interference from the other users, i.e., $T_k = \log_2(1+\rho_k)$, where ρ_k is the k-th user SNR.

Both MAX-MIN and proportional fairness power allocations require the solution of a nonconvex optimization problem, illustrated in the following eq. (19). It is well known that nonconvex optimization problems are hard to solve, in general, and prone to the existence of multiple local optima [13]. In the framework considered by this paper, such difficulty has been recognized in [12], where the author proposes a sub-optimum approach to solve the nonconvex optimization problem, consisting in the approximation of the objective function by a convex function, as discussed in Section III below.

Paper [12] is not the only reference recognizing the complexity of the fair power allocation problem. According to the survey presented in [14], the fair power allocation problem in NOMA is an open problem addressed only approximately by the fractional transmit power control algorithm [2], which selects an appropriate decaying factor to balance system throughput and user fairness.

Therefore, to the best of the author's knowledge, the fair power optimization problem in NOMA has not been solved in general, except for the two-user case with MAX-MIN fairness [15].

Two main contributions of this paper are Theorem 1 and Algorithm 1. Theorem 1 provides the necessary and sufficient conditions to solve the proportional fairness power allocation problem described in detail in (19). Algorithm 1 outlines a numerical procedure to determine the power allocation vector with complete Successive Interference Cancellation (SIC) and an arbitrary number of users. The solution of the nonconvex optimization problem is based on a nonlinear equation in a single variable (illustrated in Theorem 1), which reduces to polynomial form with MAX-MIN fairness.

Another contribution is the analysis of *limited SIC* reported in Section IV and illustrated by the results in Section V. The technique is particularly effective with a large number of users and, to the best of the author's knowledge, it has not been proposed in the literature yet. Theorem 1 is applicable also in this case (actually, it is applicable in a more general case) so that proportional fairness optimum power allocation can be found by applying eqs. (22). However, Algorithm 1 is no more applicable but the nonlinear eqs. (22) can be solved in a different way, leading to Algorithm 2.

A. DISCUSSION

A large body of scientific literature has been published in recent years to target the issues and the applications of NOMA to 5G/6G wireless communication systems. The general classification reported in the paper [1] (published in 2017) is confirmed by more recent survey papers like [18] (2022). Focusing on Power-Domain (PD) NOMA, which is the primary subject of this paper, there are a few recent papers addressing very similar topics. One of them is [19], where the authors consider both code-domain (CD) and PD-NOMA for a beam hopping satellite communication systems. Their analysis leads to a nonconvex objective function, which is handled by resorting to Dinkelbach's transform and variable relaxation. User fairness is handled by minimizing the quadratic offset of the users' achievable rates with respect to certain target user traffic demands. The resulting iterative power optimization algorithm requires to solve a mixed integer nonconvex programming (MINCP) system and is more complex than the method proposed in this paper. Another characteristic of this study is the consideration of a unified framework for NOMA including its CD and PD declinations. This approach has been proposed earlier in [20]. CD-NOMA was also studied in [21] for heterogeneous cellular networks, in [22] to contrast Doppler in orthogonal frequency division reusing, and in [23] in conjunction with intelligent reflecting surface technologies. PD-NOMA was also studied in [24], [25], [26], and in [27] and [28] for satellite networks. A heuristic approach to the determination of the power distribution for PD-NOMA is presented in [29]. In contrast with the information-theoretic approach considered in the previous papers, some works proposed a different approach based on specific QAM modulation designs for PD-NOMA, such as [30] and [31].

B. ORGANIZATION OF THE PAPER

The paper is organized as follows. Section II describes the downlink NOMA transmission channel model, corresponding to the multiuser Gaussian broadcast channel, in accordance with the definitions from [8], [16], and [17]. [1] The section provides introductory insight relevant to the two-user case, which paves the way to the extension to the case of an arbitrary number of users (Section II-B). Then, Section III introduces the concept of proportional fairness, according to the definition in [12], and provides the main result of the paper, namely, Theorem 1, which provides necessary and sufficient conditions for the optimization of the proportional fair user rate. These conditions are relevant to a nonconvex optimization problem, whose solution is generally hard and may lead to multiple optima. Theorem 1 shows that the solution of the derived equations always exists and is unique. Algorithm 1 reports an iterative method for the numerical solution of eqs. (22) stemming from the results stated by Theorem 1. The MAX-MIN fairness case (which is a special case of proportional fairness) is specifically addressed to show that the equations are equivalent to a single polynomial equation in one variable, and a few examples are outlined. Next, Section IV approaches the SIC process from a limited complexity perspective and suggests the possibility of containing the receiver complexity in the presence of a large number of users by limiting the number of interference cancellation steps. It is clarified in the section (and, subsequently, by numerical results) which interference steps are more convenient to maximize the user rate. The resulting limited SIC policy is still handled by Theorem 1, and bridges the extreme SIC policies ranging from the case of no SIC to the case of complete SIC. Section V provides a set of numerical results based on two different user scenarios. The former considers a moderate number of users (i.e., 10 users). The MAX-MIN and proportional fairness user rate optimization are considered first from the point of view of the system operator by showing the penalty on the sum-rate of the multiuser network (Figs. 3 to 5) One of these figures (Fig. 4) presents the negative impact of an alternative interference cancellation order on the sum-rate when limited SIC is considered. The next two Figs. 6 and 7 illustrate the system performance according to the user's point of view by showing the behavior of the user rate versus the SNR with different levels of limited SIC in the MAX-MIN and proportional fairness cases, respectively. A second scenario with 1000 users is also illustrated in Figs. 8 to 11. The performance results are quite similar to the 10-user scenario. This scenario has been reported to show the efficiency of the optimization algorithm proposed (Algorithm 2), which can process a single optimization for 1000 users in about 10 ms.

II. SYSTEM MODEL

This section describes the system model for the NOMA transmission scheme considered and its capacity region. A single terminal (the base station) transmits the superposition of K signals $X_1, \ldots, X_K$ addressed to K users with channel gains $h_1, \ldots, h_K$. The users are labeled so that the inequalities $|h_1| \geq |h_2| \geq \cdots \geq |h_K|$ hold. According to the information

[1] The case of uplink NOMA can be handled in a similar way.

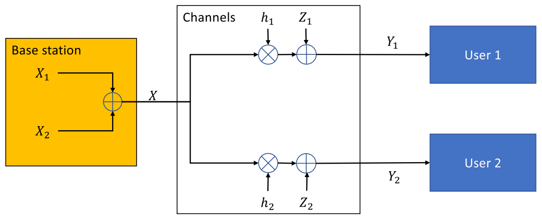

FIGURE 1. Block diagram of a two-user downlink NOMA scheme (broadcast channel).

theoretical analysis in [8] and [17], this system is described as a *Gaussian broadcast channel* and the transmitter implements superposition encoding. The channel equations are reported as follows.

$$X = X_1 + \ldots + X_K, \tag{5}$$
$$Y_k = h_k X + Z_k, \quad Z_k \sim \mathcal{CN}(0,1), \tag{6}$$
$$\mathbb{E}[|X_k|^2] \le P_k = \alpha_k P_x, \; \rho_k = |h_k|^2 P_k, \; \sum_{k=1}^{K} \alpha_k = 1. \tag{7}$$

The receiver resorts to Successive Interference Cancellation (SIC) to achieve the following rates [8], [16], [17]:

$$R_k \le \log_2\left(1 + \frac{\alpha_k}{\rho_k^{-1} + \sum_{\ell=1}^{k-1} \alpha_\ell}\right) \tag{8}$$

for $k = 1, \ldots, K$, where

$$\rho_k \triangleq |h_k|^2 P_x \tag{9}$$

are the user SNRs, ordered as

$$\rho_1 \ge \rho_2 \ge \cdots \ge \rho_K. \tag{10}$$

The capacity achieving distributions are $X_k \sim \mathcal{CN}(0, \alpha_k P_x)$. The α_k are called power allocation coefficients and are normalized by $\sum_{k=1}^{K} \alpha_k = 1$. Fig. 1 illustrate the multiuser downlink NOMA scheme with $K = 2$ users.

Remark 1: The achievability of the rate inequalities (8), based on inequalities (10), is justified as follows. For each $k = 1, \ldots, K$, the k-th user can successfully decode the ℓ-th user signal by using SIC as long as $k \le \ell$ (and hence $\rho_k \ge \rho_\ell$). In fact,

$$\begin{aligned} R_\ell &\overset{(a)}{\le} \log_2\left(1 + \frac{\alpha_\ell}{\rho_\ell^{-1} + \sum_{i=1}^{\ell-1} \alpha_i}\right) \\ &\overset{(b)}{\le} \log_2\left(1 + \frac{\alpha_\ell}{\rho_k^{-1} + \sum_{i=1}^{\ell-1} \alpha_i}\right). \end{aligned} \tag{11}$$

Inequality (a) derives from (8). Inequality (b) derives from $\rho_k \ge \rho_\ell$ since $k \le \ell$. The rhs of (b) is the achievable rate of the ℓ-th user signal at the k-th user receiver.

A. USER FAIRNESS

It is plain to check that the maximum achievable sum-rate $R \triangleq \sum_{k=1}^{K} R_k$ is attained by the most unfair power allocation, i.e.,

$$\alpha_1 = 1, \alpha_2 = \cdots = \alpha_K = 0,$$

and corresponds to $R = R_1 = \log_2(1 + \rho_1)$. In this case, only the strongest user (user 1) transmits, while all weaker users remain silent. [2] Though this is a valid rate vector in the capacity region, it is also maximally unfair.

To improve user fairness, even though at the price of reducing the total throughput, different optimization criteria can be used, like the following ones:

- Equal rates $R_1 = R_2 = \cdots = R_K$.
- Maximum minimum rate ($\max\min\{R_1, \ldots, R_K\}$).

It will be shown that the power allocation vectors satisfying these two optimization criteria coincide (Theorem 1 with $T_k = 1$ for all $k = 1, \ldots, K$). The simple case of $K = 2$ users is addressed first, and then extended to the general case of $K > 2$.

B. TWO-USER POWER OPTIMIZATION

Before considering the general case, the two-user downlink broadcast NOMA channel is addressed, as illustrated in Figs. 1. Let $\alpha \triangleq \alpha_1$ and $\alpha_2 = 1 - \alpha$, so that, at the boundary of the capacity region, the following equations hold:

$$R_1 = \log_2(1 + \alpha\rho_1) \tag{12}$$
$$R_2 = \log_2\left(1 + \frac{(1-\alpha)\rho_2}{1 + \alpha\rho_2}\right). \tag{13}$$

As expected, R_1 increases with α (the power share of the first user) while R_2 decreases with α. The two rates coincide for

$$(1 + \alpha\rho_1)(1 + \alpha\rho_2) = 1 + \rho_2. \tag{14}$$

The solution of this quadratic equation yields

$$\begin{aligned} \alpha_{\text{opt}} &= \frac{\sqrt{(\rho_1+\rho_2)^2 + 4\rho_1\rho_2^2} - (\rho_1+\rho_2)}{2\rho_1\rho_2} \\ &= \frac{2\rho_2}{\sqrt{(\rho_1+\rho_2)^2 + 4\rho_1\rho_2^2} + \rho_1 + \rho_2}, \end{aligned} \tag{15}$$

where the latter expression is numerically more stable when ρ_2 is very small. If $\rho_2 \ll \rho_1$, then $\alpha_{\text{opt}} \approx \frac{\rho_2}{\rho_1}$. Then, one can see that

$$\min\{R_1, R_2\} = \begin{cases} R_1 & \alpha \le \alpha_{\text{opt}} \\ R_2 & \alpha \ge \alpha_{\text{opt}} \end{cases}. \tag{16}$$

This implies that the maximum minimum rate is achieved when $\alpha = \alpha_{\text{opt}}$, and corresponds to a fair power allocation. A numerical example for the two-user NOMA is illustrated by the results shown in Fig. 2.

The two-user case suggests that the maximum-minimum (MAX-MIN) and equal-rate power allocations coincide, which is a key insight leading to the statement of Theorem 1.

III. PROPORTIONAL FAIRNESS

This section begins by providing numerical evidence about the inadequacy of the MAX-MIN fairness criterion when the SNR dispersion is large. In this case, the MAX-MIN

[2] The attributes strong and weak refer to the level of SNR.

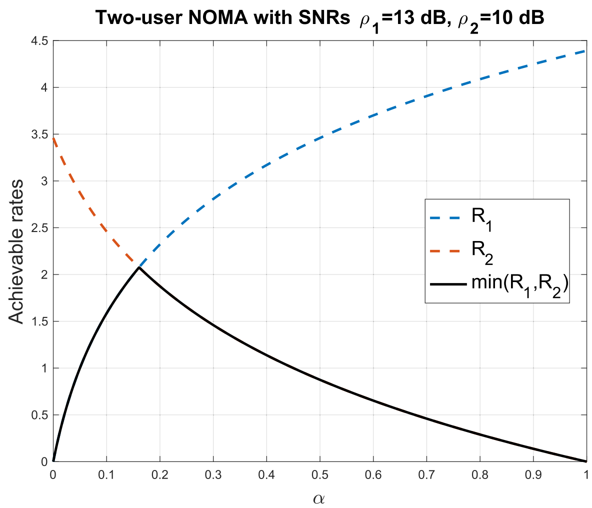

FIGURE 2. Plot of the user achievable rates R_1, R_2, and their minimum versus the power allocation coefficient α when $\rho_1 = 13$ dB and $\rho_1 = 10$ dB. In this case, $\alpha_{\mathrm{opt}} = 0.1608$.

approach, introduced in the previous section for $K = 2$ users, may entail an excessive penalty on the strong users. Consider, for example, $K = 6$ users with SNRs given by

$$\rho_1 = \rho_2 = \rho_3 = 10 \text{ and } \rho_4 = \rho_5 = \rho_6 = 1.$$

Accordingly, the MAX-MIN rate can be obtained as $R_{\mathrm{opt}} = 0.2944$ bit/channel use. However, if only the first three users are served (and the other three users are not), they would achieve $\log_2(11)/3 = 1.1531$ bit/channel use (unfair case). This entails a 74% rate reduction for the strong user rates (MAX-MIN versus unfair).

Such penalty can be alleviated by assuming, for example, that the rates be proportional to the single-user rates, $T_k = \log_2(1 + \rho_k)$. By exploiting the forthcoming results, one can see that the optimum rates are 0.6907 and 0.1996 bit/channel use for the strong and weak users, respectively. That corresponds to a strong user rate loss of 40% (from the unfair rate) and a weak user rate loss of 32% (from the MAX-MIN fair case).

More generally, one can define a set of arbitrary target rates T_k and consider the maximization of the minimum ratios between the achievable rates and the target rates. The following functions are defined:

$$\Theta_{\min}(\boldsymbol{\alpha}) \triangleq \min\left\{\frac{\mathcal{R}_1(\boldsymbol{\alpha})}{T_1}, \ldots, \frac{\mathcal{R}_K(\boldsymbol{\alpha})}{T_K}\right\}, \tag{17}$$

where

$$\mathcal{R}_k(\boldsymbol{\alpha}) \triangleq \log_2\left(1 + \frac{\alpha_k}{\rho_k^{-1} + \sum_{\ell=1}^{k-1}\alpha_\ell}\right). \tag{18}$$

This approach has been proposed in [12] for the two-user case. By setting $T_k = 1$ for $k = 1, \ldots, K$ reverts to MAX-MIN fairness. The standard form of this optimization problem is as follows [13]:

$$\begin{cases} \min & -t \\ s.t. & tT_k - \mathcal{R}_k(\boldsymbol{\alpha}) \le 0, \quad k = 1, \ldots, K \\ & \alpha_k \ge 0, \quad k = 1, \ldots, K \\ & \sum_{k=1}^K \alpha_k = 1 \end{cases} \tag{19}$$

This is a nonconvex optimization problem because of the nonconvex constraint equations. The difficulty in the solution of this problem has been recognized, *e.g.,* at the end of [12, Section III]. The paper proposes the use of the approximation

$$\exp\left\{-\beta\min\left[\frac{\mathcal{R}_1(\boldsymbol{\alpha})}{T_1}, \ldots, \frac{\mathcal{R}_K(\boldsymbol{\alpha})}{T_K}\right]\right\} \approx \sum_{k=1}^K \exp\left\{-\beta\frac{\mathcal{R}_k(\boldsymbol{\alpha})}{T_k}\right\}, \tag{20}$$

where β is a large positive number. In this way, the nonconvex optimization problem is approximately turned into a convex one, and standard convex optimization algorithms are applied to solve it.

It is remarkable that the numerical results reported in [12] consider only the case of $K = 2$ users. One may wonder if the approximation remains valid when the number of users is greater than two, and possibly very large.

The first key contribution of this paper, Theorem 1, provides necessary and sufficient conditions for the solution of the optimization problem (19). These conditions are simply amenable to a fast algorithm to find the optimum power allocation under the proportional fairness criterion, which remains numerically stable even when the number of users is very large. Additionally, Theorem 1 considers a more general case of SIC than that required in eqs. (18), which is characterized by the following more general expression:

$$\mathcal{R}_k(\boldsymbol{\alpha}) = \log_2\left(1 + \frac{\alpha_k}{\rho_k^{-1} + \sum_{\ell\in\mathcal{L}_k}\alpha_\ell}\right). \tag{21}$$

Here, it is assumed that the sets $\mathcal{L}_k$ are arbitrary but *do not include* the index k. This generalization will be used while dealing with limited SIC in Section IV.

Theorem 1: The maximum $\Theta_{\min}(\boldsymbol{\alpha})$*, defined in (17), with rate functions defined in (21), under the constraints* $\boldsymbol{\alpha} \ge \mathbf{0}$ *and* $\sum_{k=1}^K \alpha_k = 1$*, is achieved by a unique power allocation vector* $\boldsymbol{\alpha}_{\mathrm{opt}}$*, which satisfies the equations*

$$\Theta_{\min}(\boldsymbol{\alpha}_{\mathrm{opt}}) = \frac{\mathcal{R}_1(\boldsymbol{\alpha}_{\mathrm{opt}})}{T_1} = \cdots = \frac{\mathcal{R}_K(\boldsymbol{\alpha}_{\mathrm{opt}})}{T_K}. \tag{22}$$

Proof: See Appendix A. ■

The conditions expressed by eq. (22) above lead to the following equations:

$$\log_2\left(1 + \frac{\alpha_{k+1}\rho_{k+1}}{1 + \sum_{\ell=1}^k \alpha_\ell\rho_{k+1}}\right) = \frac{T_{k+1}}{T_k} \times \log_2\left(1 + \frac{\alpha_k\rho_k}{1 + \sum_{\ell=1}^{k-1}\alpha_\ell\rho_k}\right) \tag{23}$$

for $k = 1, \ldots, K-1$. The previous expressions can be turned into the following equivalent *recursive* form:

$$\alpha_{k+1} = \left\{\left(1 + \frac{\alpha_k}{\rho_k^{-1} + \sum_{\ell=1}^{k-1} \alpha_\ell}\right)^{T_{k+1}/T_k} - 1\right\} \times \left(\rho_{k+1}^{-1} + \sum_{\ell=1}^{k} \alpha_\ell\right) \quad (24)$$

Eqs. (24) yield a simple, fast, and accurate algorithm described as Algorithm 1.

Algorithm 1. Proportional Fairness Power Allocation

Require: K, number of users
Require: ρ_k for $k = 1, \ldots, K$ (User SNRs)
Require: T_k for $k = 1, \ldots, K$ (Target rates)
Solve the equation $\sum_{k=1}^{K} \alpha_k = 1$ wrt α_1
where $\boldsymbol{\alpha} = \phi(\alpha_1, K, \rho_1, \ldots, \rho_K, T_1, \ldots, T_K)$
Ensure: The solution of the previous equation, $\boldsymbol{\alpha}_{\mathrm{opt}}$

Function: $\boldsymbol{\alpha} = \phi(\alpha_1, K, \rho_1, \ldots, \rho_K, T_1, \ldots, T_K)$
for $k = 1$ to $K - 1$ **do**
Use eq. (24) to calculate α_{k+1}
end for

Remark 2: By setting $T_k = 1$ for all $k = 1, \ldots, K$, proportional fairness reduces to MAX-MIN fairness and leads to this set of recursive equations:

$$\alpha_{k+1} = \alpha_k \cdot \frac{\rho_{k+1}^{-1} + \sum_{\ell=1}^{k} \alpha_\ell}{\rho_k^{-1} + \sum_{\ell=1}^{k-1} \alpha_\ell}, \quad (25)$$

for $k = 1, \ldots, K-1$. These can be solved by finding the zeros of a K-th degree polynomial equation in α_1. For example,

- *If $K = 2$: $\rho_1^2\alpha_1^2 + (\rho_1 + \rho_2)\alpha_1 - \rho_2 = 0$*
- *If $K = 3$: $\rho_2\rho_1^3\alpha_1^3 + (2\rho_2 + \rho_3)\rho_1^2\alpha_1^2 + (\rho_1\rho_2 + \rho_3\rho_2 + \rho_1\rho_3)\alpha_1 - \rho_2\rho_3 = 0$*
- *If $K = 4$: $\rho_2\rho_3\rho_1^4\alpha_1^4 + \rho_2(3\rho_3 + \rho_4)\rho_1^3\alpha_1^3 + (3\rho_2\rho_3 + \rho_4\rho_3 + 2\rho_2\rho_4)\rho_1^2\alpha_1^2 + (\rho_1\rho_2\rho_3 + \rho_1\rho_4\rho_3 + \rho_2\rho_4\rho_3 + \rho_1\rho_2\rho_4)\alpha_1 - \rho_2\rho_3\rho_4 = 0$*

All these polynomial equations have a single root in the interval $(0, 1)$*, as implied by the uniqueness part of Theorem 1.*

In the following section, the concept of SIC is extended to meet the complexity limitation requirements affecting specifically hand-held terminals.

IV. LIMITED SIC

With standard complete SIC, every receiver decodes all the other weaker user signals, then it cancels them from the received signal, and finally decodes its own signal [8], [16], [17]. Therefore, the k-th receiver decodes each signal X_ℓ (for $\ell = K, K-1, \ldots, k+1$), cancels the corresponding interference on Y_k, and decodes its own signal X_k by

$$Y_k - h_k(X_{k+1} + \ldots + X_K) = h_k(X_1 + \ldots + X_k) + Z_k. \quad (26)$$

With limited SIC, each user cancels at most $\kappa < K$ signals. If the number of signals to cancel, $K - k$, is greater than κ, it is convenient to choose the κ weaker signals $X_K, \ldots, X_{K-\kappa+1}$ for cancellation in that exact order (first X_K, then X_{K-1}, and so on up to $X_{K-\kappa+1}$). Thus, the following signal is decoded:

$$\begin{aligned} &Y_k - h_k(X_{K-\kappa+1} + \ldots + X_K) \\ &\quad = h_k(X_1 + \ldots + X_k + X_{k+1} + \ldots + X_{K-\kappa}) + Z_k. \end{aligned} \quad (27)$$

Accordingly, the achievable rate becomes

$$\mathcal{R}_k(\boldsymbol{\alpha}) \triangleq \log_2\left(1 + \frac{\alpha_k}{\rho_k^{-1} + \sum_{\ell=1}^{k-1} \alpha_\ell + \sum_{\ell=k+1}^{K-\kappa} \alpha_\ell}\right). \quad (28)$$

Theorem 1 and eqs. (22) still hold in this case but the recursive equations (24) are no more valid because of the possible dependence of $\mathcal{R}_k(\boldsymbol{\alpha})$ on some α_ℓ with $\ell > k$. Nevertheless, the numerical solution of eqs (22) can be found, with limited complexity overhead, as illustrated in Appendix B. The results are summarized in Algorithm 2. Finally, the no-SIC case, where interference is not canceled at all, is dealt with in Appendix C.

Algorithm 2. Proportional Fairness Power Allocation With Limited SIC

Require: K, number of users
Require: ρ_k for $k = 1, \ldots, K$ (User SNRs)
Require: T_k for $k = 1, \ldots, K$ (Target rates)
Require: $\kappa \le K - 1$ (maximum number of cancellations)
if $\kappa < K - 1$ **then**
Solve equation $\sum_{k=1}^{K-\kappa}(1 - 2^{-T_k\xi}) = 1$ for $\xi > 0$
Let $\tilde{\xi}$ be the solution
else
Let $\tilde{\xi} \to \infty$
end if
Solve the equation $\sum_{k=1}^{K} \alpha_k = 1$
where $\boldsymbol{\alpha} = \phi(\xi, K, \kappa, \rho_1, \ldots, \rho_K, T_1, \ldots, T_K)$
and $\xi \in (0, \tilde{\xi})$
Ensure: The solution of the previous equation, $\boldsymbol{\alpha}_{\mathrm{opt}}$

Function: $\boldsymbol{\alpha} = \phi(\xi, K, \kappa, \rho_1, \ldots, \rho_K, T_1, \ldots, T_K)$
Use eq. (47) to calculate σ
for $k = 1$ to $K - \kappa$ **do**
Use eq. (45) to calculate α_k
end for
for $k = K - \kappa + 1$ to K **do**
Use eq. (48) to calculate α_k
end for

V. NUMERICAL RESULTS

To illustrate the performance of the proposed power allocation algorithm, a scenario characterized by the presence of K users is proposed. The user SNRs are $\rho_k = \rho_0 \cdot 10^{\Delta\rho_k/10}$, for $k = 1, \ldots, K$, where each $\Delta\rho_k$ represents an SNR displacement, expressed in dB. Accordingly, an SNR displacement

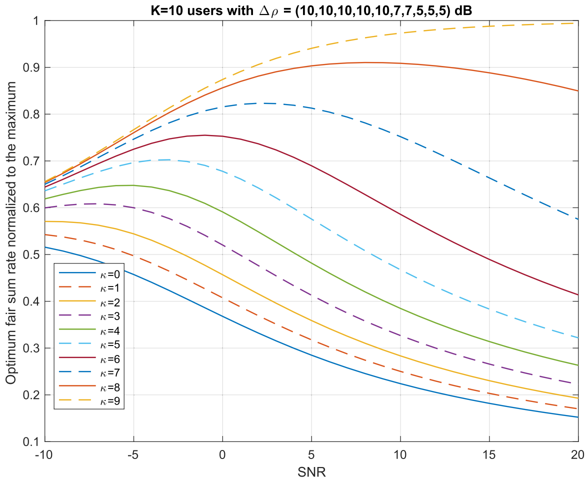

FIGURE 3. Optimum fair sum-rate with limited SIC power allocation and $T_k = 1$ (MAX-MIN) normalized by the maximum, $\log_2(1 + \rho_1)$.

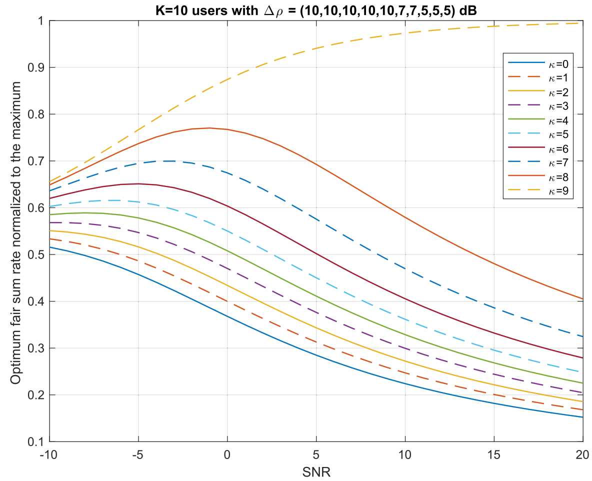

FIGURE 4. Same as Fig. 3 but canceling the stronger signals first.

vector $\Delta\boldsymbol{\rho} = (\Delta\rho_1, \ldots, \Delta\rho_K)$ is defined. Particular emphasis is given to the impact of limited interference cancellation, which is a key contribution in the analysis presented in this study.

A. IMPACT OF FAIRNESS ON THE SUM-RATE

Fig. 3 illustrates the optimum sum-rate penalty due to MAX-MIN fairness power allocation in a system with $K = 10$ users and relative SNRs

$$\Delta\boldsymbol{\rho} = (10, 10, 10, 10, 10, 7, 7, 5, 5, 5) \text{ dB}.$$

The abscissa contains the SNR ρ_0 in dB. The curves report the sum-rates normalized by $\log_2(1+\rho_1)$, which is the maximum achievable sum-rate corresponding to the transmission of only the stronger user signal. Each curve depends on the limited SIC parameter κ representing the maximum number of cancellation allowed to every user. As expected, with complete SIC ($\kappa = K - 1 = 9$), the full sum-rate can be achieved asymptotically. Lower values of κ illustrate the degradation entailed by limited SIC. The curve corresponding to $\kappa = 0$ corresponds to the case of no interference cancellation.

It is noticeable that, whenever limited SIC is implemented, there is a degradation of the relative sum-rate which decreases monotonically after a threshold SNR. The curves illustrate the trade-offs between complexity (represented by the maximum number of cancellations, κ) and performance (the relative sum-rate). Recall that, with MAX-MIN power allocation, all users achieve the same rate, which corresponds to the maximum fairness condition.

Fig. 4 reports the normalized sum-rate, as Fig. 3, but with the assumption that the stronger signals $X_{k+1}, \ldots, X_{k+\kappa}$ are canceled first (instead of the weaker ones). A sum-rate degradation for all values of κ, except 0 and $K - 1$ (corresponding to no SIC and complete SIC, respectively), can be noticed by this figure. The degradation increases with κ, the number of cancellations, and is particularly severe when the number of cancellation is close to the maximum, *e.g.*, $K - 2$. This confirms the correctness of canceling the weaker signals first.

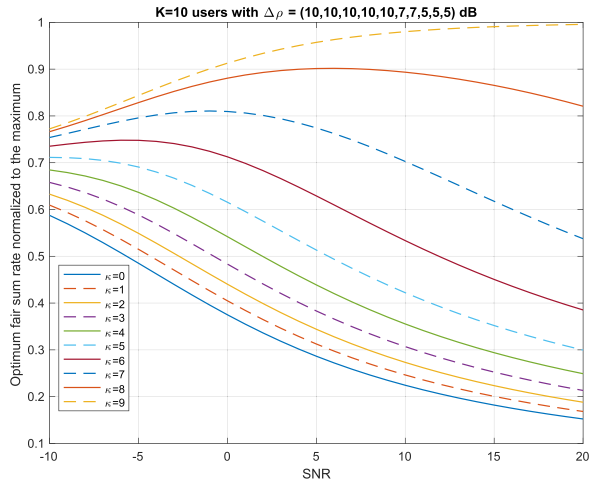

FIGURE 5. Optimum fair sum-rate with limited SIC power allocation and $T_k = \log_2(1 + \rho_k)$ normalized by the maximum, $\log_2(1 + \rho_1)$.

Fig. 5 reports the normalized sum-rate as in Fig. 3 but with proportional fairness with respect to the coefficients $T_k = \log_2(1+\rho_k)$. The motivation of this choice is providing the users with rates proportional to what they would achieve in the absence of interference. The resulting performance is qualitatively similar to that corresponding to the MAX-MIN power allocation illustrated in Fig. 3 but the sum-rate penalty is more limited.

B. USER ACHIEVABLE RATES WITH MAX-MIN AND PROPORTIONAL FAIRNESS

In a similar way, Figs. 6 and 7 report the optimum user achievable rates, i.e., $\Theta_{\min}(\tilde{\boldsymbol{\alpha}}_{\text{opt}})$, normalized by the targets $T_k = 1$ and $\log_2(1 + \rho_k)$, respectively. The curves in Fig. 6 grow monotonically with the SNR, confirming the fact that the MAX-MIN fair rate increases withe SNR for a given SNR displacement vector $\Delta\boldsymbol{\rho}$. These curves include 10 cases of limited SIC ($\kappa = 0$ to 9) and another curve corresponding to equal power allocation, which is discussed in the following, and consist of the plot of eq. (30). Thus, the absence of power allocation optimization entails a degradation, with respect to

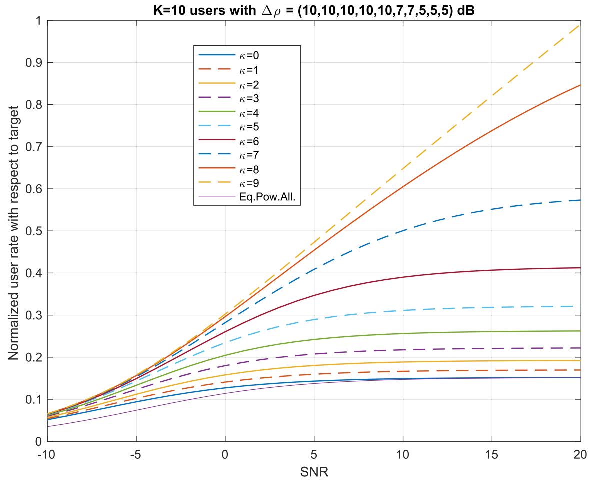

FIGURE 6. Optimum user rates normalized by the targets $T_k = 1$ (MAX-MIN) with limited SIC power allocation.

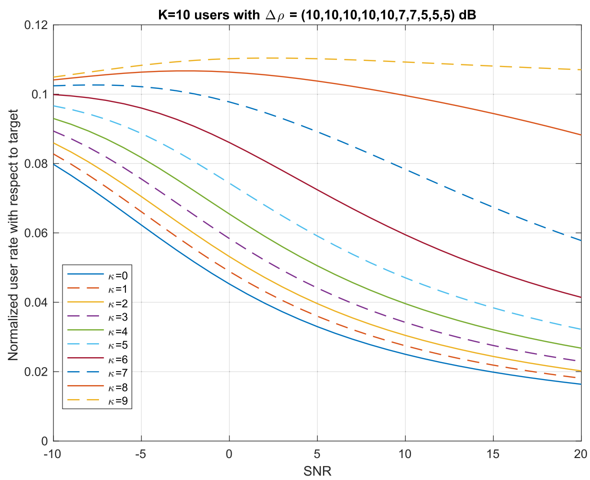

FIGURE 7. Optimum user rates normalized by the targets $T_k = \log_2(1 + \rho_k)$ with limited SIC power allocation.

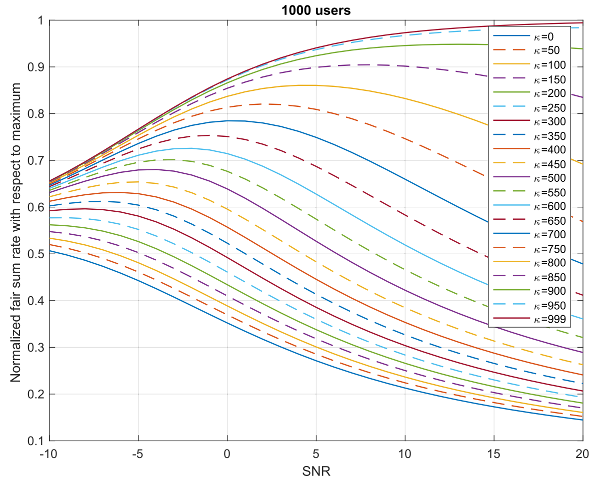

FIGURE 8. Optimum fair sum-rate with limited SIC power allocation, $\Delta\rho$ specified in (31), and $T_k = 1$ (MAX-MIN) normalized by the maximum, $\log_2(1 + \rho_1)$.

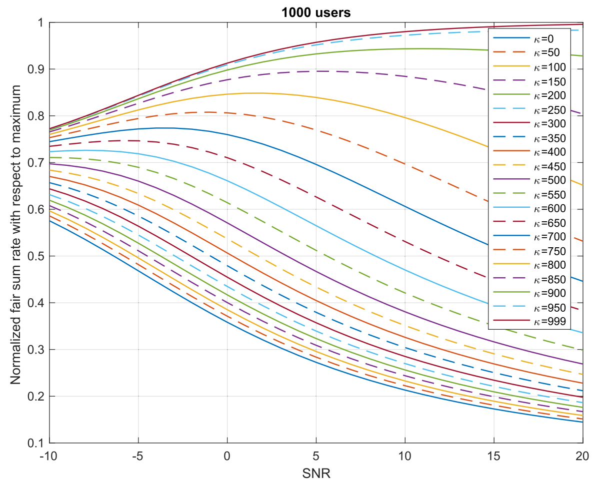

FIGURE 9. Same as Fig. 8 but $T_k = \log_2(1 + \rho_k)$.

the case of no SIC and optimum MAX-MIN power allocation, which is more noticeable at low SNR and tends to vanish at high SNR.

Changing the target rates from $T_k = 1$ to $\log_2(1 + \rho_k)$, the curves are no more monotonically increasing with the user SNRs, as illustrated in Fig. 7. In this case, the initial growth is followed by a decrease because the user rates cannot keep pace with the increasing targets.

C. CONSTANT POWER ALLOCATION

All previous results (except one of the curves in Fig. 6) are based on optimum power allocation, according to the proportional fairness criterion. Constant power allocation leads to different results. In fact, with constant power allocation, $\mathcal{R}_k(\boldsymbol{\alpha}_{\text{const}}) =$

$$\log_2\left(1 + \frac{1}{K\rho_k^{-1} + k - 1 + \max(0, K - \kappa - k)}\right), \quad (29)$$

since $(\boldsymbol{\alpha}_{\text{const}})_k = 1/K$. Accordingly,

$$\min_{1\le k\le K} \mathcal{R}_k(\boldsymbol{\alpha}_{\text{const}}) = \log_2\left(1 + \frac{\rho_K}{K + (K-1)\rho_K}\right). \quad (30)$$

Eq. (30) is independent of the number of canceled signals, and corresponds to the case of no SIC.

D. EFFECTIVENESS OF THE POWER ALLOCATION ALGORITHM IN THE PRESENCE OF A VERY LARGE NUMBER OF USERS

The multiuser scenario considered so far was based on a limited number of users, i.e., $K = 10$. Now, consider another multiuser scenario with $K = 10^3$ users and

$$\Delta\rho = (\underbrace{10,\ldots,10}_{500\text{ times}},\ \underbrace{7,\ldots,7}_{200\text{ times}},\ \underbrace{5,\ldots,5}_{300\text{ times}})\text{ dB}. \quad (31)$$

In spite of the very large number of users, Algorithm 2 is still amazingly fast and solves the optimization equations in a small fraction of a second .[3] The results are reported in Figs. 8 to 11. They are qualitatively similar to those in Figs. 3 to 7, so that they are not illustrated in detail.

[3]More precisely, a single optimization (calculating 1000 power allocation coefficients) is processed in about 10.4, 13.6, 22.5 ms for $\kappa = 100, 200, 999$, respectively, on a MacBook Pro equipped with Apple M1 Max CPU and 64 GB RAM running MacOS 12.6 and Matlab R2022B.

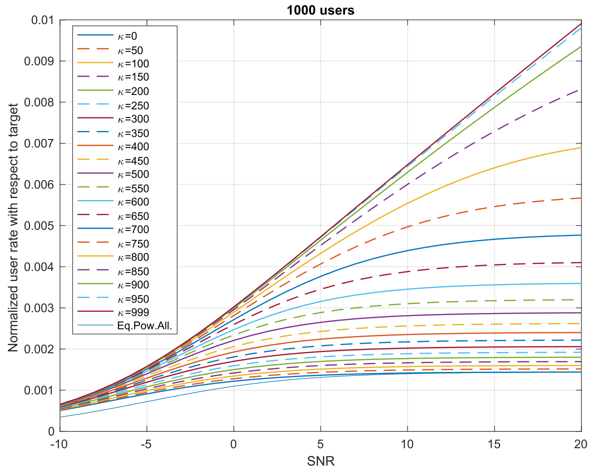

FIGURE 10. Optimum user rates normalized by the targets $T_k = 1$ (MAX-MIN) with limited SIC power allocation.

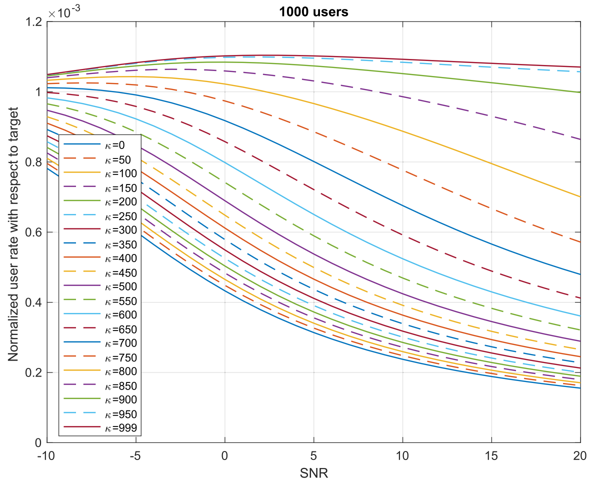

FIGURE 11. Same as Fig. 10 but $T_k = \log_2(1 + \rho_k)$.

VI. CONCLUSION

This paper proposed an analytic method to derive the optimum power allocation for a NOMA system with a proportional fairness objective function. The method is based on the solution of a nonconvex optimization problem consisting of maximizing the minimum ratios between the user achievable rates and certain proportionality coefficients representing the user rate demands or targets. The main difficulty of this nonconvex optimization problem is due to the fact that these problems do not have in general a unique solution and multiple local optima may exist. In this special case, it was shown that the solution exists and is unique, and the characterizing equation has been derived.

The subject has been considered in the literature but most results refer to the two-user case or resort to approximations which avoid the intricacies of nonconvex optimization. The application of two-user results is rather limited since, in general, base stations address a large number of users simultaneously. Thus, a two-user restriction represents a considerable limitation.

A key contribution of this work is the solution of the nonconvex optimization problem resulting from the maximization of the proportional fairness objective. Specifically, an algorithm is provided to derive the optimum power allocation according to an arbitrary proportional fairness criterion, without limitations on the number of users. Numerical results have shown that the number of users can be pushed to very large values (*e.g.*, $K = 1000$) and the algorithm remains stable.

Additionally, the impact of limiting the number of allowed SIC steps on the performance results has been carefully assessed. This option is very important when interference cancellation is implemented over low-complexity terminals, such as hand-held devices. In such cases, in conjunction with a large number of terminals served by the same base station, the full SIC operation appears to be exceedingly complex but using limited SIC allows the low-complexity terminals to harvest a fraction of the benefits available over the NOMA downlink.

A key result, Theorem 1, is valid in all cases of limited SIC, ranging from the absence of SIC to the case of complete SIC. This theorem derives the solution of the nonconvex optimization problem considered in this work. It provides the characterizing equations and proves its existence and uniqueness.

Numerical results illustrate the transitional behavior of the optimum achievable rate performance in this range of limited SIC. Among them, it is notable to consider how much of the available sum rate is achievable due to the limitation consisting in the proportional fairness objective. The fraction of the sum rate is increasing with the SNR except in the cases of limited SIC, where a sizable penalty may be entailed by not completing the SIC itself.

The results are applicable to 5G/6G wireless systems with a large number of hand-held terminals with limited computational capabilities, where complete interference removal is not affordable over the downlink NOMA broadcast channel. As already mentioned, the number of users served by one of these base stations is commonly much larger than two, so that most literature results on PD-NOMA hardly apply. Removing this limitation is a merit of the proposed approach. Additionally, allowing the option of limited SIC and evaluating its impact is a second nontrivial contribution of this work.

All the results obtained in this work are based on achievable rates, so that further studies are required to evaluate these techniques with finite coded modulations. In this framework, one shall select a number of modulation and coding options and decide which ones to apply in order to optimize the system fairness (either MAX-MIN or proportional). The information-theoretical analysis is asymptotically valid but shall be corroborated by simulation results in order to account for the performance limitations due to finite length codes.

APPENDIX A PROOF OF THEOREM 1

Lemma 1: The functions

$$\phi_k(\boldsymbol{x}) \triangleq \mathcal{R}_k\left(\frac{x_1}{\sum_{i=1}^K x_i}, \ldots, \frac{x_K}{\sum_{i=1}^K x_i}\right), \tag{32}$$

with $\mathcal{R}_k(\cdots)$ defined in (21) and $\boldsymbol{x} \in \mathbb{R}_+^K \backslash \{\mathbf{0}\}$, increase with each x_k and decrease with any x_ℓ with $\ell \neq k$.

Proof: A straightforward calculation yields:

$$\phi_k(\boldsymbol{x}) = \log_2 \left(1 + \frac{x_k}{\rho_k^{-1} \sum_{i=1}^K x_i + \sum_{i \in \mathcal{L}_i} x_i}\right) \tag{33}$$

A direct calculation of the partial derivatives of $\phi_k(\boldsymbol{x})$ with respect to the variables $x_1, \ldots, x_K$ yields the following results:

$$\frac{\partial \phi_k}{\partial x_\ell} = \psi_k(\boldsymbol{x}) \cdot \begin{cases} \rho_k^{-1} \sum_{i \neq k} x_i + \sum_{i \in \mathcal{L}_k} x_i & \ell = k \\ -(\rho_k^{-1} + 1) x_k & \ell \in \mathcal{L}_k \\ -\rho_k^{-1} x_k & \text{otherwise} \end{cases}. \tag{34}$$

In the previous expressions, each $\psi_k(\boldsymbol{x})$ is defined as

$$\frac{\log_2 e}{(\rho_k^{-1} \sum_{i=1}^K x_i + \sum_{i \in \mathcal{L}_k} x_i)(\rho_k^{-1} \sum_{i=1}^K x_i + x_k + \sum_{i \in \mathcal{L}_k} x_i)}. \tag{35}$$

Since $\psi_k(\boldsymbol{x}) > 0$ for all $k = 1, \ldots, K$, the partial derivatives $\partial \phi_k / \partial x_\ell$ are positive for $\ell = k$ and negative otherwise, which proves the statement of this lemma. ■

Now, the proof of Theorem 1 is reported as follows.

Proof: Since $\Theta_{\min}(\boldsymbol{\alpha})$ is a continuous function defined on the compact set

$$\mathcal{A}_K \triangleq \{\boldsymbol{\alpha} : \boldsymbol{\alpha} \geq \mathbf{0}, \sum_{k=1}^K \alpha_k = 1\}, \tag{36}$$

the Heine-Borel Theorem (see, *e.g.*, [32, Cor. 2.37, 2.39]) shows that it attains the minimum value at some $\tilde{\boldsymbol{\alpha}} \in \mathcal{A}_K$. We claim that the power allocation vector $\tilde{\boldsymbol{\alpha}} = (\tilde{\alpha}_1, \ldots, \tilde{\alpha}_K)$ satisfies the following equations:

$$\Theta_{\min}(\tilde{\boldsymbol{\alpha}}) = \frac{\mathcal{R}_1(\tilde{\boldsymbol{\alpha}})}{T_1} = \cdots = \frac{\mathcal{R}_K(\tilde{\boldsymbol{\alpha}})}{T_K}. \tag{37}$$

In fact, assume (on the contrary) that eqs. (37) are not satisfied, so that, for some $k \in \{1, \ldots, K\}$,

$$\frac{\mathcal{R}_k(\tilde{\boldsymbol{\alpha}})}{T_k} > \Theta_{\min}(\tilde{\boldsymbol{\alpha}}). \tag{38}$$

Then, applying Lemma 1 and the continuity of the functions $\phi_k(\boldsymbol{x})$, $\tilde{\alpha}_k$ can be decreased by a sufficiently small amount δ, so that

$$T_k \Theta_{\min}(\tilde{\boldsymbol{\alpha}}) < \phi_k(\tilde{\boldsymbol{\alpha}} - (0, \ldots, \delta, \ldots, 0)) < \phi_k(\tilde{\boldsymbol{\alpha}}) \tag{39}$$

and, for all $\ell \neq k$,

$$\phi_\ell(\tilde{\boldsymbol{\alpha}} - (0, \ldots, \delta, \ldots, 0)) > \phi_\ell(\tilde{\boldsymbol{\alpha}}) \geq T_\ell \Theta_{\min}(\tilde{\boldsymbol{\alpha}}) \tag{40}$$

These inequalities imply that

$$\Theta_{\min}(\tilde{\boldsymbol{\alpha}}) < \Theta_{\min}\left(\frac{\tilde{\boldsymbol{\alpha}} - (0, \ldots, \delta, \ldots, 0)}{1 - \delta}\right), \tag{41}$$

contrary to the optimality of $\Theta_{\min}(\tilde{\boldsymbol{\alpha}})$.

The proof of uniqueness of the optimum is derived as follows. Assume there is another vector $\boldsymbol{\alpha}' \neq \tilde{\boldsymbol{\alpha}}$ such that $\Theta_{\min}(\boldsymbol{\alpha}') = \Theta_{\min}(\tilde{\boldsymbol{\alpha}})$. Then, there is a smallest index ℓ such that $\alpha'_\ell < \tilde{\alpha}_\ell$ and $\alpha'_k \geq \tilde{\alpha}_k$ for $k = 1, \ldots, k-1$. Thus, $\mathcal{R}_\ell(\boldsymbol{\alpha}') < \mathcal{R}_\ell(\tilde{\boldsymbol{\alpha}})$, which implies that $\Theta_{\min}(\boldsymbol{\alpha}') < \Theta_{\min}(\tilde{\boldsymbol{\alpha}})$, contrary to the assumption that $\tilde{\boldsymbol{\alpha}}$ minimizes $\Theta_{\min}(\boldsymbol{\alpha})$. ■

APPENDIX B SOLUTION OF EQS. (28) FOR LIMITED SIC

Recalling the previous eqs. (28), hereafter reported for the sake of easy reference,

$$\mathcal{R}_k(\boldsymbol{\alpha}) \triangleq \log_2 \left(1 + \frac{\alpha_k}{\rho_k^{-1} + \sum_{\ell=1}^{k-1} \alpha_\ell + \sum_{\ell=k+1}^{K-\kappa} \alpha_\ell}\right), \tag{28}$$

one can notice that the second term in the denominator vanishes when $k \geq K - \kappa$. Thus, two ranges for k can be distinguished in order to specialize eqs. (28) as follows:

- For $1 \leq k \leq K - \kappa$,

$$\mathcal{R}_k(\boldsymbol{\alpha}) = -\log_2 \left(1 - \frac{\alpha_k}{\rho_k^{-1} + \sum_{\ell=1}^{K-\kappa} \alpha_\ell}\right) \tag{42}$$

- For $K - \kappa < k \leq K$,

$$\mathcal{R}_k(\boldsymbol{\alpha}) = \log_2 \left(1 + \frac{\alpha_k}{\rho_k^{-1} + \sum_{\ell=1}^{k-1} \alpha_\ell}\right) \tag{43}$$

Then, by setting

$$\sigma \triangleq \sum_{\ell=1}^{K-\kappa} \alpha_\ell, \tag{44}$$

one can see, from eqs. (42) for $1 \leq k \leq K - \kappa$, that

$$(\boldsymbol{\alpha}_{\text{opt}})_k = (\rho_k^{-1} + \sigma)(1 - 2^{-T_k \xi}). \tag{45}$$

Here, setting $\xi = \mathcal{R}_k(\boldsymbol{\alpha}_{\text{opt}})/T_k$ for every $k = 1, \ldots, K$, in accordance with eqs. (22). Then, by definition (44), the following equation is derived:

$$\sigma = \sum_{k=1}^{K-\kappa} (\rho_k^{-1} + \sigma)(1 - 2^{-T_k \xi}). \tag{46}$$

Its solution is given by

$$\sigma = \frac{\sum_{k=1}^{K-\kappa} \rho_k^{-1}(1 - 2^{-T_k \xi})}{1 - \sum_{k=1}^{K-\kappa}(1 - 2^{-T_k \xi})}. \tag{47}$$

Next, eqs. (43), for $K - \kappa < k \leq K$, yield:

$$\alpha_k = \quad (2^{T_k \xi} - 1)\left(\rho_k^{-1} + \sigma + \sum_{\ell=K-\kappa+1}^{k-1} \alpha_\ell\right). \tag{48}$$

Finally, the unknown ξ can be found by solving

$$\sum_{k=1}^K \alpha_k = \sigma + \sum_{k=K-\kappa+1}^K \alpha_k = 1. \tag{49}$$

APPENDIX C POWER OPTIMIZATION WITHOUT SIC

This is a special case of the limited SIC developed in App. B and corresponding to $\kappa = 0$. Since $\kappa = 0$, $\sigma = 1$ and

$$(\boldsymbol{\alpha}_{\text{opt}})_k = (\rho_k^{-1} + 1)(1 - 2^{-T_k \xi}). \tag{50}$$

The unknown ξ can be found by solving

$$\sum_{k=1}^{K} (\rho_k^{-1} + 1)(1 - 2^{-T_k \xi}) = 1. \tag{51}$$

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