Received 30 June 2023; revised 29 August 2023; accepted 2 September 2023; date of publication 5 September 2023; date of current version 3 October 2023.

Digital Object Identifier 10.1109/TQE.2023.3312284

Improving Urban Traffic Mobility via a Versatile Quantum Annealing Model

ANDREA MARCHESIN[1] (Graduate Student Member, IEEE), BARTOLOMEO MONTRUCCHIO[2] (Senior Member, IEEE), MARIAGRAZIA GRAZIANO[3], ANDREA BOELLA[4], AND GIOVANNI MONDO[4]

[1]Department of Electronics and Telecommunications, Politecnico di Torino, 10129 Turin, Italy
[2]Department of Control and Computer Engineering, Politecnico di Torino, 10129 Turin, Italy
[3]Department of Applied Science and Technology, Politecnico di Torino, 10129 Turin, Italy
[4]Department of Technology Innovation, Telecom Italia Lab, 10148 Turin, Italy

Corresponding author: Andrea Marchesin (e-mail: andrea.marchesin@polito.it).

The work of Andrea Marchesin was supported by TIM S.p.A. through the Ph.D. scholarship.

ABSTRACT The growth of cities and the resulting increase in vehicular traffic pose significant challenges to the environment and citizens' quality of life. To address these challenges, a new algorithm has been proposed that leverages the quantum annealing paradigm and D-wave's machines to optimize the control of traffic lights in cities. The algorithm considers traffic information collected from a wide urban road network to define activation patterns that holistically reduce congestion. An in-depth analysis of the model's behavior has been conducted by varying its main parameters. Robustness tests have been performed on different traffic scenarios, and a thorough discussion on how to configure D-wave's quantum annealers for optimal performance is presented. Comparative tests show that the proposed model outperforms traditional control techniques in several traffic conditions, effectively containing critical congestion situations, reducing their presence, and preventing their formation. The results obtained put in evidence the state of the art of these quantum machines, their actual capabilities in addressing the problem, and opportunities for future applications.

INDEX TERMS Quadratic unconstrained binary optimization (QUBO), quantum annealing, quantum computing, urban traffic optimization.

I. INTRODUCTION

The significant growth of the global population over the past century has led society to organize large urban centers to welcome a high density of people and provide them with many public and commercial services. However, the amount of resources needed to sustain these realities has progressively increased costs, both from an economic and an environmental perspective. Recently, information technologies have become widespread in the urban fabric, trying to improve the quality and efficiency of the services present locally, and the concept of a smart city raised [1]. From the alienating modern cities, the attention of legislators begins to refocus on the life and needs of their citizens, and thanks to the information derived from the mutual cooperation between sensors' networks [2] and distributed elaboration systems [3], [4]; many ideas are born to improve the quality of life and the quality of service. Reference principles in these advancements are enhanced safety and security, low environmental impact, both in terms of emissions and needed resources, and improved educational facilities and public health. On the other hand, the impacts of adopting such technologies also involve the companies and institutions themselves, allowing them to introduce even better and more sustainable services while containing the expenditure.

Research in this area is broad and interdisciplinary, and many subfields have been derived over the years, including smart mobility, to which the present article refers. The aims are to address the reduction of urban traffic, which significantly affects citizens' lives and has consequences on the environment [5], to guarantee more safe, efficient, and sustainable movements.

In order to optimize the vehicle's travel across the road networks, control strategies through traffic signals are adopted. Looking at the future [6], smart cities are expected to be provided with automatic urban traffic management and control systems, which will be capable of working without the assistance of humans. In a more holistic view, more services will be considered and optimized at once, such as managing together the vehicles and pedestrian flows [7], or the routing of vehicles [8] to reach specific destinations. There are also proposals for cooperative techniques between connected vehicles [9] and even self-driving ones [10].

This article introduces a novel model to optimize traffic conditions in smart cities, taking the benefit of quantum computing. In particular, the problem of optimizing the control patterns of urban traffic lights has been considered, leveraging the information that may be made available by distributed sensor networks. Differently from traditional control techniques [6], the model is based on a comprehensive view of the road network state, which allows to provide better combinations of control signals and, therefore, to reduce critical congestion events. The algorithm is intended to be implemented on D-Wave Systems, Inc., quantum annealers (QAs) and, in particular, the tests to prove its effectiveness have been conducted on *Advantage* machines.

Few other works proposing quantum annealing algorithms with the same aim are present in the literature. Inoue et al. [11] attempted to mitigate traffic congestion by minimizing the number of vehicles passing through the roads connecting adjacent intersections. They assigned a binary variable to each intersection, enabling traffic to flow in the longitudinal direction or toward the transversal direction. However, this approach has limitations in representing more complex scenarios, as intersections may have more than two roads crossing at a given point with several independent traffic lights. Furthermore, the authors do not provide guidance on how to define prioritized travel directions, which is crucial when controlling the primary arterials of the network.

Hussain et al. [12] propose a formulation that assigns a set of variables to each intersection, representing various traffic light configurations. The cost function aims to maximize the number of vehicles passing through each intersection. The model also includes a synchronization strategy between neighboring traffic lights. However, the authors focused solely on the number of vehicles flowing per activation, and the control patterns do not consider actual traffic conditions on the road network. Moreover, this choice may disadvantage smaller roads, potentially resulting in directions never being served.

Despite showing intriguing results and insights, the above investigations are limited in scope, hindering their ability to handle more realistic application scenarios. Therefore, a more flexible representation of the control problem is necessary to address the above-mentioned criticalities. The algorithm discussed in this article offers such a solution, which is structurally different and represents a novel approach to tackling the problem with an eye to practical applications. Any direct comparison with previous models would be unfair. Hence, the discussion will mainly focus on the model's description, characteristics, and application perspectives.

II. BACKGROUND

Nowadays, many challenges are still open in solving complex optimization problems, of primary importance also for smart city applications, and the limited computational capabilities of the available processing systems represent a significant hurdle. Overcoming the performance of classical machines is a more general problem that is attracting scientists from another emerging research domain, quantum computing, which has received new vital lymph over the past decade due to the technological progress that has led to the birth of the first quantum machines. The fledgling quantum computers try to address different kinds of problems, and D-wave's QAs [13] specifically work on optimization tasks. These machines consider quantum annealing [14], [15] to solve combinatorial problems, exploiting quantum mechanics phenomena and trying to obtain the best solution in a limited amount of time, with a higher likelihood than the traditional processing systems. Several applications of quantum annealing formulations have been recently proposed, some more theoretical, such as for solving the factorization [16] and the graph partitioning [17] problems, some more application specific, such as optimizing the routing of vehicles in urban centers to reduce traffic [18], the assignment of nurses to shifts [19] (namely the nurse scheduling problem), and introducing new ways to speed up the machine learning techniques to distinguish Higgs–Boson decay in experimental physics [20].

As the present research proposes an optimization algorithm to be solved by D-wave's advantage systems, the formalism underlying quantum annealing is discussed, dwelling on the main characteristic aspects. Historically, quantum annealing is the evolution of simulated annealing (SA) [21], a widespread heuristic algorithm that, since its formulation in the 1980s, has proved successful in many application scenarios. The latter was born, thanks to the intuition of implementing an optimization strategy that mimics thermal annealing processes, where metal is cooled in a heat bath to allow crystals to form large structures with low defects associated with small energies. Therefore, by expressing an optimization problem in terms of an energy function, the solver is able to find solutions that approximate the global optimum. About 20 years later, the formulation of quantum annealing was proposed to obtain even better solutions to problems mapped on energy landscapes by replacing the effects of temperature with quantum fluctuations and, in particular, exploiting the tunnel effect to escape local minima. The problem to be optimized is embedded in a quantum mechanical system, which evolves over time following a trend described by Schrödinger's equation

$$i\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle \tag{1}$$

where $\psi(t)$ is the state vector associated with the quantum system, and $H(t)$ is the Hamiltonian that describes its evolution over time.

At the basis of the quantum annealing algorithm, there is the adiabatic theorem, which states that the ground state of the problem under analysis (i.e., its solution), with which the Hamiltonian H_p is associated, can be reached by a quantum

IEEE Transactions on Quantum Engineering

system that is set initially to a known ground state, described by a reference Hamiltonian $H(0)$, if and only if the term $H(t)$ changes with a slow transient τ from $H(0)$ to H_p.

In particular, the transformation referring to this behavior can be expressed as follows, considering the Lenz–Ising model to make the energy functions explicit:

$$\begin{aligned} H(t) &= \frac{t}{\tau}H_p + \left(1 - \frac{t}{\tau}\right)H(0) \\ &= -\sum_{\langle ij\rangle} J_{ij}\sigma_i^z\sigma_j^z - \Gamma(t)\sum_i \sigma_i^x \end{aligned} \tag{2}$$

where $\langle ij\rangle$ are the neighboring lattice points, σ_i^x and σ_i^z are the discrete variables associated with the Ising spin, J_{ij} are the values of the coupling between interacting spins, and $\Gamma(t)$ is the time-dependent amplitude of the external transversal field that makes the $H(0)$ term even less relevant on the overall energy contribution.

While the Ising model is very effective in representing physics problems as H_p, in the computer science domain, an equivalent expression is considered, named quadratic unconstrained binary optimization (QUBO), which is obtained simply by applying a variable substitution. Specifically, the spin variables $\sigma_i \in \{-1; 1\}$ are reported in binary form, considering $\sigma_i \mapsto 2x_i - 1$, where $x_i \in \{0, 1\}$. The expression so revised is reported in (3), while an exhaustive demonstration of its equivalence with that of Ising can be found in the literature [22].

$$\begin{aligned} f(x) &= c_0 + \sum_{\langle ij\rangle} c_{ij}x_ix_j \\ &= c_0 + \sum_i c_{i,i}x_i + \sum_{i<j} c_{i,j}x_ix_j \text{ with } x_ix_i = x_i \end{aligned} \tag{3}$$

This QUBO formulation can be further simplified by considering the vector x as the expression of all the binary terms x_i and gathering all the coefficients c_{ij} into an upper-triangular matrix Q

$$f(x) = x^{\dagger}Qx. \tag{4}$$

The objective of the optimization process over this model is conceived as to find the combination of parameters x_i associated with the minimum energy of $f(x)$, formally

$$x_{\min} = \arg\min f(x). \tag{5}$$

In conclusion, it is worth noting that a problem expressed in the form of a QUBO model can be solved with both quantum annealing and SA, opening the possibility of directly comparing the two techniques. This aspect is also relevant for other reasons, more practical in terms of near-term application scenarios. In fact, SA is typically considered a way to address the computational limitations of current quantum machines by allowing for suboptimal but reasonable solutions to problems that would, otherwise, not be solvable via today's available hardware. Furthermore, the same algorithm can be applied to speed up the study and validation procedures of a reference QUBO model, leaving to real quantum computers only the task of solving its *refined version*.

III. METHODS

In this section, the formulation of the QUBO model is discussed in detail, together with the main elements that shape the optimization task. The proposed algorithm is *"traffic signal concentric"*; therefore, it is not bound to specific conformations of the road network. To properly characterize the problem, two kinds of information are necessary, static and dynamic. The former coincides with the geometrical and structural properties of the reference road network, the location of the available traffic lights, and the directions controlled by each of them. The latter, instead, is related to the distribution of vehicles and their movements, both quantities described through a fluid dynamics approach. In particular, the amount of vehicles arriving at an intersection is expressed in the form of a density value (i.e., the average number of vehicles moving per unit of time) and its behavior is statistically characterized by how the traffic flows, where vehicles go straight ahead or make turns. Fig. 1 presents a graphical representation of the reference situation that can be optimized by the model on a sample four-way intersection.

First, the road network map is reported on a graph where nodes represent the characteristic intersections and the oriented edges model the relative road connections. To give a spatial reference to the structure obtained, nodes are the assigned values of *latitude* ϕ and *longitude* λ, while each connection is assigned a unique label i. The traffic lights inside are identified as $\mathrm{X}_n^{(\phi,\lambda)}$, with $n \in \mathbb{N}$. Then, the variables of the optimization problem $x_{i,j}$ are instantiated to represent the various traffic routes at intersections, where i are the source roads and j are the destination ones. As a single signal can allow the vehicles to proceed in multiple directions (e.g., for right-hand drive, go straight, and turn right), each traffic light controls a set of variables $x_{i,j}$. Therefore, when a traffic light is turned ON, all the related variables are activated, allowing vehicles to move in the permitted directions.

Proceeding with the optimization process, the model evaluates the consequences of the traffic light activation in relation to the state of the vehicular density in the surrounding area. Specifically, the decision to enable a signal is influenced by the traffic waiting behind it together with that present on the downstream road segments. In this way, the QUBO is able to analyze the impacts of vehicles moving from departure to exit roads in order to decrease incoming traffic and to avoid increasing congestion at nearby intersections. To accomplish the task, variables are always considered in pairs of successive routes, $x_{i,j}$ and $x_{j,k}$, so as to understand whether the vehicles coming from road i and heading to road j will accumulate there or can possibly proceed toward road k. Combining all the couples of variables available, the optimization model can comprehensively evaluate the consequences of activating a specific traffic light with respect to the state of all the others,

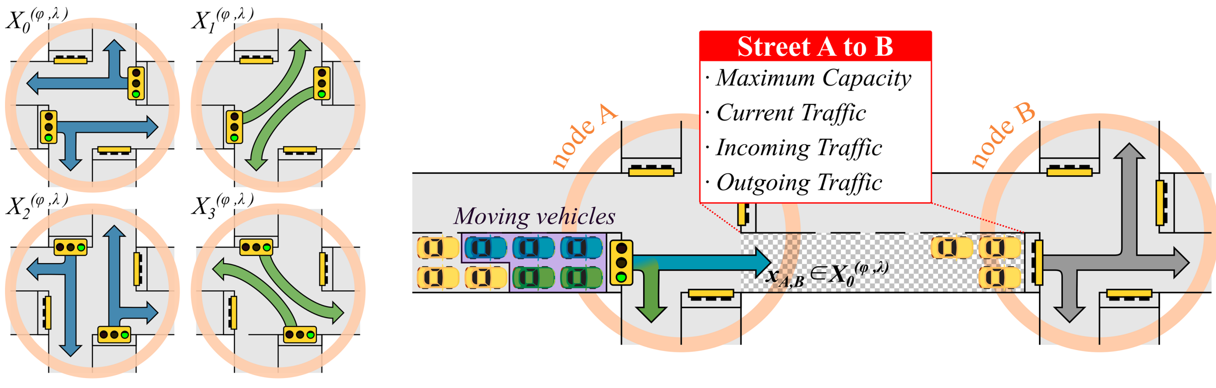


FIGURE 1. **Visual representation of the key quantities involved in the optimization problem formulation on a reference four-way intersection. (a) Example of how variables $X_n^{(\phi,\lambda)}$ are assigned at a typical intersection is shown. A variable gathers a couple of traffic lights and the related controlled directions. (b) Information required to assess congestion in one street where vehicles are allowed to move is displayed. Blue vehicles will move in direction $x_{A,B}$ and join the existing ones. The model considers two scenarios to evaluate congestion on the road: with or without any active traffic lights downstream.**

predicting the best strategies to mitigate the critical traffic conditions on the overall road network.

A. DETAILS OF THE QUBO MODEL

The formulation proposed for the optimization task is reported as follows:

$$f(x) = \sum_{i,j}\sum_{j,k}\left\{x_{i,j}\left(1 - x_{j,k}\right)W'_{i,j} + x_{i,j}x_{j,k}W''_{i,j,k}\right\} + P\sum_{i,j} x_{i,j}\left[\alpha_{i,j} + \sum_{s,t} x_{s,t}\beta_{i,j,s,t}\right] + B. \quad (6)$$

The expression is composed of a first term, which describes the impact of the activation of the available traffic lights on local congestion, and other additional terms, which refer to constraints and penalties to limit the solution space to those permitted.

As for the first part, the two terms $W'_{i,j}$ and $W''_{i,j,k}$ are the weight coefficients, $W'_{i,j}$ is associated with the condition in which the traffic light regulating the direction $x_{i,j}$ is switched ON and the subsequent one, acting on $x_{j,k}$ is not; $W''_{i,j,k}$ is associated with a Green-wave condition, i.e., when both traffic lights on $x_{i,j}$ and $x_{j,k}$ are active. The latter represents the most convenient situation, as it avoids the stagnation of vehicles in between. They are defined through hyperbolic functions to progressively increase the relative weights to the formation of localized traffic congestion

$$W'_{i,j} = \frac{C_j}{C_j - \left[D_i S_{i,j} T_{i,j} + D_j\right]} \quad (7)$$

$$W''_{i,j,k} = \frac{C_j}{C_j - \left[D_i S_{i,j} T_{i,j} + D_j\left(1 - S_{j,k}T_{j,k}\right)\right]} \quad (8)$$

where

C_j maximum capacity of the road j, expressed in terms of traffic density;
D_i traffic density of the road i;
$S_{i,j}$ turn statistics of the traffic density coming from road i and confluent into road j;
$T_{i,j}$ transfer factor associated with $S_{i,j}$.

In particular, for $T_{i,j}$, it represents the portion of vehicles, which will be able to move from road i to road j with a single activation of the control traffic light. No assumptions are made in the definition of the road network. Therefore, when considering specific application cases, only the intersections and road segments available in the network are characterized by the whole set of parameters.

As for the constraints and penalties, the second term modifies the weights associated with each variable to prevent or disadvantage unwanted solutions, promoting those close to practical application scenarios. However, they are obtained as the expression of two different requirements: the uniqueness constraint and the sequence penalty. The former avoids the activation of multiple concurrent traffic signals at a single intersection at the same time. The latter promotes reaching solutions that define activation sequences at each intersection, reducing the risk of occurring in conditions of not served traffic directions. The decision to distinguish the constraint from the penalty lies in the need to explore with flexibility the solution space and to allow the definition of variable switching timing depending on specific traffic patterns. The weights associated with these terms (i.e., P, α, β, *and* B) must be evaluated at execution time, considering the information from the road network under analysis and their definition is to be treated carefully; as QUBO formulations represent energy landscapes where to find the optimal points of the problems considered, any additional term modifies its shape inevitably influencing the quality of the obtainable outcomes. Therefore, the methodology to derive them is discussed in detail in Section III-B.

The objective of the optimization process is to find the best patterns to apply to the traffic signals present inside a reference road network in order to minimize congestion situations. As discussed, a single traffic light can control multiple travel directions $x_{i,j}$; therefore, when the QUBO is defined, this set of variables is grouped into one variable, whose associated weight is the sum of the partial terms. This approach can also be applied when considering pairs of traffic lights that share the same lighting sequence (e.g., traffic lights acting in opposite directions on two-way roads). The technique is helpful to reduce to a minimum the number of variables to be optimized.

B. ANALYSIS OF THE CONSTRAINTS

There are no strict strategies for defining constraints, and only methodological considerations are made, demanding their characterization to the numerical analysis of the experimental tests applied to each problem tackled.

A general distinction is made when considering the requirements of a specific application. In particular, constraints can be categorized as hard or soft, depending on the consequences of their eventual violation. Hard constraints cannot be broken; otherwise, no acceptable solutions are obtained. Soft ones are more proper penalties, as their violation leads to little advantageous solutions. Considering the two constraints applied to the QUBO model, the uniqueness one is hard, while the sequence is soft.

Different strategies have been adopted to define weights associated with them, which will be detailed, but first, it is important to notice that they both share a weight term P whose value has to be evaluated for every single state of the road network. From the literature [23], it is suggested to scale it in a range between [0.75, 1.5] of the average energy associated with the variables describing the QUBO problem. As this value cannot be known precisely except through a broad numerical analysis, unfeasible given the complexity of the problem, it is necessary to estimate its value. Therefore, few simplifications have been adopted to derive P.

Considering again the proposed QUBO formulation in (6) and introducing the following equivalent:

$$f(x) = g(x) + Pu(x) \tag{9}$$

the term P can be expressed as follows:

$$P \approx \nu \overline{\epsilon}_{x \in g(x)}, \text{ with } \nu \in [0.75, 1.5] \tag{10}$$

where the energy quantity $\overline{\epsilon}_{x \in g(x)}$, intended as the average weight associated with the variables belonging to function $g(x)$, is unknown. For what concerns the tests conducted, an approximation of the average energy of $g(x)$, namely $\overline{\epsilon}_{g(x)}$, has been obtained classically solving the function by imposing a set of *ten* uniformly distributed random patterns to the available variables $x_{i,j}$ and evaluating the energy result through an arithmetic mean.

Finally

$$\overline{\epsilon}_{x \in g(x)} = \frac{\overline{\epsilon}_{g(x)}}{N} \Rightarrow P = \frac{\overline{\epsilon}_{g(x)}}{N} \nu \tag{11}$$

with N that represents the number of traffic lights in the considered road network.

1) UNIQUENESS CONSTRAINT

Recalling the QUBO formulation from (6), its contribution is defined as follows:

$$\text{Uniqueness} := P \sum_{i,j} x_{i,j} \sum_{s,t} x_{s,t} \beta_{i,j,s,t} + B. \tag{12}$$

The requirement of having only an allowed active direction can be expressed as follows:

$$\forall \text{ Intersection } I, \text{ Penalty} = \left[\sum_{\langle ij \rangle \in I} x_{i,j} - 1 \right]^2 . \tag{13}$$

For its definition, reference has been made to the techniques adopted in previous works [23]. The variables $\beta_{i,j}$ and B can be obtained by collecting the coefficients from the resulting expression. As can be observed, the penalty term introduced is null if one and only one traffic light is active for each intersection; otherwise, the relative penalty increases the energy profile of the QUBO function so that the solver will discard these solutions as not advantageous.

2) SEQUENCE PENALTY

Recalling the QUBO formulation again, its contribution is defined as follows:

$$\text{Sequence} := P \sum_{i,j} \alpha_{i,j} x_{i,j}. \tag{14}$$

The idea is to add to the individual variables a penalty proportional to the priority assigned to the control traffic light, related to the activation history of all the local traffic lights at each intersection. The parameter $\alpha_{i,j} \in \mathbb{N}$ is associated with a counter specific for each signal, which is incremented every time it is activated. The higher the value of $\alpha_{i,j}$, the lower the priority. Initially, all counters are set to zero, and no penalties related to the sequence constraint are introduced.

As the optimization process proceeds, their value should be monitored to avoid diverging conditions and, therefore, when

$$\forall \text{ Intersection } I, \ A = \min \bigcup_{\langle ij \rangle \in I} \alpha_{i,j} > 0 \tag{15}$$

all the terms $\alpha_{i,j}$, with $\langle ij \rangle \in I$, are updated by removing the bias A.

C. REFERENCE CLASSICAL MODEL

A reference algorithm has been formulated to define comparative analyses between the proposed QUBO model and a classical solution. The algorithm is inspired by the paradigm of "vehicle actuate isolated junctions" [6], where the current number of vehicles circulating in a local intersection is considered to determine the best activation sequence of the traffic lights present. Therefore, each network intersection is analyzed separately, and a higher priority is given only to signals that control roads with a traffic density above a threshold σ. During the next activation cycle, the priority value establishes which signal should be enabled. In the event that several traffic lights reveal suitable, the choice falls randomly on a single element as a consequence of applying a uniform probability distribution on the available set. Furthermore, in a series of subsequent optimizations, once a signal is activated, it cannot be turned back on until all the others have been.

By inspecting intersection per intersection, the algorithm can provide a complete control pattern, specific for a particular traffic condition, for the road network to be optimized.

IV. RESULTS

First, the proposed QUBO model has been fully analyzed to study its behavior in different application scenarios. These preliminary tests aimed to determine the model's capability to represent large problems and find the best values to apply to the available parameters. To verify its effectiveness in optimizing the congestion of reference urban road networks, several traffic conditions have also been taken into account. These experiments involved, as a solver, the SA algorithm that runs on a classical machine. As stated in Section I, a heuristic algorithm is useful to "prototype" a model before implementing it in real quantum devices. The approach can fasten the characterization of a QUBO formulation, reducing, at minimum, the experiments on quantum computers directly impacting the implementation costs. In fact, access to these machines and their resources is limited and intended for consideration.

Once the most suitable parameters have been identified, tests on real quantum computers have been executed, specifically on D-wave's advantage systems. Here, additional analyses have been addressed to define the best strategies to configure the specific target machines, observing how they influence achievable results. These will be discussed in detail to present a methodology for characterizing and implementing a general QUBO problem for optimal execution on a QA.

In the end, test cases on a QA have been addressed on a road network characterized by an initial distribution of traffic. The QUBO model has been applied iteratively, simulating, at each optimization step, the evolution of vehicles determined by the traffic light patterns obtained. The process has also been replicated by considering the SA and the reference optimization model. The derived comparisons highlighted the effectiveness of the proposed model over a classical solution but also the limits of the current quantum hardware that, despite representing showpieces of today's technology, are affected by limited computational capabilities and nonidealities that reduce the benefits with respect to the solver running on classical machines.

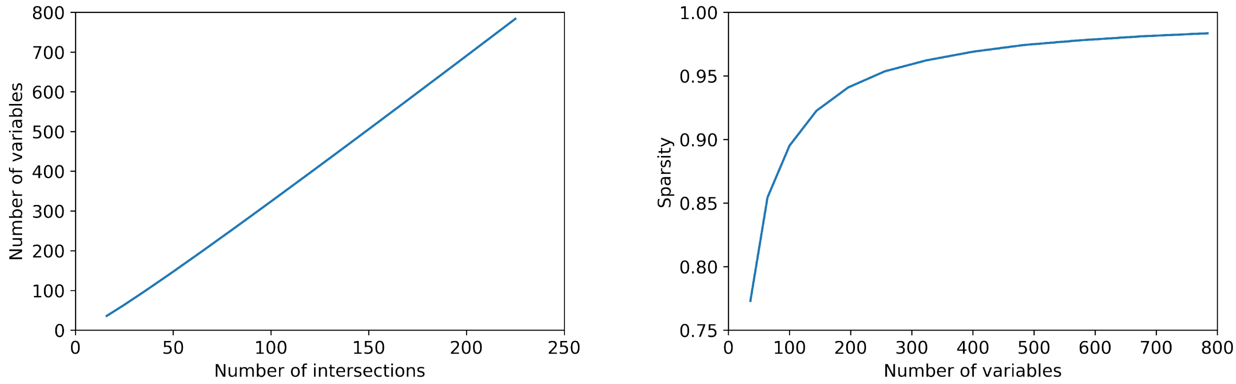
In order to evaluate the efficacy of the QUBO formulation, a specific metric has been employed to compare its solutions with those of the reference classical model. The chosen metric is based on the average difference between the traffic densities across the network's roads that experience heavy traffic accumulation. For the preliminary analysis presented in this article, the information on traffic density has been deemed sufficient, given the strong correlation between a reduction in traffic density and an increase in the average velocity of vehicles [24]. The adoption of more detailed measures of efficiency is demanded for future works and applications to real-world test cases.

A. STATIC ANALYSES OVER THE PROBLEM FORMULATION

One of the most important characteristics of an optimization problem is related to its scalability by increasing the number of variables. For the proposed QUBO model, the variables needed are proportional to the number of traffic lights regulating vehicular traffic. Recalling the formulation, the QUBO model collects in a single variable the traffic lights regulated by the same timing signal at each intersection; therefore, the dependence between the number of traffic lights and the number of QUBO variables is at most linear, thus expressing a *sublinear tendency*, as shown in Fig. 2(a).

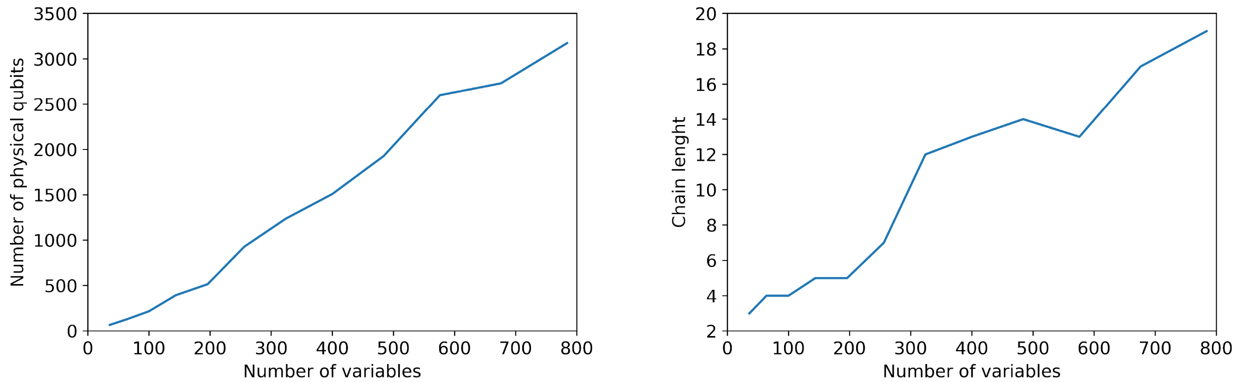
Other important static properties are directly related to the QUBO model characteristics that impact the implementation into the target machine. In particular, the physical architectures at the basis of QAs can be interpreted as sparse graphs, where each node represents a qubit and the associated edges express the relationships between one and another. Different topologies have been proposed over the years, and the one considered during all the tests conducted is called Pegasus [25], implemented specifically on D-wave advantage systems. These machines are equipped with more than 5000 qubits, each with 15 connections. This information is of primary interest, as it directly affects the possibility of running experiments by limiting their intrinsic complexity. In fact, the graph representing the QUBO problem must be mapped on the equivalent graph of the QA. The translation called *minor embedding* is needed to conjugate an abstract representation of a reference problem with its physical implementation, constrained to the hardware specifications. Therefore, variables are associated with qubits and the characteristic relationships are expressed through the available interconnects. This process modifies the original shape of the problem graph with larger requirements of resources. By way of example, to guarantee the physical connection of variables distant from one another, a series of subsequent physical qubits between them is used to define a sort of bridge to connect the two distant points. The allocation of multiple qubits for representing one logic variable is also adopted when its relationships are higher than the available physical interconnects. Hence, the embedding process reduces the effective number of qubits.

A metric to evaluate the effects of the minor embedding at an early stage is the degree of sparsity of the QUBO matrix: the more sparse the matrix, the lower the overhead of physical qubits on the target machine. Considering the application of the original formulation to optimize road networks with increasing dimensions, the sparsity of the resulting QUBO matrices revealed high, increasing the number of intersections. This is consistent with the optimization problem definition: the range of one traffic light is limited to its neighboring



(a) (b)

FIGURE 2. Mathematical analyses on the proposed QUBO model, by varying the road network dimensions. (a) Number of logic variables needed to represent the optimization problem is reported. (b) Behavior of the density of the associated QUBO matrix.



(a) (b)

FIGURE 3. Analysis of the effects of the minor embedding process on the proposed QUBO model with an increasing number of variables. (a) Number of physical qubits needed to represent the problem on a D-wave's advantage 4.1 system was addressed. (b) Maximum chain length of the variables.

intersections. Therefore, the "locality" of these relationships explains well the tendency of the graph, as shown in Fig. 2(b).

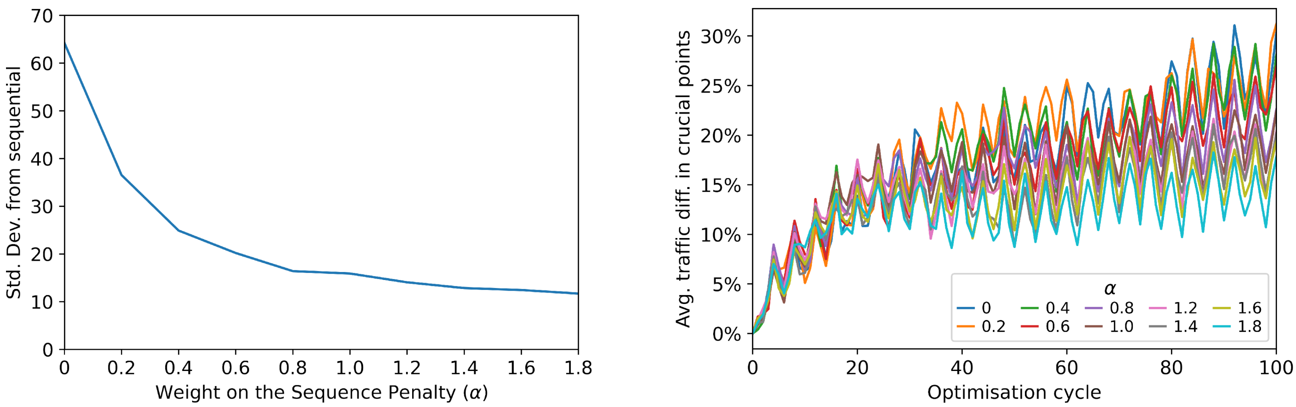
The minor embedding was implemented through the *minorminer* tool provided by the D-Wave Ocean SDK, a heuristic algorithm. The used QA was an Advantage 4.1 system with 5627 qubits.

The overhead of physical variables necessary to represent the QUBO problem has been quantified through the "chain length" parameter, expressing the set of subsequent physical qubits representing the same logical variable (i.e., the bridges introduced before). Considering D-wave's documentation, this value should be kept in a low range, typically around 6–7, and the distribution of the different chain lengths should not be widespread. In fact, the chains are basically physical qubits representing the same logic variable and associated with physical connections, which have to guarantee the coherence of the values represented. Therefore, in the same physical circuit, the original problem should be mapped together with support qubits and connections that act as physical constraints, with the result that the longer the chains make it more difficult to find the optimal solution for the annealer machines.

The evolution of the chain length has been analyzed by varying the dimensions of the problem and by applying the *minorminer* tool. The results, as shown in Fig. 3(b), highlight that the quantum machine's physical topology further reduces the dimensions of the proposed QUBO problem that can be directly implemented. In particular, considering chains with a maximum length of eight qubits, the maximum number of QUBO variables that can be considered is approximately 250. Therefore, applying the same proportions considered before, the number of controlled intersections lowers to about 80. This number can still allow the representation of a problem of reasonable size, compatible with the needs of entire small cities or a significant portion of bigger ones.

B. DYNAMIC ANALYSES OVER THE PROBLEM FORMULATION

The model has also been characterized by considering the consequences of its application in optimization cycles, where there is an alternation between identifying the best traffic light pattern for a peculiar traffic condition of the road network and the consequent evolution of vehicles.



(a)

(b)

FIGURE 4. **Analysis of the effects of an incremental variation of the weight term α associated with the sequence penalty, characteristic of the model. (a) Average standard deviation between the obtained activation sequences and a constant regular scheme has been represented. (b) Comparison between the results of the resolution of the QUBO model and the ones derived by applying the classical reference model has been addressed. The crucial points are intended as the roads where traffic intensity is particularly high (i.e., with congestion greater than 80% of the related maximum capacity).**

In particular, the impact of two main quantities has been analyzed: one more related to the definition of the model, to determine the best strategy to configure it for specific needs, and one that concerns the moving statistics of vehicles, to try the resilience of the proposed optimization technique by varying the traffic conditions. The former is the sequence penalty, and the latter is the turning statistics of vehicles near intersections.

As these tests aim to characterize the model and explore its potentialities, the limits imposed by the currently available QAs are not adopted, and the SA has been considered as a solver. Therefore, a reference road network of 225 intersections has been considered, regulated by 784 independent traffic lights. This network has been randomly synthesized, starting from a squared lattice of 15×15 intersections. Each intersection has been characterized by a set of variables consistent with the representation in Fig. 1 and every road segment has been assigned a maximum capacity C_j of 50 vehicles. For the initial traffic conditions, the vehicles inside the network have been organized by allocating congestion following the combination of two normal distributions: one with mean $= 20\%$ and var $= 20\%$ and one with mean $= 90\%$ and var $= 10\%$, with a mutual weight in a ratio of 0.8–0.2. The two distributions are needed to define a base traffic condition that is not critical, where some situations of local congestion must be managed properly.

The uniqueness constraint has not been taken into account in these analyses as it represents a hard constraint. During all the conducted tests, the associated P term has been set to a minimum so that the obtained solutions would have met the requirements; in particular, after some preliminary tests, the P variable has been imposed to 50.

The optimization process has been repeated 100 times for every variation of the parameters studied. Therefore, vehicular traffic has evolved for 100 subsequent control patterns of the available traffic lights.

As for the sequence penalty term, it guarantees that all the traffic lights acting on a single intersection are switched ON every activation cycle. As previously pointed out, this is a soft constraint as nonserved conditions should be avoided, but, at the same time, we do not want to impose rigid activation sequences. The higher its value is, the less likely a single traffic light can remain active for more than a time slot within the same activation cycle. Here, the weight associated with the penalty term α has been varied between $[0 \div 1.8]$ and the main results have been represented in Fig. 4.

In Fig. 4(a), the activation sequences of the available traffic lights have been compared with the ones obtained through a pure sequential scheme, i.e., where the traffic lights on each intersection are switched ON in a constant sequence. In particular, the average standard deviation between the two sets of solutions has been evaluated. It has been observed that the lower the weight term, the more the solution diverges from a sequential scheme. It should be noted that no specific constraint limits the duration a single traffic light must be kept active. Therefore, the lower the α value, the more the QUBO model can optimize these time intervals by allocating more time to those traffic lights that regulate roads affected by intense traffic conditions.

In Fig. 4(b), the impacts of different values for the sequence penalty factor on the QUBO model performance have been analyzed. The resulting evolution of traffic conditions within the road network has been compared with the one obtained by applying the reference classical model. The graph represents the average traffic difference in roads where heavy traffic accumulates and generates congested situations above 80%. As can be seen, the benefits in the optimization process are always present, but they are even more significant when considering low values of α.

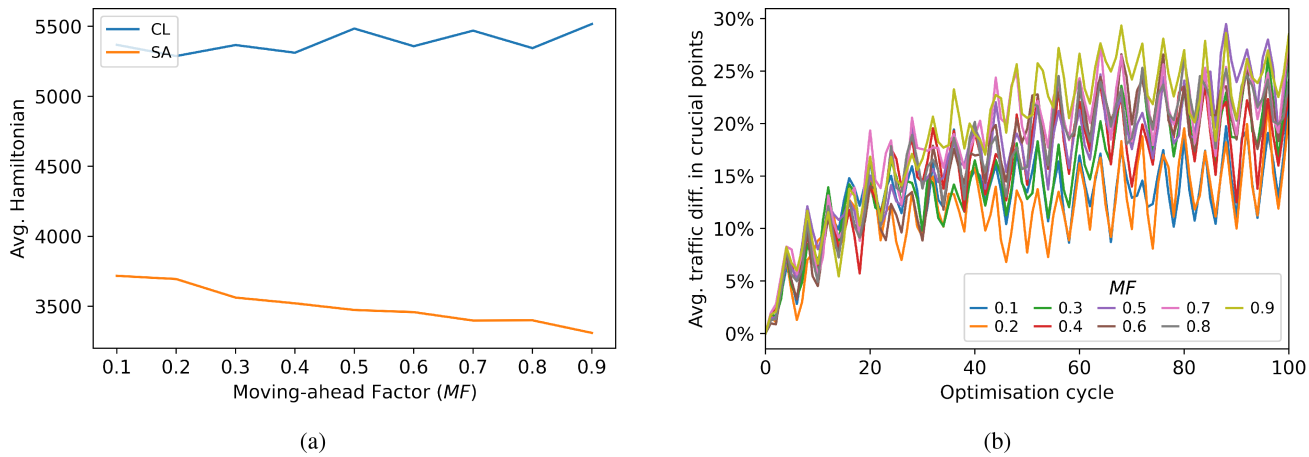


FIGURE 5. Analysis of the impacts of different values of MF for each road segment controlled by a traffic light. (a) Comparison between the solutions obtained by applying the QUBO model through SA and the reference classical algorithm (CL) is detailed. In particular, the behavior of the average Hamiltonian of the solutions on optimization cycles of 100 subsequent steps is represented. (b) Comparison has been reported directly in the domain of effects on vehicular traffic. The crucial points are intended as the roads where traffic intensity is particularly high (i.e., with congestion greater than 80% of the related maximum capacity).

Finally, in accordance with the application context, the scale factor should be chosen as a tradeoff between the performance of the optimization process and the constraints related to the laws in force. During all the subsequent studies, the value of α has been imposed to 0.2.

The other investigation focused on moving statistics, which statistically determine how the vehicles cross intersections. Recalling the formulation of the proposed QUBO model, as detailed in Section III. Vehicles' movements are characterized by the parameter $S_{i,j}$. It represents the percentage of traffic coming at an intersection, controlled by a specific traffic light, which statistically proceeds from road i toward road j. Therefore, considering a generic traffic light, the controlled access road, and its associated output directions, the following relationship holds: $\sum_j S_{i,j} = 1$.

A new quantity has been introduced to simplify the analysis of its influences, the moving-ahead factor (MF), which expresses how many vehicles statistically proceed straight at every intersection. This quantity is assigned to the corresponding $S_{i,j}$, while the other turning statistics are directly derived by equally distributing the remaining moving vehicles to the available output directions. This value has been applied to each traffic flow inside the network. Therefore, considering the regular squared lattice on which the reference roads are articulated, the MF represents the statistics that regulate vehicles' movements along the vertical and horizontal directions. Its value has been varied in a range between 0.1 (i.e., most of the vehicles turn) and 0.9 (i.e., most of the vehicles go ahead), with a step of 0.1. The results of this examination are reported in Fig. 5.

The optimization process of 100 steps has been repeated for every value of the MF and applying both the QUBO model and the classical reference method, as discussed in Section III-C. Then, the average energy associated with the obtained solutions during each test case has been evaluated, and the collection of these data has been represented in Fig. 5(a). It has to be noticed that the energy values associated with the solutions provided by the classical optimization algorithm have been estimated by applying the resulting activation patterns for the available traffic lights to an equivalent QUBO model configured to express the current traffic conditions.

A first observation that can be made regards the energy gap that separates the two sets of solutions: the ones derived through the classic reference model are always higher than those obtained through the direct resolution of the QUBO model using the SA algorithm. Therefore, the proposed optimization strategy proves to be more effective in resolving the problem of traffic congestion, as it has been conceived in the present work. Second, only for the QUBO model, does the MF value significantly impact optimization performance by decreasing the energy associated with the obtained solutions by about 11% from one end to another of the MF variation range.

In Fig. 5(b), the results of applying the QUBO model to the different MFs have been compared with those obtained using the reference classical model. Again, the average traffic difference graph in roads, where heavy traffic accumulates and generates congested situations above 80%, has been considered. Also, here, the benefits in the optimization process are always present, but they increase with higher values of the MF.

As it stands to reason, these results are achieved when vehicles move inside the road network favoring specific traffic directions, reducing their random spreading. This condition can be considered reasonably realistic since vehicles in urban environments tend to accumulate in particular traffic routes.

C. ANALYSES OF D-WAVE SYSTEMS' MACHINES' PARAMETERS

When considering physical machines, several parameters are made available to the users to modify their behavior to

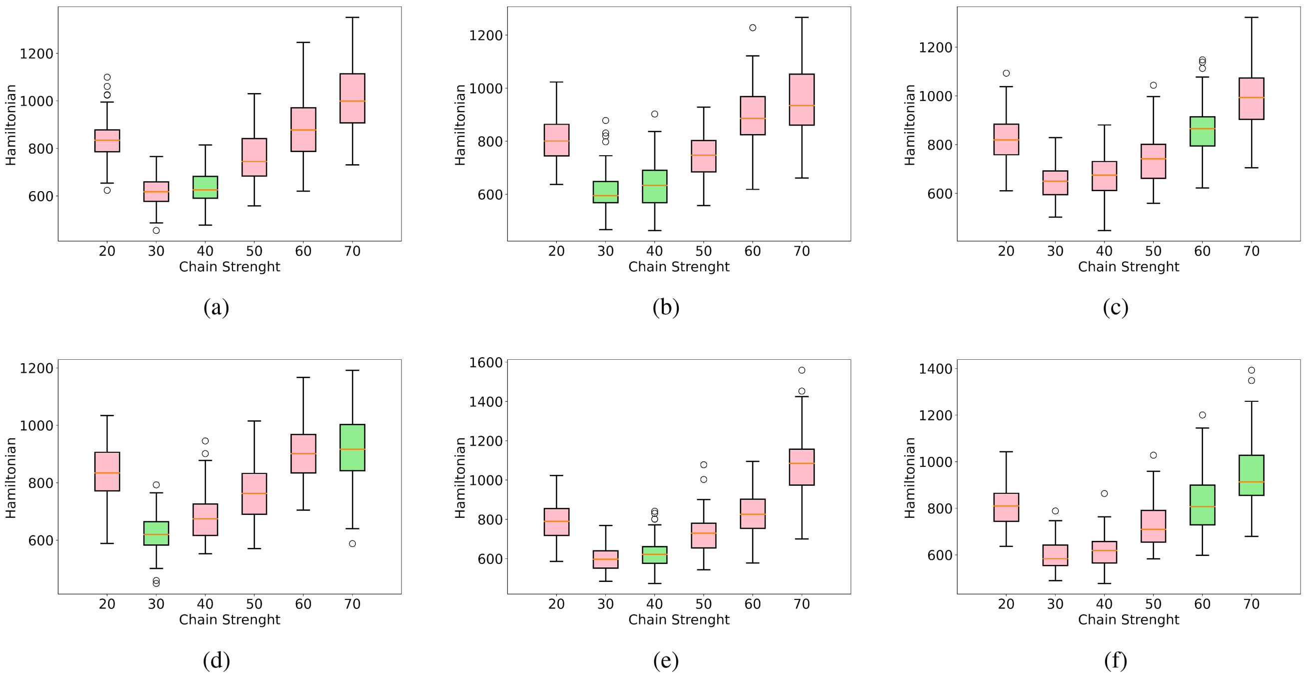


FIGURE 6. **Analysis of the joint impacts on the QUBO model of the two reference parameters of D-wave's machines, chain strength and annealing time (a.t.), by imposing a constant value of number of reads to 100. The distributions reported are the collections of samples from QAs during each optimization procedure. Green data represent valid solutions, while red ones indicate a violation of constraints. The lower values of the Hamiltonian correspond to optimal results. (a) a.t. 20 μs. (b) a.t. 30 μs. (c) a.t. 40 μs. (d) a.t. 50 μs. (e) a.t. 60 μs. (f) a.t. 70 μs.**

improve the solutions obtainable from the optimization process. D-wave's QAs allow influencing the annealing process by acting on a set of variables, among which: *number of reads*, *annealing time,* and *chain strength*.

The measurement process at the end of a quantum annealing optimization is statistics; therefore, the problem should be solved in a sequence of optimization tasks to get reliable results. Then, the solution associated with the lowest energy is selected as the optimal one. The number of reads is a parameter directly influencing the success probability of the experiments conducted. In particular, it should be large enough to obtain statistically relevant data.

The annealing time refers to the duration of the annealing process, which was explained in Section II. It is an essential parameter that should be defined with caution because it significantly impacts the ability of the quantum machine to arrive at an optimal solution. If the time is too short, the annealing process becomes ineffective, while if it is too long, the results become more affected by the quantum machine's nonidealities. In either scenario, the output obtained will be far from the optimal point.

Finally, the chain strength parameter shows how strongly physical qubits are connected to map the same logical variable. As already pointed out, since the physical hardware has limited connectivity, the minor embedding process enables solving problems with higher connectivity than the QAs physical topology. However, this requires more resources to represent the same variable. To ensure that the qubits representing the same variable assume a unique logical value when measured, the value of the parameter should be sufficiently large. This parameter functions as a coupling coefficient and has the same meaning as the weights associated with the relationships between variables described by the optimization problem. If the value is too low, the solution's consistency cannot be guaranteed, while if it is too high, the chains become rigid and penalize the annealing process, reducing optimization possibilities.

Regarding the number of reads, two different values have been used for reference: 100 and 1000. However, after conducting several tests, no differences have been found in the quality of the solutions. Therefore, to conserve resources, the number of reads has been set to 100 for all tests. The solver's behavior has been analyzed by varying the other two parameters together. The values considered have been chain strength [20; 30; 40; 50; 60; 70] units and annealing time *(a.t.)* [20; 30; 40; 50; 60; 70] μs.

For each annealing time value, experiments have been conducted with varying chain strength within the reference set. The results have been collected in Fig. 6, showing the distribution of output energy values for each annealing cycle iteration. The Green distributions represent valid solutions to the input QUBO problem.

After analyzing the results, the best solutions have been obtained using an annealing time of 30 μs and a chain strength of 40 units. These values have been used for the final tests on the QUBO model implemented on D-wave QAs, described in Section IV-D.

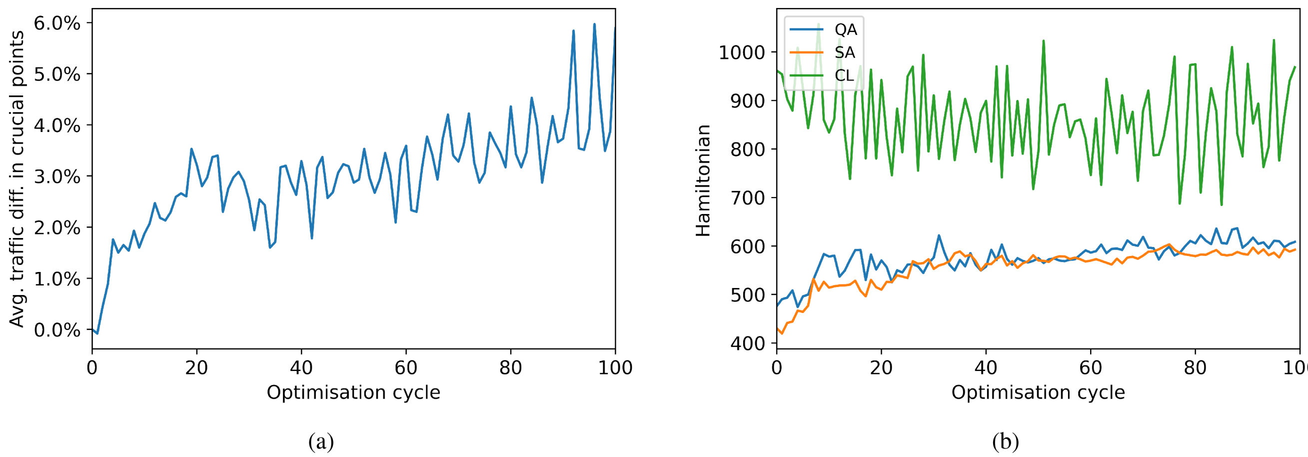


FIGURE 7. Analysis of the effects of implementing the proposed QUBO model on a D-wave's QA to optimize the activation patterns of 196 independent traffic lights. The results are compared with those provided by the classical reference model. (a) Comparison focuses on high-traffic intensity areas where congestion is greater than 80% of the maximum capacity of roads. Graph (b) displays the evolution of the Hamiltonian during the three different optimization procedures: the QUBO model solved by a QA and through SA on a classical machine, and the classical reference model (CL).

D. TRAFFIC OPTIMIZATION OVER A WIDE TIME INTERVAL WITH A QA

After having characterized the QUBO model and the behavior of the quantum annealing machines during its resolution, a comprehensive optimization process has been conducted for a road network comprising 64 intersections and 196 traffic lights, which was specifically designed to meet the requirements of quantum annealing machines. It is noteworthy that the same elementary structure, as previously considered in Section IV-B, has been applied to this network.

To accomplish this task, the initial traffic has been arranged using two different congestion distributions, with a combined weight ratio of 0.8–0.2. One of the distributions had a mean of 20% and a variance of 20%, while the other had a mean of 90% and a variance of 10%.

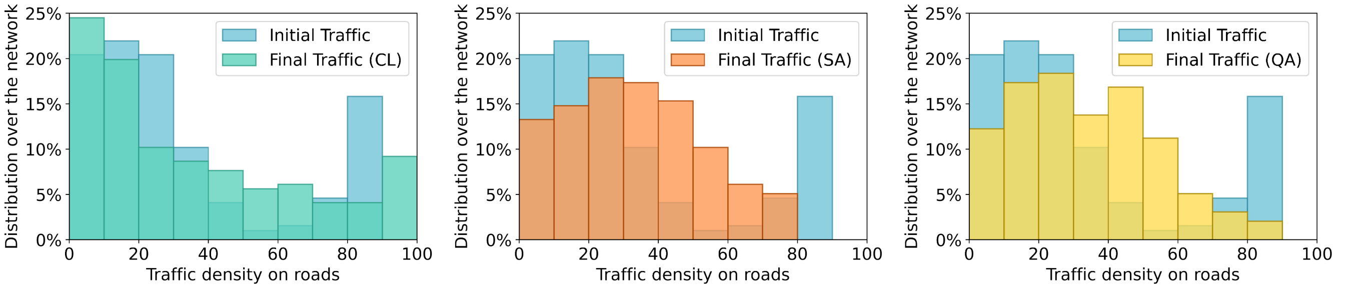
To optimize the QUBO model, the parameter P has been initialized at 50. If the solution failed to adhere to the constraints, its value would have been increased by 5, submitting again the problem to the solver. Moreover, the value of α has been set to 0.2. The MF for vehicles at intersections has been specified at 0.7. With regard to the quantum annealing machine, the following values have been established: 100 *number of reads*, 30 μs of *annealing time*, and 40 units of *chain strength*.

The optimization procedure has been repeated 100 times, simulating the evolution of vehicular traffic for each iteration. Therefore, traffic has evolved for 100 subsequent control patterns of the available traffic lights. The process has been executed three times, once for each optimization method: the classical reference, the QUBO solved throughSA on a classical computer, and the QUBO solved through quantum annealing on D-wave machines. The data obtained from the heuristic technique were used as a reference to evaluate the consistency of the quantum solver's solutions in addressing the optimization model. The results from the conducted tests have been collected in Fig. 7.

Based on the analysis of the application of the different optimization strategies in critical traffic conditions, whose effects are shown in Fig. 7(a), it has been observed that the adoption of QAs produces a more significant reduction in traffic as compared with the classical reference algorithm. This observation is supported by Hamiltonian's evolution, as depicted in Fig. 7(b), where the energy associated with the solutions generated through the proposed optimization model is consistently lower than that achieved by the classical model. The proposed technique yields clear benefits, as the average energy of the solutions is approximately 40% higher when the classical reference method is employed. Similar results have also been obtained by applying the SA algorithm; therefore, they represent an opportunity to effectively validate the methodology proposed to configure the reference quantum machine.

An additional observation can be made by directly comparing the outcomes, as obtained in Fig. 7(a), and those derived from optimizing a larger network, as depicted in Figs. 4(b) and 5(b). Regardless of the solver employed to resolve the QUBO model, it is evident that the benefits associated with the proposed optimization technique are contingent on the network size. Specifically, the advantages over the classical reference control strategy have been attenuated when dealing with a reduced-size road network. Hence, implementing the proposed model yields increasingly significant benefits as control and optimization opportunities are extended to more extensive road networks.

In order to better understand how traffic changes with different optimization strategies, the initial and final traffic distribution in the road network using various solvers have been compared. The resulting graphs are shown in Fig. 8. It has been discovered that the classical solver tended to disperse traffic throughout the road network, resulting in heavily congested roads and sparsely traveled ones. This approach has been demonstrated to be suboptimal as it failed to manage and restrict traffic concentration along crucial



(a) (b) (c)

FIGURE 8. **Comparison of the traffic density distribution at the beginning and the end of each optimization cycle, consisting of 100 subsequent optimization processes. The distribution is intended as the percentage of streets experiencing congestion about their maximum capacity, grouped in specific intervals. (a) Reference classical model (CL). (b) Proposed QUBO model through SA on a classical machine. (c) Proposed QUBO model on a D-wave's QA.**

roads efficiently. Instead, implementing the QUBO model substantially reduced congestion on critical roads and distributed vehicles on lower density streets, effectively preventing the accumulation of vehicles in specific sections of the network. These findings indicate that the deriving dynamic containment strategy proved successful during the traffic progression within the network and that applying the proposed optimization model can effectively optimize traffic conditions.

When comparing the results of quantum and SA, it is important to consider the time required to solve the QUBO problem. While the traffic conditions obtained from both methods are comparable, the data collected reveal that quantum annealing offers a significant speed advantage over SA. On average, SA took 153.31 ms, while quantum annealing took only 36.53 ms. It should be noted that the results of SA are heavily reliant on the machine being utilized, whereas quantum annealing is not. The latter primarily depends on the experiments' duration (30 μs of annealing time multiplied by a factor of 100 read cycles) and the time needed to physically implement the QUBO problem into the machine (e.g., programming time, readout time, and postprocessing operations). However, it is also important to consider the impact of the time required to access D-wave machines remotely. This time is not fixed and varies based on the speed of the remote connection and the machine's busy state, as it is shared among several users through queues. As a result, while the speed advantage of the annealer is significant, its current benefits are attenuated.

V. CONCLUSION

In conclusion, the proposed QUBO formulation presents an innovative solution for controlling traffic signals in urban environments using quantum computing techniques. The formulation enables the comprehensive control of traffic flows within large portions of an urban road network, reducing intense local traffic conditions and avoiding critical situations. The benchmark analyses and comparative results with a reference classical algorithm demonstrate the advantages of the proposed model.

The tests conducted involved synthetic data of a dimensionality compatible with the presently available D-wave QAs. Applying heuristic techniques to solve the same problem has demonstrated similar effectiveness, prompting minimal observations. It is worth noting that the dimension of the QUBO problem significantly impacts the time SA that requires to provide a solution. However, the same cannot be said of QAs, whose limitations are related to the number of available qubits. In addition, nonidealities also affect the performance of quantum machines, further limiting their results. The expected advancement of quantum machines and an increase in the dimension of the problem's complexity is expected to confer a net advantage to quantum annealing over SA.

Further tests on real-world cities will investigate the benefits of the proposed model, providing a direct comparison with the implemented controlled strategies for the traffic lights present. The discussions will also be enriched with additional figures of merit, descriptive of the specific application context.

Overall, this article contributes to the growing knowledge of implementing quantum computing techniques for complex optimization problems and presents a promising approach to managing traffic flow in modern cities.

REFERENCES

[1] B. N. Silva, M. Khan, and K. Han, "Towards sustainable smart cities: A review of trends, architectures, components, and open challenges in smart cities," *Sustain. Cities Soc.*, vol. 38, pp. 697–713, 2018, doi: 10.1016/j.scs.2018.01.053.

[2] Y. Mehmood, F. Ahmad, I. Yaqoob, A. Adnane, M. Imran, and S. Guizani, "Internet-of-things-based smart cities: Recent advances and challenges," *IEEE Commun. Mag.*, vol. 55, no. 9, pp. 16–24, Sep. 2017, doi: 10.1109/MCOM.2017.1600514.

[3] L. U. Khan, I. Yaqoob, N. H. Tran, S. M. A. Kazmi, T. N. Dang, and C. S. Hong, "Edge-computing-enabled smart cities: A comprehensive survey," *IEEE Internet Things J.*, vol. 7, no. 10, pp. 10200–10232, Oct. 2020, doi: 10.1109/JIOT.2020.2987070.

[4] R. Petrolo, V. Loscrì, and N. Mitton, "Towards a smart city based on Cloud of Things, a survey on the smart city vision and paradigms," *Trans. Emerg. Telecommun. Technol.*, vol. 28, no. 1, 2017, Art. no. e2931, doi: 10.1002/ett.2931.

[5] P. R. Stopher, "Reducing road congestion: A reality check," *Transp. Policy*, vol. 11, no. 2, pp. 117–131, 2004, doi: 10.1016/j.tranpol.2003.09.002.

[6] A. Hamilton, B. Waterson, T. Cherrett, A. Robinson, and I. Snell, "The evolution of urban traffic control: Changing policy and technology," *Transp. Plan. Technol.*, vol. 36, no. 1, pp. 24–43, 2013, doi: 10.1080/03081060.2012.745318.

[7] F. Montazeri, F. Errico, and L. Pellecuer, "Comparison of the performance of hybrid traffic signal patterns and conventional alternatives when accounting for both pedestrians and vehicles," *Sustainability*, vol. 14, no. 20, 2022, Art. no. 13667, doi: 10.3390/su142013667.

[8] C. Yu, Y. Feng, H. X. Liu, W. Ma, and X. Yang, "Integrated optimization of traffic signals and vehicle trajectories at isolated urban intersections," *Transp. Res. B, Methodol.*, vol. 112, pp. 89–112, 2018, doi: 10.1016/j.trb.2018.04.007.

[9] Y. Feng, K. L. Head, S. Khoshmagham, and M. Zamanipour, "A real-time adaptive signal control in a connected vehicle environment," *Transp. Res. C, Emerg. Technol.*, vol. 55, pp. 460–473, 2015, doi: 10.1016/j.trc.2015.01.007.

[10] Z. Li, L. Elefteriadou, and S. Ranka, "Signal control optimization for automated vehicles at isolated signalized intersections," *Transp. Res. C, Emerg. Technol.*, vol. 49, pp. 1–18, 2014, doi: 10.1016/j.trc.2014.10.001.

[11] D. Inoue, A. Okada, T. Matsumori, K. Aihara, and H. Yoshida, "Traffic signal optimization on a square lattice with quantum annealing," *Sci. Rep.*, vol. 11, no. 1, Feb. 2021, Art. no. 3303, doi: 10.1038/s41598-021-82740-0.

[12] H. Hussain, M. B. Javaid, F. S. Khan, A. Dalal, and A. Khalique, "Optimal control of traffic signals using quantum annealing," *Quantum Inf. Process.*, vol. 19, no. 9, Aug. 2020, Art. no. 312, doi: 10.1007/s11128-020-02815-1.

[13] E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, and D. Preda, "A quantum adiabatic evolution algorithm applied to random instances of an NP-complete problem," *Science*, vol. 292, no. 5516, pp. 472–475, 2001, doi: 10.1126/science.1057726.

[14] T. Kadowaki and H. Nishimori, "Quantum annealing in the transverse ising model," *Phys. Rev. E*, vol. 58, pp. 5355–5363, Nov. 1998, doi: 10.1103/PhysRevE.58.5355.

[15] G. E. Santoro and E. Tosatti, "Optimization using quantum mechanics: Quantum annealing through adiabatic evolution," *J. Phys. A, Math. Gen.*, vol. 39, no. 36, Aug. 2006, Art. no. R393, doi: 10.1088/0305-4470/39/36/R01.

[16] S. Jiang, K. A. Britt, A. J. McCaskey, T. S. Humble, and S. Kais, "Quantum annealing for prime factorization," *Sci. Rep.*, vol. 8, no. 1, Dec. 2018, Art. no. 17667, doi: 10.1038/s41598-018-36058-z.

[17] H. Ushijima-Mwesigwa, C. F. A. Negre, and S. M. Mniszewski, "Graph partitioning using quantum annealing on the D-wave system," in *Proc. 2nd Int. Workshop Post Moores Era Supercomput.*, 2017, pp. 22–29, doi: 10.1145/3149526.3149531.

[18] F. Neukart, G. Compostella, C. Seidel, D. v. Dollen, S. Yarkoni, and B. Parney, "Traffic flow optimization using a quantum annealer," *Front. ICT*, vol. 4, 2017, doi: 10.3389/fict.2017.00029.

[19] K. Ikeda, Y. Nakamura, and T. S. Humble, "Application of quantum annealing to nurse scheduling problem," *Sci. Rep.*, vol. 9, no. 1, Sep. 2019, Art. no. 12837, doi: 10.1038/s41598-019-49172-3.

[20] A. Mott, J. Job, J.-R. Vlimant, D. Lidar, and M. Spiropulu, "Solving a Higgs optimization problem with quantum annealing for machine learning," *Nature*, vol. 550, no. 7676, pp. 375–379, Oct. 2017, doi: 10.1038/nature24047.

[21] S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, "Optimization by simulated annealing," *Science*, vol. 220, no. 4598, pp. 671–680, 1983, doi: 10.1126/science.220.4598.671.

[22] A. Lucas, "Ising formulations of many NP problems," *Front. Phys.*, vol. 2, 2014, doi: 10.3389/fphy.2014.00005.

[23] F. Glover, G. Kochenberger, and Y. Du, "Quantum bridge analytics I: A tutorial on formulating and using QUBO models," *4OR*, vol. 17, no. 4, pp. 335–371, Dec. 2019, doi: 10.1007/s10288-019-00424-y.

[24] M. J. Lighthill and G. B. Whitham, "On kinematic waves II: A theory of traffic flow on long crowded roads," *Proc. Roy. Soc. London, A, Math. Phys. Sci.*, vol. 229, no. 1178, pp. 317–345, 1955, doi: 10.1098/rspa.1955.0089.

[25] K. Boothby, P. Bunyk, J. Raymond, and A. Roy, "Next-generation topology of D-wave quantum processors," 2020, *arXiv:2003.00133*, doi: 10.48550/arXiv.2003.00133.

Andrea Marchesin (Graduate Student Member, IEEE) received the B.Sc. and M.Sc. degrees in electronic engineering in 2018 and 2020, respectively, from Politecnico di Torino, Turin, Italy, where he is currently working toward the Ph.D. degree in electrical, electronics and communications engineering.

He is involved in research projects on both quantum computing algorithms and platforms, as well as advanced logic-in-memory architectures. His research interests include the quantum world, digital design, and computer-aided design tools development for the exploration of innovative electronic systems.

Bartolomeo Montrucchio (Senior Member, IEEE) received the M.S. degree in electronic engineering and the Ph.D. degree in computer engineering from Politecnico di Torino, Turin, Italy, in 1998 and 2002, respectively.

He is currently a Full Professor of computer engineering with the Dipartimento di Automatica e Informatica, Politecnico di Torino. His current research interests include image analysis and synthesis techniques, scientific visualization, sensor networks, RFIDs, and quantum computing.

Mariagrazia Graziano received the Dr. Eng. and Ph.D. degrees in electronics engineering from Politecnico di Torino, Turin, Italy, in 1997 and 2001, respectively.

Since 2002, she has been an Assistant Professor with Politecnico di Torino. Since 2008, she has been an adjunct faculty with the University of Illinois at Chicago, Chicago, IL, USA, and since 2014, she has been a Marie-Curie Fellow with the London Centre for Nanoelectronics. She works on beyond CMOS devices, circuits, and architectures for traditional and quantum processing systems.

Andrea Boella received the M.Sc. degree in electronic engineering from Politecnico di Torino, Turin, Italy, in 1995.

He is a Senior System Engineer with TIM S.p.A at the Department of Technology Innovation in Turin. Till 2018, he was a Researcher with Service Innovation Group, focusing on services for IOT, blockchain, quantum computing, and quantum communication.

Giovanni Mondo received the Ph.D. degree in robotics from the University of Genova, Genova, Italy, in 2002.

He is a Senior Research Engineer with TIM S.p.A at the Department of Technology Innovation in Turin. His activities include server administration, information visualization, and the development of quantum-compliant optimization models.