Abstract

The core of this PhD thesis is the extension and adaptation of concepts, originally envisioned in the context of Analytical Mechanics, to the study of mechanical problems—discrete, semi-discrete, and continuous—in which constraints, either internal or external, are considered. Rather than trying to address each class of problems separately, in this work we employ the enlightening language of Analytical Mechanics with the purpose of unifying such problems and contributing to a paradigmatic approach to mechanics. The main target of this PhD Thesis is to extend some fundamental concepts of the mechanics of constrained systems, such as Chetaev's conditions, quasi-velocities, quasi-coordinates, and transpositional relations, from their classical "discrete" definition to the semi-discrete continuous contexts. This adaptation demonstrates the versatility of Analytical Mechanics across biological and industrial problems, beyond enriching this discipline itself.

This PhD thesis is structured in three main parts.

Part I focuses on constrained discrete systems subjected to nonholonomic constraints, which are restrictions on system's generalized velocities that cannot be equivalently reformulated as conditions on the admissible configurations of the system itself. Constraints of this kind are rather typical in robotics, control theory, and other fields of the like but, to our knowledge, have received less attention in Continuum Mechanics. We analyze the dynamics of nonholonomic constrained systems by providing a modified version of a variational method originally suggested by Kozlov in the 1980s and subsequently expanded by other authors. Our work sets itself the scope of healing inconsistencies that arise in some problems of Classical Mechanics when some logical steps of the traditional formulations of d'Alembert and Lagrange are modified. In this respect, two problems that fall into this "gray zone", and that are analyzed in the present work, are the rolling coin problem and the *Gedankenexperiment* of a charged nonholonomic skate subjected to a magnetic field.

In Part II, we develop continuous models, inspired by the mechanics of constrained discrete systems, for mechanical problems stemming from biology, with emphasis on the biomechanics of growth and remodeling of tissues. In the case volumetric growth, we look at the mass balance law as a nonholonomic constraint on the growth tensor and, thus, we formulate a variational framework based on the Hamilton–Suslov principle, from which governing equations can be derived variationally. This perspective may provide new insights into the mechanics of growth and may pave the way to future investigations on the symmetries and conservation laws in growth mechanics. In the case remodeling, we extend Gurtin and Anand's strain-gradient theory of plasticity to the biphasic case in order to address multicellular aggregates, thereby coupling the deformation of the solid constituents with a Darcy–Brinkman model for the hydraulic part of the problem. Using the principles of Virtual Power and Maximum Dissipation, we derive a constitutive framework able to capture *boundary effects* related to the fluid flow and to remodeling.

Part III addresses semi-discrete mechanical systems in the industrial context, focusing on the dynamics of concentrated defects in elastic media, as is the case of *disclinations*. These defects, even though less studied than dislocations, are known to play a crucial role in some strengthening processes occurring in technologically relevant materials. By starting our discussion with a variational principle for the mechanical description of a plane-strain linear elastic medium in small deformations, we analyze the dynamics of a finite number of disclinations by showing how the defects interact reciprocally in a manner resembling *charged particles*. In doing so, we establish existence results of solutions, and we explore both the isotropic and the crystalline settings (with constrained, predefined "glide" directions), also through the use of numerical experiments.