

Cost and EAL based optimization for seismic reinforcement of RC structures

*Original*

Cost and EAL based optimization for seismic reinforcement of RC structures / Di Trapani, F.; Sberna, A. P.; Marano, G. C.. - In: *PROCEDIA STRUCTURAL INTEGRITY*. - ISSN 2452-3216. - ELETTRONICO. - 33:(2021), pp. 917-924. (Intervento presentato al convegno 26th International Conference on Fracture and Structural Integrity, IGF26 2021 tenutosi a ita nel 2021) [[10.1016/j.prostr.2021.10.102](https://doi.org/10.1016/j.prostr.2021.10.102)].

*Availability:*

This version is available at: [11583/2970703](https://doi.org/10.1016/j.prostr.2021.10.102) since: 2022-08-21T09:51:21Z

*Publisher:*

Elsevier

*Published*

DOI:[10.1016/j.prostr.2021.10.102](https://doi.org/10.1016/j.prostr.2021.10.102)

*Terms of use:*

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

*Publisher copyright*

(Article begins on next page)



IGF26 - 26th International Conference on Fracture and Structural Integrity

## Cost and EAL based optimization for seismic reinforcement of RC structures

Fabio Di Trapani<sup>a,\*</sup>, Antonio Pio Sberna<sup>a</sup>, Giuseppe Carlo Marano<sup>a</sup>

<sup>a</sup>*Dipartimento di Ingegneria Strutturale, Edile e Geotecnica, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Turin, Italy*

---

### Abstract

In this paper, a new genetic algorithm-based framework aimed at efficiently design multiple seismic retrofitting interventions is proposed. The algorithm focuses on the minimization of retrofitting intervention costs of reinforced concrete (RC) frame structures. The feasibility of each tentative solution is assessed by considering in an indirect way the expected annual loss (EAL), this evaluation is performed by referring to different limit states whose repairing costs are expressed as a percentage of reconstruction costs and evaluating the respective mean annual frequency of exceedance. As the EAL takes into account the overall structural performances, to involves both serviceability and ultimate limit states, two different seismic retrofitting techniques are considered. In particular, FRP wrapping of columns is employed to increase the ductility of RC elements managing life safety and collapse limit state demands. On the other hand, steel bracings are used to increase the global stiffness of the structure and mainly increase operational and damage limit states performances. The optimization procedure is carried out by the novel genetic algorithm-based framework developed in Matlab<sup>®</sup> that is connected to a 3D RC frame fiber-section model implemented in OpenSees. For both the retrofitting systems, the algorithm provides their position within the structure (topological optimization) and their sizing. Results will show that seismic retrofitting can be effectively designed to increase the overall structural safety by efficaciously optimizing the intervention costs.

© 2021 The Authors. Published by ELSEVIER B.V.

This is an open access article under the CC BY-NC-ND license (<https://creativecommons.org/licenses/by-nc-nd/4.0>)

Peer-review Statement: Peer-review under responsibility of the scientific committee of the IGF ExCo

*Keywords:* seismic retrofitting; structural optimization; genetic algorithm; expected annual loss

---

---

\* Corresponding author. Tel.: +39-011-090-5323; fax: +39-011-090-5323.

*E-mail address:* [fabio.ditrapani@polito.it](mailto:fabio.ditrapani@polito.it)

2452-3216 © 2021 The Authors. Published by ELSEVIER B.V.

This is an open access article under the CC BY-NC-ND license (<https://creativecommons.org/licenses/by-nc-nd/4.0>)

Peer-review Statement: Peer-review under responsibility of the scientific committee of the IGF ExCo

## 1. Introduction

The high seismic vulnerability of existing structures located in earthquake-prone regions has led the last decades of research activities to focus on seismic retrofitting techniques. Even though the large availability of retrofitting solutions, currently, their design is exclusively entrusted to the designers' intuition. Nowadays, there are no formal methods for assisting practitioners in designing this kind of intervention controlling the resulting seismic performance. This may lead to an over-estimation of retrofitting design with a consequent increase of intervention costs, invasiveness, and downtime.

In the last years, the scientific interest in structural optimization was mainly focused on new structures design. In the scientific literature, there are numerous studies concerning the optimization of bridges or structures focused on exploitation of structural performance through topological or shaping optimization techniques. However, the optimization of seismic retrofitting for existing structures has not been examined over the past years. Only recently, few researchers have addressed the problem of the optimization of FRP jackets (Chisari and Bedon (2016), Seo et al. (2018)) or other applications of seismic retrofitting techniques for RC buildings employing fluid viscous dampers (Pollini et al. (2017)), dissipative bracings (Braga et al. (2019)) or both (Lavan and Dargush (2009)). More recent studies have tackled the optimization of seismic retrofitting costs. Among these, Falcone et al. (2019) proposed a framework for optimizing the realization costs of FRP jacketing and steel bracings for existing RC frame structures through genetic algorithm optimization. Papavasileiou et al. (2020) faced retrofitting optimization of encased steel-concrete composite columns comparing concrete jacketing, steel jacketing, and steel bracing. A similar approach was followed by Di Trapani et al. (2020) who proposed an innovative framework based on a genetic algorithm aimed at minimizing the intervention cost for ductility deficient RC structures accomplished through steel-jacketing of columns. This last approach was further improved for the design of seismic retrofitting in both shear-critical and ductility-critical frame structures, proving that, with appropriate tweaking, genetic algorithms are effective tools for the optimization of the seismic retrofitting costs (Di Trapani et al. (2021)).

The main objective of this paper is the development of a new optimization framework aimed at minimizing service-life costs of seismic retrofitting interventions for existing RC frame structures. According to Calvi (2013), the expected annual loss (EAL) has been proved as a valid parameter for the comparison of structural seismic performance during service life. It estimates the overall behaviour of the construction in terms of expected economic annual losses associated with seismic events that could take place during the reference service life.

The aim of the proposed algorithm is to determine, for RC buildings designed without seismic detailing, the best retrofitting configuration in terms of position (topological optimization) and amount of reinforcement (sizing optimization). The framework focuses on the minimization of retrofitting realization costs indirectly taking into account the resulting EAL value. Since EAL evaluation involves different limit states assessment, the proposed algorithm has to consider multiple retrofitting techniques. In particular, for the case study of a multistorey frame RC structure, two distinct retrofitting interventions to optimize are considered; FRP jacketing of RC columns to mainly increase ductility, and steel bracings to reduce lateral deformability incrementing the stiffness of the building.

The optimization process is performed by a genetic algorithm (GA) developed in MatLab® which is connected to a fiber-section model implemented in OpenSees. The structural performance of each solution is assessed from the results of static pushover analyses in the framework of the N2 method. The validity and efficiency of the proposed method are proved by implementing its application on a case study structure.

## 2. Optimization framework

The optimization algorithm herein proposed is based on a genetic algorithm (GA) developed in MatLab®. The optimization framework relates a structural model developed in the OpenSees software platform (McKenna et al. (2000)) with the GA routine. A schematic flowchart of the proposed framework is depicted in Figure 1. The genetic algorithm is an evolutionary algorithm inspired by the evolution theory; it generates a population of individuals representing different tentative retrofitting arrangements.

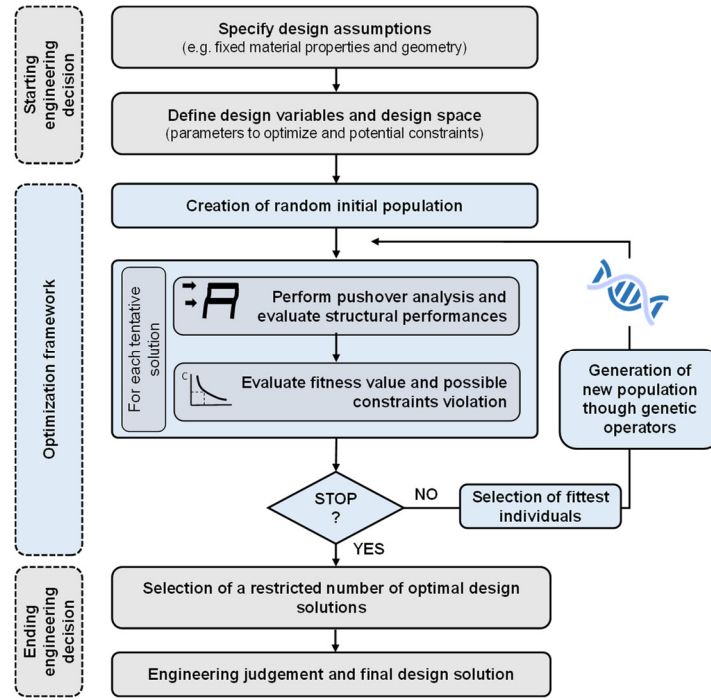


Fig. 1. Schematic flowchart of the proposed optimization framework.

Each individual handled by the algorithm is characterized by a design vector collecting all the design variables that characterize the tentative solution. The fitness of each tentative solution is evaluated by a proper objective function considering the cost associated with retrofitting intervention. The algorithm proceeds in the search for the optimal solution through the use of genetic operators (mutation and crossover) that create new individuals starting from the fittest ones.

### 2.1. Design vector definition

The framework aims at optimizing the intervention cost of two different retrofitting systems: FRP wrappings of columns and concentric steel bracings. The decision variables that encode the position and sizing of both retrofits are collected together into a so-called design vector. The selected design variables are the number of braced frame fields ( $n_{br}$ ), the diameter of braces ( $\phi_{br}$ ), the number of layers of FRP ( $n_{FRP}$ ), and the position of the columns retrofitted by the FRP. Design variables are gathered in the design vector  $\mathbf{b}$  so defined:

$$\mathbf{b} = (n_{br} \quad \phi_{br} \quad n_{FRP} \quad \mathbf{p})^T \quad (1)$$

in which the term  $\mathbf{p}$  is an array of binary numbers representing the position of the FRP retrofitted columns as:

$$\mathbf{p} = [\dots \quad \dots \quad c_{ij} \quad \dots]^T \quad (2)$$

where the general element  $c_{ij}$ , is a binary assuming the value 1 if the column is retrofitted and 0 if not. The subscript  $i$  indicates the position of a column in plan and  $j$  the storey.

The term  $n_{FRP}$ , instead, is a natural variable that represents the number of overlapping layers of FRP fabrics on each column. A heuristic repair technique is involved to modify the design vector to introduce FRP wrapping on the columns adjoining the bracing systems. This is performed to prevent the premature collapse of columns caused by the additional shear demand induced by the bracings.

## 2.2. Definition of the objective function

The objective function is aimed at the evaluating the retrofitting intervention costs considering the realization of the two retrofitting systems as:

$$F = C_{br} + C_{FRP} \quad (3)$$

where  $C_{br}$  is the cost related to the arrangement of bracings and  $C_{FRP}$  is the one for the realization of the FRP wrapping of the columns. Both terms consider the material and manpower costs and the necessary works for the demolition and restoration of adjoining plaster and masonry. The first one can be evaluated as:

$$C_{br} = \sum_{i=1}^{n_{br}} (W_{br,i} \cdot c_{br}) + n_{br} \cdot c_{br,m} \quad (4)$$

where  $c_{br}$  is the manpower and material cost per unit weight (estimated in  $c_{br} = 6 \text{ €/kg}$ ),  $c_{br,m}$  is the fixed cost related to the demolition and reconstruction of masonry (2000€ every braced frame fields), and  $W_{br,i}$  is the weight of the bracings in the  $i$ -th frame field. As regards FRP retrofitting the cost  $C_{FRP}$  is computed as follows:

$$C_{FRP} = \sum_{i=1}^{n_c} (A_{FRP,i} \cdot c_{FRP}) + n_c \cdot c_{FRP,m} \quad (5)$$

where  $n_c$  is the number of retrofitted columns taking into account also the local reinforcement of the columns adjoining the steel bracings systems as presented in the previous section,  $c_{FRP}$  is the unit cost of the FRP (estimated in  $c_{FRP} = 300 \text{ €/m}^2$ ),  $c_{FRP,m}$  is the cost per column for the demolition and reconstruction of adjacent masonries and plasters (equal to  $c_{FRP,m} = 1000 \text{ €}$ ) and  $A_{FRP}$  is the area of the FRP fabric used to retrofit the generic  $i$ -th column.

## 2.3. Optimization constraints definition

The EAL value of each tentative retrofitting arrangement is considered an indirect way as constraints of the optimization procedure. The EAL represents the percentage annual loss of economic value of a structure in its reference life considering the associated seismic risk. The assessment of the EAL value is achieved by the simplified method proposed by Cosenza et al. (2018), according to which, repair costs are expressed as percentages of the repair costs (%RC) concerning the reconstruction cost. The EAL is evaluated as the area under the curve that connect the points  $(\lambda, \%RC)$  for each limit state. For sake of simplicity, the annual rates of failure for the operational and collapse limit states can be obtained as a function of those evaluated for DL and LS limit states, thus EAL is known once obtained  $\lambda_{D,LLS}$  and  $\lambda_{L,LS}$ . For this reason, the feasibility of each solution is restrained by their simultaneous verification which implies that the EAL of retrofitted structures is lesser than the code-compliant building, namely a structure having for each limit state a capacity that is exactly equal to the demand.

$$\lambda_{D,LLS} \leq \lambda_{cb,D,LLS} \quad \& \quad \lambda_{L,LS} \leq \lambda_{cb,L,LS} \quad \Rightarrow \quad EAL \leq EAL_{ccb} \quad (6)$$

Non-penalty approach is developed to consider the feasibility (or not) of each tentative solution. It is exerted by the survival selection that accomplishes a double sorting process, first ordering the individuals with respect to the number of violated constraints and then, among the individuals with the same number of violations, for the fitness value. Selection is accomplished by choosing fittest individual that create the mating pool for the next generation.

### 3. Case study of the proposed framework

The proposed framework can be interfaced with any FE software handling non-linear static analysis. For the current application, the *OpenSees* software platform has been used. Frame elements are modelled adopting distributed plasticity force-based elements with five Gauss-Lobatto integration points present in OpenSees.

Concrete elements are modelled using a *Concrete01* uniaxial material model. In order to simulate the crushing of the cross-section fibers, *Concrete01* material is combined with *MinMax* material, which removes the contribution of a fiber when a specified strain threshold is achieved. Steel rebars are modelled using the *Steel02* Giuffrè-Menegotto-Pinto material model (elasto-plastic with linear strain hardening).

The confined concrete model adopted for RC elements with and without retrofitting is the standard confined parabola-rectangle model, evaluated according to the Italian Technical Code (2018) and Eurocode 8 (2005). The FRP reinforcement is supposed to be applied in a continuous arrangement at both ends of the columns where major ductility is required. The effect of FRP retrofitting is introduced by modifying the constitutive model of concrete fibers. Moreover, it is assumed that the effect of confinement is extended to the entire cross-section.

Steel bracings are modelled using truss elements available in OpenSees. The steel is modelled adopting *Steel02* elastic-plastic with isotropic strain hardening (Giuffrè-Menegotto-Pinto material model). Steel elements are assumed to have a circular cross-section whose diameter is defined by the decision variable  $\phi_{br}$ .

#### 3.1. Details of the reference structural and performance of the as-built structure

The effectiveness of the proposed framework is tested by performing the retrofitting optimization for an RC structure having a structural configuration typical of buildings designed before the entry into force of seismic guidelines. In detail, the building consists of a five-storey reinforced concrete frames structure presenting uni-directional frames (Fig. 2). Reinforcement details of beams and columns are reported in the following Table 1. Dimensions in plan of the structure, together with the sizes of RC elements are represented in Fig. 2b.

Table 1. Geometrical dimensions and reinforcement details of RC elements.

RC members	$b$ (mm)	$h$ (mm)	Longitudinal reinforcement	Transverse reinforcement
Beams	800	300	4+4 $\phi 18$	$\phi 6 / 200$ mm
Columns	450	450	12 $\phi 18$	$\phi 6 / 200$ mm

Reinforced concrete elements are assumed to be made of poor resistance concrete having average unconfined strength  $f_{co} = 20$  MPa and steel rebars with nominal average yielding strength  $f_y = 455$  MPa and strain hardening ratio that is assumed equal to  $\eta = 0.01$ . As regards seismic hazards, the building is supposed to be located in Cosenza (Italy), soil type C, the nominal life ( $V_N$ ) is 100 years. The structure has double symmetry in-plan, and it is regular in elevation. Vertical loads are modelled as point loads applied to top nodes of columns as a function of the respective tributary areas in-plan. Rigid diaphragm behaviour is imposed on every floor.

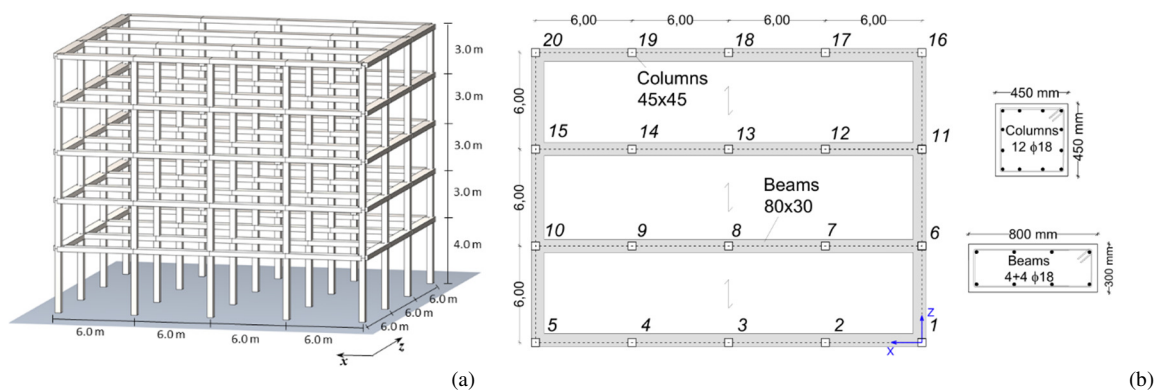


Fig. 2. Geometrical dimension of the reference structural model: (a) 3D frame view; (b) in plane dimensions.

The maximum inter-story drift ratio, at which the DLLS condition is achieved, is set to  $\delta_{i,max} = 0.005$ . Interstory drifts are monitored at each step of pushover analysis so that the damage limitation limit state is associated with top displacement of the structure which corresponds to the first exceeding of the limit value  $\delta_{i,max}$ . A preliminary assessment of the as-built structure has been carried out to test its performance with the reference earthquake loads. For the sake of simplicity, pushover analysis is performed by considering only a uniform profile for lateral loads acting along the z-direction of the structure, which is supposed to be the most vulnerable to seismic actions. Results are shown in the Table 2, showing that the as-built configuration safety factors related to DLLS and LSLs are smaller than unity ( $\zeta_{E,DLLS} = 0.9$  and  $\zeta_{E,LSLS} = 0.8$ ). This leads to an EAL value that is equal to 1.381, a value greater than the one associated to the code-compliant building ( $EAL_{ccb} = 1.13\%$ ). The structure shows both reduced ductility and vulnerability on damage limit states, therefore seismic retrofitting interventions are needed. \

The retrofitting system is composed of FRP wrapping of columns and concentric steel bracings. The FRP sheets have a thickness of  $t_{f,l} = 0.337$  mm per layer, elastic modulus  $E_f = 230$  GPa, ultimate stress referred to net area of the fibers  $f_{fib,k} = 3250$  MPa and ultimate strain  $\epsilon_{fib} = 1,3\%$ . For the implementation of FRP wrapping, it is assumed that a rounding of the column edges with a radius equal to  $r_c = 25$  mm is carried out. The bracings are supposed to be made of S275 structural steel with  $f_{yb} = 275$  MPa, elastic modulus  $E_{sb} = 210$  GPa, and strain hardening ratio  $\eta = 0.01$ . Since the structure has a double symmetry in-plane, the bracings are defined symmetrically on the two external transversal frames. In this way the  $n_{br}$  is the number of floors where the bracing systems are defined, starting from the ground floor.

Table 2. GA analysis parameters set up for the case study.

$\zeta_{E,DLLS}$	$\zeta_{E,LSLS}$	$\lambda_{DLLS}$	$\lambda_{LSLS}$	EAL [%RC]
0.906	0.812	0.0263	0.0027	1.381

To decrease the design space dimension reducing the computational effort, the analysis has been constrained to a limited number of columns for the confinement systems and a restricted number of frames for the bracings. The following hypotheses are being placed:

- i) The optimization space for retrofitted columns by FRP jacketing is limited to the first two floors.
- ii) The maximum number of FRP layers is 4.
- iii) The design space for the bracings is restricted to the central transversal frames.
- iv) Bracing diameter optimization range is 20-100 mm, and it varies with a minimum step size  $\Delta\phi_{br}$  of 10 mm.

### 3.2. Optimization results

The analysis was carried out starting from a first-generation containing 100 tentative solutions randomly generated. The algorithm proceeds by creating 100 new children every generation choosing the parents through a tournament selection on three randomly picked parents. In the following Table 3, a summary of the GA framework parameters is reported.

Table 3. GA analysis parameters set up for the case study.

Generation dimension	Number of offspring	Tournament size $k$	Max generations	Max stall
100	100	3	20	5

Results of the optimization are shown in Figure 3 in terms of pushover and EAL curves. The optimal solution is characterized by only steel bracing retrofitting on the external frames for the first two floors while no FRP interventions are required. The bracings of the optimal configuration have a diameter ( $\phi_{br} = 50$  mm) which is equivalent to a cross-section area of  $A_{br} = 19,6$  cm<sup>2</sup>.

Among the two intervention systems considered for this application, the bracings are those designed to increase the lateral deformation stiffness, so they are the only ones that allow increasing the DLLS safety factor. This, in addition to the elevated cost of the FRPs, has led the algorithm to prefer a reinforcement configuration that considers only the first retrofitting technique.

The overall cost of this intervention configuration is 31299€. The increase in stiffness due to the retrofitting

system leads to a reduced displacement demand, which combined with the ductility provided by the steel bracing and FRP on the adjoining columns allows the structure to satisfy both LS and DL limit states. As reported in Table 4, the safety factor related to damage limit state is barely close to the unity ( $\zeta_{E,DLLS} = 1.025$ ) whereas the safety factor related to LSLS is  $\zeta_{E,LSLS} = 1.585$ . This condition can also be easily observed from Fig. 4a where the capacity curve is depicted together with the bilinear equivalent curve in the acceleration-displacement response spectrum plane.

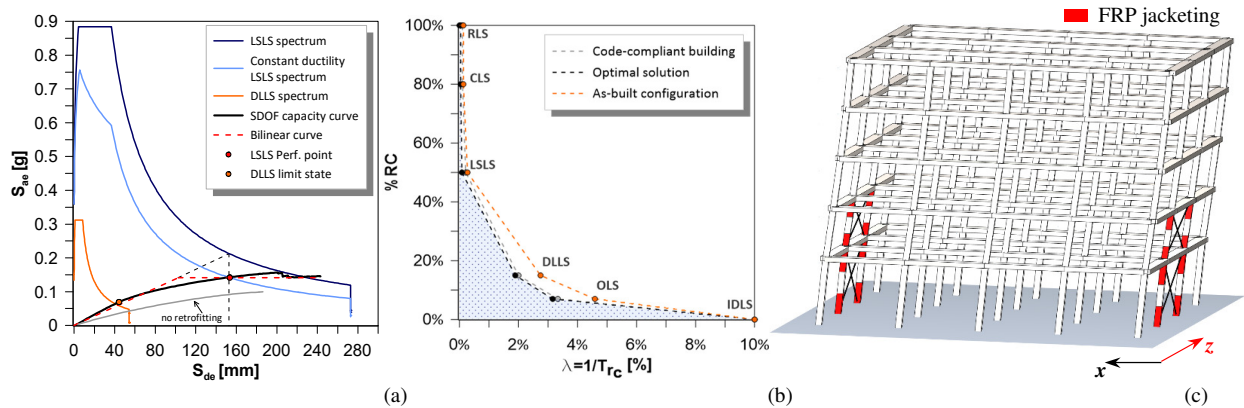


Fig. 3. Optimal solution: (a) pushover curve in ADRS plane; (b) EAL curve; (c) Retrofitting configuration (deformed shape).

The EAL curve displayed in Fig. 3b shows a noteworthy reduction with respect to as-built configuration, resulting in  $EAL = 1.01\%$ . The proposed framework has significantly improved the quality of the retrofitting design by providing a cost-optimized intervention with a control on the EAL.

Table 4. Optimization analysis results

$n_{FRP}$	$n_{br}$	$\Phi_{br}$ (mm)	$\zeta_{E,DLLS}$	$\zeta_{E,LSLS}$	$EAL$ (%RC)	$Fitness$ (€)
1	2	50	1.025	1.585	1.009	31 229

#### 4. Conclusions

The paper has presented a novel optimization framework aimed at minimizing the seismic retrofitting intervention costs on RC frame structures. The framework is based on a genetic algorithm developed in *MatLab*<sup>®</sup> which is connected with a 3D fiber-section model realized in *OpenSees*. The performance of each tentative solution is evaluated starting from the results of non-linear static analysis in the framework of the N2 method. Two different typologies of retrofitting systems are considered: FRP jacketing of columns and steel bracings.

The main target of the algorithm is to seek the retrofitting configuration that optimizes the intervention costs considering in an indirect way the expected annual loss value referring to that requested by the reference technical code. In the end, through the implementation of a case study structure, the effectiveness of the proposed algorithm has been proved.

Vast usage of this proposed framework will improve the sustainability of the seismic retrofitting interventions reducing the invasiveness and, by better management of the funds allocated for the retrofitting of existing structures, the overall structural safety of building heritage.

#### References

- Basone, F., Cavaleri, L., Di Trapani, F., Muscolino, G., 2017. Incremental dynamic based fragility assessment of reinforced concrete structures: Stationary vs. non-stationary artificial ground motions. *Soil Dyn. Earthq. Eng.* 103, 105 – 117.
- Braga, F., Gigliotti, R., Laguardia, R., 2019. Intervention cost optimization of bracing systems with multiperformance criteria. *Engineering Structures* 182, 185-197.



- Calvi, G.M., 2013. Choices and criteria for seismic strengthening. *Journal of Earthquake Engineering* 17(6), 769-802.
- Cavaleri, L., Di Trapani, F., Ferrotto, M.F., Davì, L., 2017. Stress-strain models for normal and high strength confined concrete: Test and comparison of literature models reliability in reproducing experimental results. *Ingegneria Sismica* 34(3-4), 114-137.
- Chisari, C., Bedon, C., 2016. Multi-Objective Optimization of FRP Jackets for Improving the Seismic Response of Reinforced Concrete Frames. *American Journal of Engineering and Applied Sciences* 9(3), 669-79.
- Cosenza, E., Del Vecchio, C., Di Ludovico, M., Dolce, M., Moroni, C., Prota, A., Renzi, E., 2018. The Italian guidelines for seismic risk classification of constructions: technical principles and validation. *Bulletin of Earthquake Engineering* 16, 5905–5935.
- Di Trapani, F., Bolis, V., Basone, F., Preti, M., 2020. Seismic reliability and loss assessment of RC frame structures with traditional and innovative masonry infills. *Eng Struct* 208, 110306.
- Di Trapani, F., Ferro, G.A., Malavisi, M., 2020. Definition of inelastic displacement demand spectra for precast industrial facilities with friction and fixed beam-to-column joints. *Soil Dynamics and Earthquake Engineering* 128, 105871.
- Di Trapani, F., Giordano L., Mancini, G., 2020. Progressive Collapse Response of Reinforced Concrete Frame Structures with Masonry Infills. *J. Eng. Mech.* 146(3), 04020002.
- Di Trapani, F., Malavisi, M., Marano, G.C., Sberna, A.P., Greco, R., 2020. Optimal seismic retrofitting of reinforced concrete buildings by steel-jacketing using a genetic algorithm-based framework. *Engineering Structures* 219, 110864.
- Di Trapani, F., Sberna, A.P., Marano G.C., 2021, A new genetic algorithm-based framework for optimized design of steel-jacketing retrofitting in shear-critical and ductility critical RC frame structures. *Engineering Structures* 243, 112684.
- Eurocode 8. Design of structures for earthquake resistance - Part 3: Assessment and retrofitting of buildings, European Committee for Standardization, 2005.
- Fajfar, P., 2000. A Nonlinear Analysis Method for Performance-Based Seismic Design. *Earthquake Spectra* 16(3), 573-92.
- Falcone, R., Carrabs, F., Cerulli, R., Lima, C., Martinelli, E., 2019. Seismic retrofitting of existing rc buildings: a rational selection procedure based on genetic algorithms. *Structures* 22, 310–326.
- Lavan, O., Dargush, G.F., 2009. Multi-objective evolutionary seismic design with passive energy dissipation systems. *Journal of Earthquake Engineering* 13(6), 758-90.
- McKenna, F., Fenves, G.L., Scott, M.H., 2000. Open system for earthquake engineering simulation. University of California Berkley.
- Norme tecniche per le costruzioni. Decreto ministeriale 17 gennaio 2018, Ministero delle infrastrutture e dei trasporti, 2018.
- Papavasileiou, G.S., Charmpis, D.C., Lagaros, N.D., 2020. Optimized seismic retrofit of steel-concrete composite buildings. *Engineering Structures* 213, 110573.
- Pollini, N., Lavan, O., Amir, O., 2017. Minimum-cost optimization of nonlinear fluid viscous dampers and their supporting members for seismic retrofitting. *Earthquake Engineering & Structural Dynamics* 46, 1941–61.
- Seo, H., Kim, J., Kwon, M., 2018. Optimal seismic retrofitted RC column distribution for an existing school building. *Engineering Structures* 168, 399-404.