Stability analysis of systems under Parametric Excitation

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Unwanted vibrations in mechanical structures may gradually lead to failures of the components and consequently the operating system. Such oscillations could be induced by different external or internal excitation sources depending on the working condition. A particular form of excitation is generated due to the variation of physical characteristic parameter(s) of the operating system with time and is known as 'Parametric Excitation'. Parametric excitation is observable in various mechanical structures such as a simple swinging pendulum, rotating machineries, offshore structures, etc. The main property of a parametrically excited system is the occurrence of unstable zones, containing diverging responses, which are separated from the stable zones by means of the so – called 'Transition Curves'. Such instabilities arise due to pumping extra energy to the operating system when the frequency of the parametric excitation approaches the Twice of the natural frequency/ies or the Combination (summation or difference) of the two different natural frequencies. The regions of instability formed around the twice and combination of the natural frequencies are respectively known as 'Primary' and 'Combination' parametric resonances.

In this research work, the state-of-the-art stability analysis approaches applicable to the study of parametrically excited systems i.e. Floquet Theory, Hill's method, Harmonic Balance Method (HBM), Multiple Scales Method, and recently developed method named Jacobian Based Approach (JBA) are explained in detail. The implementation procedure and the applicability of each stability analysis tool have been explained in detail via an example of a Mass – Spring – Damper. In this chapter, the accuracy and efficiency of JBA have been evaluated by comparing the results and the computational time outputted from this method and the state-of-the-art stability analysis approaches.

A detailed study of the HBM for the stability analysis of a parametrically excited system is performed in the third chapter of the thesis. A Jeffcott rotor supported by Rolling Elements Bearing (REB) is adopted as a case study. Here, the REB due to its varying compliance has been modeled as a time – varying stiffness, introducing the parametric excitation in the system. In this study, an improved form of HBM named 'Trained HBM (THBM)' to obtain the stability plot has been proposed. However, it has been proven that THBM is not efficient to implement for bigger systems. In this chapter, a detailed study of the newly proposed method i.e.

Jacobian Based Approach (JBA) is given and its efficiency and applicability have been further investigated.

The study of the JBA's applicability for stability analysis of a more complex system under parametric excitation and an experimental study of the instability triggered by the parametric excitation are performed in chapter 4. For this work, a cantilever beam model mounted on a spring with time – varying stiffness is adopted as a demonstrator. By comparing the stability plots obtained by JBA and Hill's method, the performance of the JBA has been examined. A test rig consisting of a cantilever beam and an electromagnet unit is designed for the experimental activity. In this study, it has been demonstrated, experimentally and numerically, that the electromagnet generates a time –varying stiffness where by tuning the frequency of the electromagnet to the combination parametric resonance frequency a diverging response of the beam is observed. The validity of this study is justified by validating the numerical results by the results from the experiment at the combination parametric resonance frequency.