Argumentation and interaction: Discussing a new theoretical perspective

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Argumentation is an essentially interactive process that is closely related to language. At this regard, a framework underlining these aspects of argumentation processes could be helpful for research. In this theoretical paper we focus on the argumentative processes that are developed in the interactional context of the mathematics classroom. In this context, argumentations depend on the specific characteristics of mathematics and on the specific goals of the educational setting, among which the improvement of understanding is crucial. Related to this goal, we discuss the different functions that argumentations can play and introduce a theoretical perspective, proposed by Mercier and Sperber (2017) that values the interactional and linguistic aspects of argumentation.

Keywords: Argumentation, interaction, language, explanation, reason.

Introduction

In recent years argumentation, as an interdisciplinary competence, has gained a more prominent role in several institutional documents, such as the Recommendation of the Council of the European Union (2018), which influenced curricula and national guidelines of European Countries. Many mathematics education researchers investigate the role of argumentation in mathematics classroom from multiple perspectives (Stylianides et al., 2016). Various aspects of argumentation can be taken into consideration, and several definitions have been proposed. In this paper we focus on argumentative processes that are developed in a mathematical classroom, in a framework of social construction of knowledge (Vygotsky, 1978). From this perspective, teachers and peers are not only the main addressees of a certain argument, but they influence and co-develop the argumentative process (Krummehuer, 1995). The argumentative process (hereinafter *argumentation*) includes not only the initial production of an argument, but also the subsequent interlocutors' interventions, the speaker's responses to the interventions, and the subsequent modifications of the proposed argument, and so on (Johnson, 2000). It is hence highly context dependent. According to Johnson (2000, p. 181) the final product (hereinafter *argument*) can then be regarded as a codification of lines of reasoning that have emerged in the dialectical process and cannot be understood apart from the process that produces it, nor yet from the participants to that process.

In this interactional perspective, language plays a crucial role, as it provides the resources to give shape to the argument. The specific language adopted, and its properties are not neutral but affect the organization of the argument. For example, the occurrence of implicit parts in an argument, which is a fundamental aspect in the evaluation of argument, is a typically linguistic issue that needs to be addressed with linguistic tools. In the opinion of some authors (e.g. Doury, 2016), it is not possible to distinguish the argumentative thinking from the text that expresses it. Doury claims that argumentation is not just accidentally linguistic: it is not above all a logical structure that would have to be stripped of its linguistic trimmings to reach its true essence. Argumentation develops in and through language, it is coextensive with it and its analysis requires the knowledge developed within linguistics. As far as the development of argumentation in the classroom is concerned, we want to highlight the functions it plays in the processes of teaching-learning mathematics, and to stress the role of interactions with the teacher and between peers, which, in our opinion, is not always adequately considered. In this regard, we introduce a theoretical perspective on argumentation that has been proposed by Mercier and Sperber (2017). We will discuss the opportunity of adopting it in mathematics education research, highlighting its pro and cons. First, we try to make explicit some functions that argumentation can play in mathematics classrooms. Second, we introduce the new theoretical perspective, and its integration in the educational context, which we discuss with the help of two examples. Finally, we briefly discuss the introduction of this perspective, pointing out some innovative and critical aspects. Through this theoretical paper, we consider that an argument is a text (i.e. a set of written or spoken clauses) and that language competence plays a relevant role in supporting or hampering argumentations.

Functions of argumentation

Argumentation has been widely studied in a variety of contexts. It is important for us to refer to the context of mathematics and its specific characteristics. Even more important is to consider that we are in an educational context that has specific purposes, different from those of other contexts. In our opinion, it is crucial to understand the role of argumentation in relation to these specific educational purposes. Some of the functions of argumentation can be traced back to the functions of proof. In fact, research on argumentation in mathematics education has taken up themes and ideas from that on proof. See for example Hanna (2018) for concise overviews, or Reid and Knipping (2010) for a more comprehensive one. In particular, the discussions have mainly focused on some specific functions of proof. i.e., proving (in the sense of certifying, verifying), convincing and explaining, although some authors have identified different functions for proofs. For example, it has been suggested that proofs very often highlight relationships between statements and theories, allowing for a better understanding of the meanings involved. Although it is not easy to draw clear-cut lines between argumentations and proofs, it can be assumed that the idea of argumentation is somewhat more general than that of proof, if only because it is studied in various contexts, even outside mathematics. Also, argumentations are often linked to the functions of proving (certifying, verifying), convincing and explaining. The idea of certification seems to recall a stable acquisition and to refer to a norm rather than a dialogical context. For this, in our opinion, this interpretation is not the most suitable for modelling argumentative processes at school and in the mathematical field, as they can achieve increasing levels of clarification but can only rarely arrive at a permanent certification. The convincing function seems to be the one on which the agreement is greatest (Harel & Sowder 1998; Stylianides et al., 2016). The classic study by Harel and Sowder (1998) analyses some arguments, which they consider persuasive for students, and shows their diversity and, in some cases the misalignment with what is accepted in the mathematics classroom. As we understand it, the process of conviction is primarily relational. An argumentation that is convincing for one interlocutor may not be so for another. Similar considerations hold for explanation, which is a relational process too. Stylianides et al. (2016), contrast this vision with the one of someone who links the function of explanation with immutable characteristics of arguments, which do not depend on the interlocutors, such as, for example, the proof that explains in the sense of Hanna (2018). In this work we are particularly interested in educational contexts, where understanding and the collective construction of knowledge are at the centre. From our point of view, the argumentations that provides an explanation to classmates play a major role. Outside the mathematical context, other functions are recognized as related to argumentation. Doury (2016) in addition to persuasion also recognizes other functions, related to cognition, identity and interpersonal relationships.

Argumentation in Mercier and Sperber's perspective

According to Mercier and Sperber (2017), social interactions and relations play a crucial role in the evolution of human beings. In their view, arguments are mainly developed to socially support a certain conclusion. People may use them to explain and justify themselves, to evaluate others' reasoning or behavior, or to convince those who think differently. According to the authors

> When we give reasons for our actions, we not only justify ourselves, we also commit ourselves. In the first place, by invoking reasons, we take personal responsibility for our opinions and actions as described by us, that is, as attitudes and behavior that we had reasons to adopt. We thereby indicate that we expect others to either accept that we are entitled to think what we think and do what we do or be ready to challenge our reasons. (Mercier & Sperber, 2017, p. 126)

In simpler terms, arguments are primarily meant for social consumption. To be socially shared, they have to be communicated. Communication is recognised to play a fundamental role in social interactions and to be extremely beneficial to human beings. However, communication can also be a source of misinformation. To decide whether accepting what a speaker is saying, Sperber et al. (2010) suggest that we enact a controlling process, called "epistemic vigilance". Epistemic vigilance relies on two different mechanisms: the first one is the control on information source; the second is the control of information content. Being a mechanism of control, it is costly. Argumentations could help the (non) acceptance of a certain position, making communication more reliable. On the one hand, argumentations help the audience to better evaluate information they would not accept on trust. On the other hand, they allow the speaker to reach a more cautious audience. According to Mercier and Sperber (2017), a good argumentation should show the coherence relationships between the claim of the speaker and the system of beliefs held by the addressees, and it should allow the audience to evaluate these relationships on their own. Moreover, argumentative processes are intended as being interactive. The addressees have an active role, not only in deciding whether to accept or to refuse a certain conclusion, but actively participating in the argumentative process. In fact, they can indicate whether they have understood, and they can actively guide the effort of the speaker, for example posing questions or requiring further clarifications. According to Mercier and Sperber (2017), it would be unnecessarily burdensome to constantly assume the other side's perspective to anticipate possible objections. Due to the interactive nature of dialogue, justifications and arguments are refined with feedback from the interlocutors and with consideration of any objections that are raised. This is a useful strategy of cognitive labor division.

Integration of the perspective in the educational context

Some additional remarks are however necessary if we want to consider this definition of a good argumentation in mathematics classroom. Firstly, we are interested in the speaker's claims which are related to problems' resolutions or mathematics conjecture. Secondly, this perspective considers the addressee to be *vigilant*, in the sense of epistemic vigilance (Sperber et al., 2010; Mercier & Sperber, 2017). This is not always the case in the classrooms, where students sometimes accept the information given by the teacher, classmates or taken from the textbook without checking it. Nevertheless, we do think this is a fundamental aspect in developing fruitful argumentative activities in classrooms. For this reason, we focus on classroom argumentations involving classmates as the main addressees of arguments. Thirdly, the classroom context, its participants and the teaching goals involved are crucial to influence the different ways to show the "coherence relationships", that, in a mathematical classroom, could involve, for example, different representations or the reference to previous knowledge. Finally, as system of beliefs we intend both students' knowledge and abilities in respect to mathematics, or in respect to other aspects involved, such as linguistics ones, and students' assumptions about essential features of arguments or of problem situations. We try to better clarify by means of two examples. The goal of the analysis is to exemplify how the model works in the two class situations considered. At this regard, we are aware that the translation of students' excerpts from a language to another may substantially change some features of the text they have produced. Anyway, here we are just interested in seeing on which ideas they were focused.

Example 1

The first example is from a didactical sequence aimed to introduce classic probability theory and to prompt argumentations, that was implemented in an upper secondary school classroom (19 students). Students are asked to solve the "division of the stakes problem" (Paola, 2019):

Two players A and B play heads or tails with a fair coin. Each game, corresponding to each coin toss, is won by A if the outcome of the toss is heads and by B if the outcome is tails. A and B give 12 euros each. The stake is 24 euros. The player who first wins 6 rounds wins the game, and thus the entire stake. A always bets on "heads" and B on "tails". The game is interrupted at the score 1- 0 for A. How should the stakes be fairly divided i.e., that it gets both players to agree?

Students solved the problem working in small groups, before any probability theoretical concept was introduced by the teacher. They then shared their solutions to their classmates. The following episode concerns the discussion about two of the resolutions proposed by the students. Group 1's resolution is based on the equality of the theoretical probability of winning the game (i.e. getting to six points) and of winning in one throw (i.e. that in one throw heads / tails comes out). In both cases in fact a player has a 50% chance of winning. If a player wins six rounds, he wins the stakes. Then, by analogy, they chose to give all the stakes to the player who won the first round. Group 2 tried to take into consideration the possible developments of the game and to count how many times a player could have won. However, the students did not know operationally how to count the matches. After the presentation, students reflected on others' resolution in small group, and then they shared their reflection collectively. The transcript starts with Andrea (Group 1) commenting Group 2's resolution. In the transcript, we indicate if students belong to Group 1 or 2, to aid the readers' understanding.

Andrea (G1): You considered all possible cases after the first throw, but in fact the game ends at the first throw. So, only the first throw should be considered. From the situation of zero to zero you have to consider only the first throw.

Adele (G2): Yes, we considered all, because we wanted to find the way to fairly divide the stakes, considering the number of games.

Giorgio (G2): Because the deal was that if I get to six, I get 24 euros, not if I win one match, I get 24 euros. That's why we went all the way.

[...]

Andrea (G1): We actually intended fair division based on the probabilities on the first throw, it doesn't make sense to go and evaluate the probabilities on subsequent throws because the game ends on the first one anyway. […] in the problem, we are told that after the first throw, the game is stopped. So, if we reason about a fairly division of the stakes, according to the probability, it is always that the one who won take all the money because the probability is always the same.

Fabrizio: But if the probability is the same, the other one has the same probability of winning, anyway.

Agata (G1): But one loses and the other wins.

Andrea (G1): But he's already won, so it's over.

[...]

Giorgio (G2): Let's read again the problem for a moment, because I think we have different points of view because we understood differently the text. So, let's try to understand it together. Here it says [reads the text of the problem] the player who first gets to win six rounds wins the game and therefore the entire stake. Six games. So, the fact is that whoever wins six matches is entitled to receive 24 euros. But exactly because no one won six rounds, no one can win all the 24 euros. […] I give an example, it's one thing if you play 12 euros, it's another thing if you play a higher stake and you say: "I'm not going to give you all the money!". Since the deal was different you can't say "I won one then I won it all".

Group 1 and Group 2 approach the problem differently. They understand other group resolution, but none recognize the other as correct. In our view, this is connected with the fact that none of the two resolutions is able to connect to the interlocutor's belief system. In the discussion, the students show that they conceive their positions as incompatible, and they feel the need to discuss their own beliefs about the problem. In the last reported intervention, Giorgio (G2) presents its position, starting from the reading of the text of the problem. His position, which is shared by most of the class, will lead the Group 1 students to abandon their resolution. Giorgio's position is based on considering the implication "whoever wins takes €24" as an implicature (Ferrari, 2004) and concluding that if one does not win six rounds, one does not take €24. In our opinion, students exchange goes beyond the text. They confront their idea of and their identification with the problem situation. It is maybe possible to identify the core of the discussion as identifying what data to consider from the text. The discussion also involves the change of perspective that is required by students at the beginning of a probability course: the willingness to deal with what might happen. Without considering interlocutors convictions, it is difficult to go further with the collective argumentation. The exchange of interventions is crucial to better clarify the two positions. For example, Andrea (G1) explicit his idea that the game is over after the first round, to respond to his peer. This consideration had not emerged in their previous exposition. Moreover, Giorgio (G2) makes explicit his text interpretation, in response to Group 1's objections. The dialogic nature of the exchange allows students to explicit those aspects that are linked with interlocutors' feedback. Although the problem was not completely solved, the argumentation was effective in moving the students forward in the discussion.

Example 2

The second example concerns primary school students, who engage in a discussion about the resolution of the following problem. This example is taken from Ferrari (2021).

> *Ali and Fatima watch a caravan of donkeys and horses go by. There are also some men, all on horseback. On each horse there is only one man with a chest behind him. There are only two chests on each donkey. Ali counts the paws of the animals and finds 52. Fatima counts the boxes: there are* 21 *in all. How many men are in this caravan? Explain your answer.*

Here some excerpts of an interaction among three fifth graders (Rudy, Davide, Silvio). Figure 1 is a drawing produced by Rudy. Perhaps he guessed that the distribution between horses (in Italian, cavalli) and donkeys (asini) had to be as equal as possible (7-6) but then he understood that it didn't work and changed: the sequence of the three numbers at the bottom from '1 2 1 ' is corrected to '2 2 2', even though a line remains between the first number in the sequence and the word 'horses'.

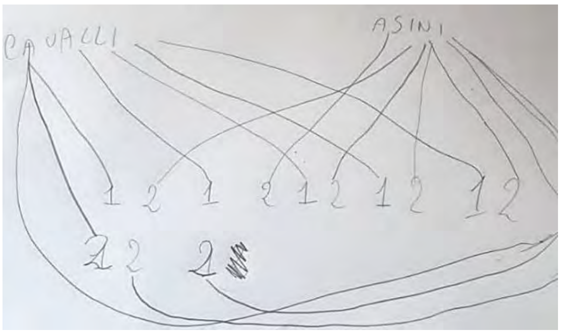

Figure 1: Rudy's drawing

Here some excerpts from the interaction that followed. Rudy starts explaining his strategy.

Rudy: I wrote horses on one side and donkeys on the other. Then I wrote 1, which was the parcel for a horse and 2 for donkeys. And so I wrote 1 and then I put with a dart to horses, then 2 to donkeys, then 1 to horses and 2 to donkeys and I came to the conclusion, slowly, doing this, that the horses are [count] one, two, three, four, five … there are five horses and eight donkeys.

Davide: In my opinion, then I made fifty-four legs, no, how many were there? Fifty-two. Then I made fifty-two and divided them all by four legs, divided by four, then I did, then, I did, I made six donkeys with two boxes and seven donkeys with one box.

Rudy: May I tell you something? But in the end you had to add up all these numbers and you had to get twenty-one, but you got nineteen. That there are twenty-one boxes and you have only put in eighteen … nineteen boxes.

Davide: We did a different thing because, since thirteen is odd, you made eight of two and five of ... no ... eight of two ...

Rudy: However, here it does not say that it must necessarily be even, here it says that there are twenty-one parcels and they must be divided between horses and those ...

Davide: However, since thirteen is odd, we have divided it in different ways …

Rudy: And yet there's only one that should come of a result, which is twenty-one, understand? … the total number of boxes.

Silvio: I made fifty-two legs by four ... and I divided it by four ... which resulted in thirteen. … Thirteen, which was odd, I divided it … and then I divided it into two numbers which were seven and six. Then I made sure that the donkeys had two boxes and the horses one…

Davide:	If there are two boxes on a donkey ... there are fewer donkeys because ... as they say ... because they occupy more boxes. I find nineteen, because I was like Silvio, six ... six donkeys and ... seven ... seven horses.
Silvio:	Right now, I tried dividing into two other numbers which were five and eight.
Rudy:	Here you are!
Silvio:	And I made eight times two sixteen ... first I saw if the two numbers added thirteen, and it came thirteen ... then I made eight, which were the donkeys, and I multiplied it by two and it came sixteen ... then the horses that had a box, which were five, …
Rudy:	And it's twenty-one!

Here at the beginning Rudy is perfectly sure that his solution is correct since it all adds up. He doesn't need any other reasons. His initial argument does not explicitly consider the view adopted by his fellows. Conversely, in discussing Davide's strategy, he is not satisfied with repeating his own, and tries to refute his schoolmate's wrong assumptions (that is, the idea that the number of donkeys and horses should be as close as possible). In other words, he tries to adapt his argument to Davide’s apparent beliefs. Rudy’s change of explanatory strategy is strictly related to the interaction with Davide. The linguistic organization of the texts produced by Rudy is aimed at facilitating understanding, as his interventions are much more detailed at the beginning and shorter at the end, when a common ground is implicitly established.

Discussion and conclusion

To sum up, we consider argument as emerging from a dialogic interaction, which is the core of the argumentative process. This reflects what we observed during classroom activities, in primary school and upper secondary school. In fact, interventions of teacher and classmates and the interactions between them strongly influence the argumentative process, which is ineluctably linguistic. In addition, we observe the crucial role that educational goals play in mathematical classrooms, and we point out the importance of the function of explaining in these contexts. We then introduce the perspective of Mercier and Sperber (2017), which allows us to shift the focus from the product to the process. Moreover, it allows us to look at argumentations that are collectively developed, and to take into consideration the different contributions of participants in the process. Since the model is dynamic, rather than static, and it refers to an interactive process, it allows researchers and educators to adjust their idea of "good argumentation" according to different features, such as interlocutors’ knowledge and beliefs and the function of argumentation. So, as we show by the two examples, we think it is possible to adapt the approach to different school grades. Obviously, linguistic aspects are fundamental in this perspective, as, for example, the organisation of the text in the second classroom episode. If the lack of a static definition of argument and of “good argumentation” is a strength in terms of adaptability, yet it could cause some difficulties, particularly in assessing. Further research is needed to characterise the coherence relationships and the ways of showing them that are effectively used in mathematics classrooms, including the influence of the mathematical knowledge at play. Despite the unavoidable difficulties and limitations, we believe such process analysis is relevant in a socio-cultural view.

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