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1 **A NEW SEMI-EMPIRICAL FORMULA FOR EVALUATING THE**
2 **EQUILIBRIUM SCOUR DEPTH AT BRIDGE PIERS**

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15 **ABSTRACT**

16 Estimating the equilibrium scour depth (y_{se}) at bridge piers is essential for both bridge foun-
17 dation design and flood risk assessment. Traditional methods typically rely on empirical formulas
18 derived from data-fitting procedures, which often result in significant overestimations of the local
19 scour phenomenon. While such conservative predictions may be acceptable for design purposes,
20 they are less suitable for accurate risk evaluation. In this paper, we propose a new formula for
21 predicting y_{se} , grounded in a theoretical framework previously developed by one of the authors.
22 This foundation is enhanced with empirically derived functions, obtained from laboratory data,
23 to account for complex phenomena that remain difficult to capture theoretically, such as sediment

24 non-uniformity. We demonstrate that the resulting semi-empirical formula consistently outperforms
25 five of the most widely used local scour methods in the literature. Predictive accuracy is evaluated
26 through four different performance metrics, using over 600 laboratory data and some selected field
27 data whereby the formula could be confidently applied. The proposed approach provides unbiased
28 estimates and avoids the systematic overestimations that characterize the reference methods, thus
29 making it particularly suitable for risk assessment applications. Furthermore, the potential exten-
30 sion of the model to bridge design is discussed, proposing the introduction of an appropriate safety
31 factor. Finally, we discuss the limitations of the proposed model and outline potential directions
32 for future improvements.

33 **Author keywords:** Bridge piers; Local scour; Equilibrium scour depth; Non-uniform sedi-
34 ments; Risk assessment.

35 INTRODUCTION

36 Local scouring at piers is considered a major cause of bridge-failure and is a primary concern for
37 transportation agencies and local authorities worldwide (Wardhana and Hadipriono 2003; Maddison
38 2012). It is widely accepted that the initiation and evolution of pier local-scour is primarily driven
39 by the erosive power of the (so-called) horseshoe vortex that develops at the interface between the
40 erodible bed and the base of the pier (Melville and Coleman 2000; Lee and Sturm 2009; Ettema
41 et al. 2011). As the scour hole deepens, the erosive capacity of the vortex gradually decreases
42 until a condition of equilibrium is reached (Melville and Chiew 1999). Accurately estimating
43 the so-called equilibrium scour depth (y_{se}) is essential for bridge design and flood-related risk
44 assessments, hence significant research efforts have been dedicated to developing formulas to
45 predict y_{se} , mainly adopting an empirical approach. There is general consensus that, for cylindrical
46 piers with circular cross-section under steady flow conditions, the non-dimensional scour depth of
47 equilibrium, expressed as y_{se}/a (where a represents the pier diameter), depends on the following
48 non-dimensional parameters:

$$49 \frac{y_{se}}{a} = f\left(\frac{h}{a}, \frac{a}{d}, \frac{U}{U_c}, Fr_a, Re_a, \frac{\rho_s}{\rho}, \sigma_g\right), \quad (1)$$

50 where, f is an unknown functional relation, h is the flow depth, d the characteristic sediment
51 diameter (i.e. d_{50}), U the cross-sectional average velocity of the flow upstream of the pier, U_c the
52 sediment critical velocity, $Fr_a = U/\sqrt{ga}$ the pier Froude number, g the gravitational acceleration,
53 $Re_a = Ua/\nu$ the pier Reynolds number, ν the fluid kinematic viscosity, ρ_s and ρ the sediment and
54 fluid densities, and σ_g the geometric standard deviation of the sediment particle size distribution
55 (Ettema et al. 1998; Ettema et al. 2006; Ettema et al. 2011).

56 Most of the currently available formulas for predicting y_{se}/a (e.g. Froelich 1988; Melville
57 1997; Richardson and Davis 2001; Sheppard and Miller 2006; Sheppard et al. 2014) were derived
58 from data-fitting procedures used on (mainly) laboratory data, with the aim of establishing relations
59 between y_{se}/a and each individual governing parameter reported on the right-hand side of Eq. 1
60 (or other different parameters depending on how dimensional analysis was carried out in different
61 studies). However, such empirical approaches may be affected by scale issues, which can arise from
62 the difficulties in simultaneously satisfying all the similarity conditions between laboratory and field
63 scales. For example, matching both Fr_a and Re_a is generally not feasible without introducing a
64 trade-off in model accuracy, as reproducing Fr_a of the prototype scale typically results in too low
65 Re_a at the laboratory scale. In addition, these empirical formulas were developed to be quite
66 conservative meaning that, very often, they provide significant overestimations of y_{se} when applied
67 to both field and laboratory data, leading to a poor agreement between measured and estimated
68 values of y_{se} . While such conservative predictions may be acceptable for design purposes, they
69 are less suited for risk assessment procedures. In engineering practice, when over-conservative
70 formulas for estimating local scour are applied in bridge risk assessments, they often classify most
71 or all bridges as high-risk. This limits the ability to rank structures accurately, making it difficult for
72 asset owners to prioritize, and hence optimize, resources dedicated to maintenance and improved
73 flood-resilience efforts.

74 Manes and Brocchini (2015), hereafter referred to as MB15, employed a physically grounded
75 framework based on turbulence phenomenology to derive scaling relations between y_{se}/a and
76 some of the governing parameters listed in Eq. 1 (i.e. $a/d, U/U_c, Fr_a, \rho_s/\rho$). The scaling

77 relations proposed by MB15 were developed for uniform sediments in both clear-water and live-
78 bed conditions, under the hypotheses of large values of Re_a and relative coarseness a/d . As
79 this approach directly stems from theoretical considerations rather than from physical modelling
80 or empirical data-fitting, it is virtually free from scale-effects. Coscarella et al. (2020), hereafter
81 referred to as CGM20, extended the applicability of MB15 by accounting for Re_a -effects and low
82 relative coarseness (i.e. low a/d) in clear-water conditions. Moreover, CGM20 discusses the role
83 played by h/a , deeming it as negligible.

84 The aim of the present study is to build upon the theoretical foundations laid out by MB15
85 and CGM20 to: (i) identify empirical functions accounting for the effects of low a/d -values in
86 live-bed conditions and of non-uniform sediments (as quantified by σ_g) on scour; (ii) retrieve a
87 semi-empirical formula accounting for all the relevant non-dimensional parameters appearing in
88 Eq. 1; and (iii) demonstrate that the retrieved semi-empirical formula outperforms fully-empirical
89 methods available from the literature when predicting local scour. Such a formula is developed
90 to reproduce experimental measurements without overestimation biases and hence it targets risk-
91 assessment applications. We will demonstrate, though, that when appropriate safety factors are
92 employed, it can be used also for design purposes.

93 In what follows, we first review the formulations proposed by MB15 and CGM20. We then
94 extend the latter to account for low a/d values under live-bed conditions and the influence of
95 non-uniform sediments. The performance of the newly proposed formula is subsequently evaluated
96 against the most widely used empirical formulas from the literature, using both laboratory and field
97 experimental data. Finally, we provide a comprehensive discussion of the proposed formula, high-
98 lighting its limitations and outlining potential research-avenues for further enhancing its predictive
99 capability.

100 **SUMMARY OF MB15 AND CGM20**

101 In clear-water conditions ($0.5 \leq U/U_c \leq 1$), equilibrium occurs when the shear stress within
102 the scour hole approaches the critical shear stress for the sediments (Ettema et al. 2011) and y_{se}
103 can be either defined as the maximum scour depth reached at a given finite time (Melville and

104 Chiew 1999) or asymptotically for time approaching infinity (Sheppard et al. 2004; Lança et al.
 105 2013). This condition, applied to the deepest point of the scour hole, is expressed by the relation
 106 $\tau = \tau_c \sim (\rho_s - \rho)gd$, where τ is the bed shear stress and τ_c is the critical shear stress of the
 107 sediments. Building on a series of hypotheses and employing Kolmogorov's theory to characterize
 108 turbulence within the scour hole, MB15 refined the equilibrium condition to derive the following
 109 scaling relationship for y_{se} :

$$110 \quad \frac{y_{se}g}{U^2} \sim \left(\frac{\rho}{\rho_s - \rho} \right) (C_d)^{2/3} \left(\frac{a}{d} \right)^{2/3}, \quad (2)$$

111 where C_d is the drag coefficient of the part of the pier exposed to scouring. There are four
 112 assumptions underpinning Eq. 2: (i) the flow within the scour hole is in the fully rough regime;
 113 (ii) the large-eddies generated within the scour hole are comparable in size to y_{se} ; (iii) the mean
 114 sediment diameter d belongs to the inertial range of turbulence, namely $\eta \ll d \ll y_{se}$ (where η
 115 is the Kolmogorov's length scale); and (iv) the power of the horseshoe vortex (i.e. turbulent kinetic
 116 energy per unit mass and unit time required to feed the horseshoe vortex) can be estimated from the
 117 product of the drag force acting on the part of the cylinder exposed to scour and the velocity used to
 118 estimate such a drag force. Due to these assumptions and the data employed for its validation, Eq.
 119 2 is applicable exclusively to flow regimes characterized by large values of Re_a where viscosity
 120 effects are negligible (i.e. $d/\eta > 5$) and deep scour holes, i.e. $d \ll y_{se}$. MB15 report that if a is
 121 used as an a-priori estimation of y_{se} (i.e. its order of magnitude), experimental observations show
 122 that the proposed scaling works very well for $a/d > 20$.

123 In live-bed conditions, equilibrium is determined by a mass balance of sediments moving in and
 124 out of the scour hole (Melville 1984), with y_{se} defined as the time average of the maximum scour
 125 depth, which fluctuates over time due to the passage of bed forms. Mathematically, this condition
 126 can be expressed by the relation $Q_{in} = Q_{out}$, where Q_{in} and Q_{out} are the average fluxes of sediment
 127 entering and exiting the scour hole, respectively at equilibrium conditions. MB15 assumed that
 128 Q_{in} is predominantly in the form of bed load, hence it can be estimated as a function of U/U_c .

129 Then, by applying the same set of hypotheses as used for the analysis in clear-water conditions,
 130 they obtained the following live-bed scour function S_e :

$$131 \quad S_e = \frac{\frac{y_{se}g}{U^2}}{\frac{\rho}{\rho_s - \rho} (C_d)^{2/3} \left(\frac{a}{d}\right)^{2/3}} = \Phi\left(\frac{U}{U_c}\right), \quad (3)$$

132 where the flow intensity function Φ was found empirically. The results were satisfactory because
 133 the scour function S_e , when plotted against U/U_c , allowed for an excellent data-collapse, except
 134 for data points with low relative coarseness, i.e. $a/d < 20$ as observed for clear-water conditions.
 135 Manipulating Eq. 3, MB15 obtained a formula for y_{se} normalized by the pier diameter a , i.e.

$$136 \quad \frac{y_{se}}{a} \sim \Phi\left(\frac{U}{U_c}\right) (Fr_a)^2 \left(\frac{\rho}{\rho_s - \rho}\right) (C_d)^{2/3} \left(\frac{a}{d}\right)^{2/3}, \quad (4)$$

137 with $\Phi(U/U_c) = 1$ for $U/U_c \leq 1$. Eq. 4 establishes a relation between y_{se}/a and all the governing
 138 non-dimensional parameters reported in Eq. 1, except for h/a and Re_a . However, MB15 argued
 139 that, these two parameters may be partly accounted for through their indirect effects on C_d .

140 CGM20 revisited the work of MB15 to broaden its applicability in clear-water conditions by
 141 addressing viscous and low relative-coarseness effects on local scouring. Coupling theoretical and
 142 empirical arguments, they derived two corrective functions, $f_1(d/\eta)$ and $f_2(a/d)$. The corrective
 143 function $f_1(d/\eta)$, which accounts for viscous effects on y_{se} , is defined as

$$144 \quad f_1\left(\frac{d}{\eta}\right) = F_1 \left\{ 0.22 \left(\frac{d}{\eta}\right)^{-0.6} + 0.12 \exp\left[-17.77 \left(\frac{d}{\eta}\right)^{-0.6}\right] \right\}, \quad (5)$$

145 where $\eta = (\nu^3 a / (C_d U^3))^{1/4}$ is the Kolmogorov's length scale and F_1 a coefficient that compensates
 146 for the different definitions of equilibrium in clear-water conditions ($F_1 = 28$ if y_{se} is defined at
 147 a given finite time or $F_1 = 22$ if y_{se} is defined asymptotically). Eq. 5 retains the mathematical
 148 formulation proposed by Brownlie (1981) to fit the experimental data from Shields (1936) and
 149 describes the behavior of a non-dimensional critical shear stress τ_c (i.e. the so-called Shields
 150 parameter) as a function of d/η , used as a surrogate of the roughness Reynolds number. On the

151 other hand, the corrective function $f_2(a/d)$ accounts for low a/d effects and is defined as

$$152 \quad f_2\left(\frac{a}{d}\right) = 1 + 6\exp\left(-0.18\frac{a}{d}\right). \quad (6)$$

153 Physically, Eq. 6 expresses that if $a/d < 20$, τ_c within the scour hole increases if a/d decreases
154 (and consequently y_{se} decreases, see Eq. 7). CGM20 indicates that this is in line with the standard
155 theory of incipient sediment motion where the low relative submergence for uniform open channel
156 flows (i.e. h/d) plays the same role as a/d in the game of clear-water scouring around piers.
157 Despite these encouraging results, CGM20 warns that low values of a/d are often encountered for
158 large sediment diameters d and hence very permeable beds. Unfortunately permeability effects are
159 known to affect τ (Manes et al. 2011; Manes et al. 2012) and possibly τ_c but these effects are not
160 accounted for in the theoretical approach developed by MB15 and CGM20. On top of this, CGM20
161 also warns that low values of a/d can be encountered for relatively small scour holes whereby
162 the horseshoe vortex is not fully buried, which is an important hypothesis underpinning the MB15
163 approach. Hence, it was concluded that f_2 may incorporate the lumped effect of several causes.

164 By integrating the work of MB15 with these new insights, CGM20 proposed an updated formula
165 for modeling local scouring in clear-water conditions with uniform sediments, i.e.

$$166 \quad y_{se} = \frac{\frac{\rho}{\rho_s - \rho} \frac{U^2}{g} (C_d)^{2/3} \left(\frac{a}{d}\right)^{2/3}}{f_1\left(\frac{d}{\eta}\right) f_2\left(\frac{a}{d}\right)}, \quad (7)$$

167 where $f_1(d/\eta)$ and $f_2(a/d)$ are defined in Eq. 5 and Eq. 6 respectively.

168 EXTENSION OF CGM20 TO LIVE-BED CONDITIONS

169 According to MB15, in live-bed conditions, y_{se} can be related to a scour function S_e , which
170 resembles a Shields mobility parameter and is dependent on the flow intensity U/U_c through a
171 flow intensity function Φ (see Eq. 3). The function $\Phi(U/U_c)$ is here refined using a slightly
172 larger dataset compared to MB15, encompassing a total of 207 laboratory experiments conducted
173 in live-bed conditions with water flowing over uniform sediments and impinging on a circular

174 pier. This dataset (see Table 1) was extracted from the works of Shen et al. (1969), Chee (1982),
 175 Chiew (1984), and Sheppard and Miller (2006). There are two primary differences between the
 176 present dataset and that used in MB15: (i) 13 data points characterized by $U/U_c < 1$ were excluded
 177 (see Fig. 6 in MB15) to ensure consistency with the definition of live-bed conditions; and (ii) 54
 178 additional measurements from Shen et al. (1969) and Chee (1982) were incorporated to expand the
 179 dataset and strengthen the validation of $\Phi(U/U_c)$. All data considered in this analysis refer to the
 180 time-averaged maximum scour depth, thereby the transient effects caused by migrating bedforms
 181 are filtered out, consistently with the definition of equilibrium in live-bed conditions. Throughout
 182 the paper, we adopted constant values for fluid and sediment densities ($\rho = 1000 \text{ kg m}^{-3}$ and ρ_s
 183 $= 2650 \text{ kg m}^{-3}$, respectively), assumed $C_d = 1.2$ for circular piers, and used as sediment critical
 184 velocity U_c the values reported in the relevant references (see Table 1).

185 When S_e is plotted versus U/U_c (see Fig. 1a) most of the experimental data nicely collapse on
 186 a curve described by the following flow intensity function:

$$187 \quad \Phi\left(\frac{U}{U_c}\right) = \beta\left(\frac{U}{U_c}\right)^\gamma, \quad (8)$$

188 with $\beta = 0.41$ and $\gamma = -1.75$. Because of the modified dataset considered herein, these values
 189 slightly differ from those reported by MB15 (i.e. 0.47 and -1.89, respectively).

190 MB15 observed that the data points which most deviate from Eq. 8 (i.e. the black markers)
 191 are characterized by $a/d < 20$, as observed by CGM20 for clear-water conditions. To compensate
 192 for the effects of low relative coarseness in clear-water conditions, CGM20 proposed to use the
 193 corrective function $f_2(a/d)$ (see Eq. 6). Herein, we propose to use the same approach and include
 194 $f_2(a/d)$ in the formulation of S_e to develop a corrected live-bed scour function S_e' , yielding:

$$195 \quad S_e' = \frac{\frac{\gamma_{se} g}{U^2} f_2\left(\frac{a}{d}\right)}{\frac{\rho}{\rho_s - \rho} (C_d)^{2/3} \left(\frac{a}{d}\right)^{2/3}} = \Phi\left(\frac{U}{U_c}\right). \quad (9)$$

196 In Fig. 1b, the corrected scour function S_e' is plotted versus U/U_c . Interestingly, the introduction
 197 of $f_2(a/d)$, which CGM20 calibrated exclusively using clear-water data, results in a much better

198 alignment also of the live-bed data, independently of the relative coarseness. When interpreting this
199 result, it is worth noting that the live-bed data with low relative coarseness included in the dataset
200 are limited to $U/U_c = 1.2 - 1.8$ (that is, a narrow range just above the transition from clear-water
201 to live-bed). Therefore, $f_2(a/d)$ should be checked, in future experimental studies, for larger flow
202 intensities U/U_c .

203 It should be noted that the corrective function $f_1(d/\eta)$ (see Eq. 5), which accounts for viscous
204 effects, is not included in our live-bed scour framework. This is because the equilibrium scour
205 depth in live-bed conditions is dictated by the balance of bed-load sediment fluxes going in and
206 out of the scour hole. Classical bed-load transport formulas (e.g. Meyer-Peter and Müller 1948)
207 relate to the excess shear with respect to a non-dimensional critical shear stress, which is always
208 assumed to be constant, i.e. independent of viscous effects. Consistently with classical bed-load
209 formulations, here, we make the same assumption.

210 **NON-UNIFORM SEDIMENTS EFFECTS**

211 Previous studies have shown that sediment non-uniformity can significantly impact local scouring,
212 often resulting in a considerably lower y_{se} compared to uniform sediments (see e.g. Raudkivi
213 and Ettema 1983). Such a reduction is mainly attributed to the effect of armoring, with the flow
214 selectively removing from the scour hole the finer portion of the sediment mixture and leading to
215 the creation of a protective layer of coarser particles. This armored layer limits sediment transport
216 by shielding the finer particles, preventing their entrainment, and reducing the overall amount of
217 eroded material, thereby decreasing y_{se} (Chiew 1991). The process becomes more pronounced as
218 sediment non-uniformity increases (Ettema 1980). The degree of sediment gradation is usually
219 quantified by the geometric standard deviation of the particle size distribution σ_g . Given that
220 the sediment size distribution in rivers often follows the log-normal probability distribution (Dey
221 2014), σ_g can be defined as the ratio of the sediment diameters exceeding 84% and 50% of the
222 bed material, respectively, i.e., $\sigma_g = d_{84}/d_{50}$. The threshold between uniform and non-uniform
223 sediments is commonly set at $\sigma_g = 1.5$ (Chiew 1984), with higher degrees of sediment gradation
224 indicating non-uniform conditions. The analysis presented hereafter aims to develop a model ca-

225 pable of efficiently estimating y_{se} for high degrees of sediment gradation, first in clear-water and
226 then in live-bed conditions.

227 **Clear-water conditions**

228 In clear-water conditions, the rate of sediment supply into the scour hole is zero. As a result,
229 the non-uniform sediment bed undergoes a gradation process due to the erosion of the finer fraction
230 caused by the increased bed shear stress until it is fully armored (Chiew 1991). Thus, sediment
231 gradation influences both the scaling of the flow resistance (i.e. τ) and the initiation of sediment
232 motion (i.e. τ_c). Concerning the flow resistance, Gioia and Bombardelli (2001) proposed a model
233 that directly relates it to the statistical distribution of roughness element sizes. In contrast, classical
234 formulas (e.g. Strickler's formula) relate resistance to the size of the coarser sediment fraction,
235 assuming that the largest particles dictate the flow behavior. As for the initiation of sediment
236 motion, Parker et al. (1982) illustrated a slight increase in the critical shear stress for a sediment
237 mixture as the grains become coarser and related this tendency to hiding-exposure effects.

238 Given the complexities involved in theoretically modeling the effects of sediment gradation on
239 τ and τ_c separately, we account for their combined action on y_{se} , through an empirical sediment
240 gradation function $f_3(\sigma_g)$ that fits the available experimental data. The function $f_3(\sigma_g)$ is defined
241 from the scour formulation proposed by CGM20 (see Eq. 7), rearranging it into a clear-water scour
242 function $S_{e,CW}$, as follows:

$$243 \quad S_{e,CW} = \frac{\frac{y_{se}g}{U^2} f_1\left(\frac{d}{\eta}\right) f_2\left(\frac{a}{d}\right)}{\frac{\rho}{\rho_s - \rho} (C_d)^{2/3} \left(\frac{a}{d}\right)^{2/3}} = f_3(\sigma_g), \quad (10)$$

244 where the characteristic diameter for non-uniform sediments d coincides with d_{50} and f_3 must
245 be found empirically. Towards this end, we employed a collection of data from the works of
246 Ettema (1976), Molinas (2004), Sheppard et al. (2004), Raikar and Dey (2005) and Guo et al.
247 (2012), which comprises 119 experiments conducted in clear-water conditions with non-uniform
248 sediments characterized by values of σ_g up to 4.6 (see Table 1). The measurements extracted from
249 Ettema (1976), Molinas (2004), Raikar and Dey (2005), and Guo et al. (2012) report values of scour

250 depths recorded at finite times while Sheppard et al. (2004), determined y_{se} by extrapolating the
 251 curve describing the temporal evolution of the maximum scour depth to an infinite time. Following
 252 CGM20, we assigned $F_1 = 28$ to the former group of data and $F_1 = 22$ to the latter. It is worth
 253 noting that a few experiments of Raikar and Dey (2005) were performed with a square pier and for
 254 these cases we assumed $C_d = 2$.

255 The experimental data plotted in Fig. 2 show the expected reduction in scour depth attributed
 256 to armoring effects, with $S_{e,CW}$ approaching 1 as $\sigma_g = 1$ and decreasing as σ_g increases. The
 257 observed trend suggests that a suitable fitting curve may be represented by a sigmoid function as
 258 follows:

$$259 \quad f_3(\sigma_g) = \frac{1}{c_4 + \exp\left(\frac{\sigma_g - c_1}{c_2}\right)} + c_3, \quad (11)$$

260 where $c_1 = 2.21$, $c_2 = 0.17$, $c_3 = 0.15$ and $c_4 = 1.18$.

261 Eq. 11 effectively captures the overall trend of the experimental data. The parameters c_3 and
 262 c_4 were determined by imposing two boundary conditions: (i) when the sediment bed is uniform
 263 ($\sigma_g < 1.5$), $f_3(\sigma_g)$ tends to 1, so that Eq. 10 coincides with the formula for uniform sediments (Eq.
 264 7); and (ii) as the degree of sediment gradation increases, the estimated scour depth progressively
 265 decreases due to armoring, reaching a maximum reduction of approximately 85%, following the
 266 corrective function proposed by Raudkivi (1986) (see Fig. 6 in the cited paper). Notably, there is
 267 a certain degree of uncertainty associated with this latter hypothesis, as the available experimental
 268 data for highly non-uniform sediments are limited, making precise quantification challenging.
 269 Nevertheless, Raudkivi (1986) provides the most reasonable indication of the expected maximum
 270 scour depth reduction. Moreover, the few data points with $\sigma_g > 3$, show an acceptable agreement
 271 with this assumption. Regarding the estimation of parameters c_1 and c_2 , their values were obtained
 272 by applying a best-fit procedure, once the other two parameters were fixed.

273 **Live-bed conditions**

274 Chiew (1984) reported that the effect of sediment gradation on y_{se} in live-bed conditions appears
275 to be negligible for $\sigma_g < 2$, implying that the scour depth can be reliably estimated from Eq. 9, as
276 if sediments were uniform. It follows that only when the degree of sediment gradation increases
277 ($\sigma_g \geq 2$), the occurrence of armoring alters the scour mechanism reducing the scour depth. For
278 what concerns the flow intensity, it is important to note that when dealing with non-uniform
279 sediments, U_c represents the critical velocity referred to d_{50} of the sediment mixture. When the
280 flow can only erode the finer fraction (say $U/U_c \gtrsim 1$) and the supply of sediments cannot keep up
281 with the material being entrained, armoring occurs, closely resembling what happens in clear-water
282 conditions. Intuitively, as U/U_c increases, the intensified bed shear stress leads to the dislodgment
283 of all sediment grains regardless of their size so much so armoring effects (or, more in general, non-
284 uniform sediments effects) are expected to become negligible (Melville and Sutherland 1988). This
285 intuitive reasoning is supported by Fig. 3a, which displays the corrected live-bed scour function
286 for uniform sediments S_e' (Eq. 9) plotted against U/U_c for all experiments performed in live-bed
287 conditions. The 108 experimental data were extracted from Shen et al. (1969) and Chiew (1984),
288 covering a range of σ_g up to 5.5 (see Table 1). Consistently with the approach adopted for uniform
289 sediments, we considered only those data in which the equilibrium scour depth is defined as the
290 time-averaged of the deepest measured scour depth (time averaging is used to filter out bed-form
291 effects). The observed trend supports the idea that the effect of sediment gradation in live-bed
292 conditions changes depending on flow intensity, with data showing two distinct behaviors. Once
293 U/U_c exceeds a velocity threshold, experiments with non-uniform sediments show a good match
294 with those with uniform sediments (data of experiments with uniform sediments are included
295 in Fig. 3 for reference), nicely collapsing on the same curve. This confirms that the effect of
296 sediment gradation becomes insignificant at high flow intensities. However, when U/U_c is below
297 the threshold, data for non-uniform sediments tend to stratify horizontally at S_e' . The degree of
298 stratification appears to depend on σ_g , with an observed reduction in scour depth with increasing
299 sediment non-uniformity, consistently with the occurrence of armoring as observed in clear-water

300 conditions. We define this threshold as the critical armoring condition and characterize it using
 301 $U_{c,d_{84}}$, which is the critical velocity associated with d_{84} . We estimated it following a two-step
 302 procedure. First, we computed the critical shear stress, and the associated critical shear velocity
 303 related to d_{84} as per Brownlie (1981). Then, we estimated the critical velocity $U_{c,d_{84}}$ employing
 304 the logarithmic law of the wall, as per Melville (1997). To assess how the critical armoring
 305 condition is affected by the exact definition of $U_{c,d_{84}}$, we conducted a dedicated sensitivity analysis,
 306 using other approaches available from the literature to estimate it (see Appendix SI and Fig. S1
 307 in Supplemental Material for more details). The results from this analysis indicate a maximum
 308 variation of approximately 10% in $U_{c,d_{84}}$ across the approaches tested.

309 In light of the data-stratification observed at low U/U_c in Fig. 3a, we defined a new live-bed
 310 scour function $S_{e,LB}$ based on S_e' and including the sediment gradation function $f_3(\sigma_g)$ as modeled
 311 in clear-water (Eq. 11), i.e.

$$312 \quad S_{e,LB} = \frac{\frac{\gamma_{se} g}{U^2} f_2\left(\frac{a}{d}\right)}{\frac{\rho}{\rho_s - \rho} (C_d)^{2/3} \left(\frac{a}{d}\right)^{2/3} f_3(\sigma_g)} = \Phi' \left(\frac{U}{U_{c,d_{84}}} \right). \quad (12)$$

313 When $S_{e,LB}$ is plotted against $U/U_{c,d_{84}}$ (Fig. 3b), data for non-uniform sediments show a clear
 314 transition in regimes at $U/U_{c,d_{84}} \approx 0.8$, which can be identified as the critical armoring condi-
 315 tion. Below this threshold, experimental data consistently align around $S_{e,LB} \approx 1$, supporting
 316 the assumption that sediment gradation affects scour depth as in clear-water conditions when the
 317 critical armoring condition is not reached. Therefore, for $U/U_{c,d_{84}} < 0.8$, $S_{e,LB}$ appears suitable
 318 for estimating the scour depth. Conversely, above the critical armoring condition the scour depth
 319 is independent of σ_g and appears to depend only on the flow intensity (Fig. 3a) as for uniform
 320 sediments.

321 In Fig. 3c the corrected live-bed scour function S_e' (Eq. 9) is recalculated for both uniform
 322 and non-uniform sediments datasets, focusing on experiments characterized by $U/U_{c,d_{84}} \geq 0.8$.
 323 Given the excellent alignment displayed, we deduce that the function $\Phi(U/U_c)$ (Eq. 8) estimated
 324 for uniform sediments is appropriate to model the impact of flow intensity on scour depth in

325 fully developed live-bed conditions independently of the sediment mixture (solid line in Fig. 3c),
 326 confirming that in such conditions the effect of armoring becomes almost irrelevant. A slight
 327 residual dependence of data on σ_g is present for $U/U_c < 3$. This residual armoring effect could
 328 be easily modeled, however it appears to be weak and can be safely neglected, resulting in a
 329 conservative approach due to the scour-reducing effect of armoring.

330 DISCUSSION

331 Based on the results of our analysis, we can establish a final formulation for y_{se} , which
 332 varies depending on the flow conditions and sediment transport regime. In clear-water conditions
 333 ($U/U_c \leq 1$), y_{se} is determined from Eq. 10 as

$$334 \quad y_{se} = \frac{\frac{\rho}{\rho_s - \rho} \frac{U^2}{g} (C_d)^{2/3} \left(\frac{a}{d}\right)^{2/3} f_3(\sigma_g)}{f_1\left(\frac{d}{\eta}\right) f_2\left(\frac{a}{d}\right)}, \quad (13)$$

335 whereas in live-bed conditions ($U/U_c > 1$), y_{se} is determined by one of the following equations:

$$336 \quad y_{se} = \frac{\frac{\rho}{\rho_s - \rho} \frac{U^2}{g} (C_d)^{2/3} \left(\frac{a}{d}\right)^{2/3} f_3(\sigma_g)}{f_2\left(\frac{a}{d}\right)}, \quad (14a)$$

$$337 \quad y_{se} = \frac{\frac{\rho}{\rho_s - \rho} \frac{U^2}{g} (C_d)^{2/3} \left(\frac{a}{d}\right)^{2/3} \Phi\left(\frac{U}{U_c}\right)}{f_2\left(\frac{a}{d}\right)}. \quad (14b)$$

338 Eq. 14a (derived from Eq. 12) applies when armoring occurs, that is, below the critical armoring
 339 condition ($U/U_{c,d_{84}} < 0.8$) with high sediment gradation ($\sigma_g \geq 2$), whereas Eq. 14b (derived from
 340 Eq. 9) applies when the effects of armoring are negligible (i.e. $\sigma_g < 2$ and/or $U/U_{c,d_{84}} \geq 0.8$). The
 341 various functions $f_1(d/\eta)$, $f_2(a/d)$, $\Phi(U/U_c)$ and $f_3(\sigma_g)$ employed in Eqs. 13-14 are reported in
 342 Eq. 5, Eq. 6, Eq. 8 and Eq. 11, respectively. Application examples of the formula can be found in
 343 Appendix SII.

344 Overall performance of the proposed formula

345 In this section, we assess the reliability of the proposed formula through a systematic comparison
 346 of its performance with that of the most employed approaches proposed in the literature. Towards

347 this end we employ several metrics that quantify the predictive accuracy of y_{se} . This validation
348 procedure is initially conducted using laboratory data and then extended to some of the available
349 field data from the literature.

350 The reference formulas selected for comparison are those proposed by Sheppard et al. (2014)
351 (S/M), Richardson and Davis (2001) (HEC-18), Sheppard and Miller (2006) (SM06), Froelich
352 (1988) (F88), and Melville (1997) (M97). Sheppard et al. (2014) demonstrated that S/M is the most
353 accurate model among the 23 evaluated. HEC-18, SM06, and M97 are recognized by Ettema et al.
354 (2011) as leading methods for pier scour estimation. F88 is considered one of the most reliable
355 formulations derived from field measurements (Sheppard et al. 2011). Detailed descriptions of
356 all models are provided in the Supplemental Material (Appendix SIII). It is worth noting that
357 S/M, SM06 and F88 do not incorporate correction factors to account for sediment non-uniformity,
358 presumably in view of adopting a conservative approach that neglects the reduction of local scour
359 caused by armoring-effects. We based our comparative analysis between the measured scour $y_{se,m}$
360 and the predicted scour depth $y_{se,p}$ on four complementary performance metrics:

- 361 1. the mean relative error $\overline{E}_m = \frac{1}{N} \sum \frac{|y_{se,p} - y_{se,m}|}{y_{se,m}} \times 100$, where N is the number of data points,
362 as employed by MB15 and CGM20;
- 363 2. the sum of squared relative errors $SSE = \frac{\sum (y_{se,p} - y_{se,m})^2}{\sum (y_{se,m})^2} \times 100$, following Sheppard et al.
364 (2014);
- 365 3. the Nash–Sutcliffe efficiency coefficient $NS = 1 - \frac{\sum (y_{se,m} - y_{se,p})^2}{\sum (y_{se,m} - \bar{y}_{se,m})^2}$, as adopted by Franzetti
366 et al. (2022), which evaluates the accuracy of a predictive model in reproducing experimental
367 data ($NS = 1$ indicates perfect agreement, whereas $NS < 0$ indicates that the model perform
368 worse than simply using the mean of the experimental measurements) ; and
- 369 4. the percentage of predictions, P_{out} , falling outside a 30% error margin, inspired by the
370 approach of Franzetti et al. (2022).

371 The dataset of laboratory experiments involved in the analysis encompasses a total of 622 mea-
372 surements, approximately balanced between clear-water and live-bed conditions, with over 200

373 experiments involving non-uniform sediments up to $\sigma_g = 5.5$ (see Table 1). A thorough literature
374 review was carried out to include a large number of published data neglecting only those that were
375 not consistent with the assumptions underlying our model, such as clear-water data collected far
376 from equilibrium conditions (i.e. in short-duration experiments), or live-bed data not filtered from
377 the effects induced by the passage of bedforms.

378 Fig. 4 shows a comparison between the measured scour depth $y_{se,m}$ in laboratory experiments
379 and the predicted scour depth $y_{se,p}$ estimated according to our formula (panel a), S/M (panel b),
380 HEC-18 (panel c), SM06 (panel d), F88 (panel e) and M97 (panel e). Table 2 reports the values
381 of the performance metrics, evaluated over specific subsets, the full laboratory dataset and the
382 selected field data. At a first glance, it is evident that our formula leads to an improved data
383 collapse with respect to the others. The improved performance remains consistent across both
384 uniform and non-uniform sediment conditions, as well as across clear-water and live-bed regimes.
385 In general, the other formulas exhibit a lower performance across all the metrics. Exceptions are
386 noted for S/M, SM06 and M97 in clear-water conditions with uniform sediments which have a
387 few comparable or slightly better indicators. As previously discussed, S/M, SM06 and F88 are
388 specifically designed for uniform sediments. Accordingly, their accuracy diminishes when applied
389 to non-uniform sediments data, particularly in clear-water conditions. S/M exhibits a performance
390 very similar to SM06, since it retains the same underlying structure while introducing minor
391 adjustments that reduce underprediction and make it more robust as a design tool. F88 shows a
392 good accuracy in predicting scour depth in live-bed conditions with uniform sediments, reflecting
393 the fact that it was derived from field data under such conditions. HEC-18 also demonstrates
394 greater accuracy for uniform than for non-uniform sediments. This discrepancy arises from the
395 limited applicability of the corrective function used, designed to account for armoring effects only
396 for very coarse sediments. Given that few data respect this criterion, the HEC-18 formula exhibits
397 significantly reduced performance when applied to non-uniform sediments. M97 accounts for the
398 occurrence of armoring by introducing a characteristic armoring velocity that marks the transition
399 between clear-water and live-bed conditions in the presence of non-uniform sediments. However,

400 even this model exhibits limited accuracy when applied to non-uniform sediments particularly in
401 clear-water conditions.

402 Another key factor contributing to the lower performance of the reference formulas is their
403 inherently conservative nature. Being primarily intended for design purposes, these models in-
404 tentively tend to overestimate scour depths and prioritize accuracy at large values of $y_{se,m}$ (i.e.
405 $y_{se,m} > 0.2$ m), which are more relevant to practical applications. Our formula, instead, appears
406 more suitable for the assessment rather than the design phase, as it intends to predict, rather than
407 over-predict, scour depths. Therefore, to extend the applicability of the model to the design phase,
408 we would recommend the introduction of a safety factor. Specifically, following the approach of
409 (Froelich 1988), we observe that the normalized measured scour depth $y_{se,m}/a$ is consistently lower
410 than the normalized predicted scour depth $y_{se,p}/a$ (given by Eqs. 13-14) plus 1. Consequently,
411 adding the value of the pier diameter a to the predicted scour depth systematically overestimates
412 the entire laboratory dataset, thereby providing an appropriate safety factor.

413 We now extend the application of our formula to a series of scour measurements collected in the
414 field. It is important to note that field data typically exhibit greater uncertainty due to factors such
415 as the unknown maturity of the scour hole and the inherent challenges in data collection. However,
416 field measurements offer the significant advantage of eliminating scale effects (Sheppard et al.
417 2014). Data were extracted from the appendix of Sheppard et al. (2011) and subsequently filtered
418 to retain only those compatible with the application of our model. Specifically, we excluded all data
419 points identified as outliers by Sheppard et al. (2014), as well as those with missing or ambiguous
420 key parameters. Regarding this latter filter, when multiple cases shared identical geometric,
421 hydraulic, and granulometric variables, but reported different scour depths, we retained only the
422 measurement corresponding to the highest recorded scour depth. Additionally, cases involving
423 complex pier shapes (i.e. shapes other than circular or square) were omitted. This filtering step is
424 required because the available information for non-standard geometries are insufficient to determine
425 a reliable drag coefficient C_d . Moreover, it is important to note that, in our theoretical framework,
426 C_d refers specifically to the portion of the pier exposed to scour, which, in most cases, corresponds

427 to the foundation elements for which, however, details of the geometry are also not reported in the
428 literature. This makes it difficult to choose an appropriate value of C_d . As a first approximation,
429 we employed the C_d of the pier above the foundation elements assuming it to be prismatic in and
430 out of the sediment bed as encountered in most laboratory data sets. This filtering process resulted
431 in a final dataset of 68 measurements (see the field data subset in Table 1), all characterized by
432 non-uniform sediments, as expected in field conditions. We computed the critical velocity U_c as
433 described in the previous sections of the paper, and we modified the drag coefficient for squared
434 piers as function of their aspect ratio, following Çengel and Cimbala (2013).

435 Fig. 5 presents the comparison between our model and the previously introduced literature
436 formulas in predicting scour depth for the field dataset, using the same panel arrangement as in Fig.
437 4. Performance metrics for these field data are summarized in Table 2. Despite the approximations
438 used for estimating C_d , the results appear to be very promising, as the newly proposed formula
439 outperforms all the others across all considered metrics. As per laboratory data, the alternative
440 formulas tend to significantly overestimate the local scour depth. In contrast, our formula provides
441 less biased estimates. Clearly, with respect to laboratory data, uncertainties of predictions are
442 significantly higher and, given the complexity of field conditions as well as challenges in data
443 collection, it is hard to explain why with confidence. Nevertheless, in what follows, the limitations
444 of the proposed formula are identified and discussed with a view to assess its applicability in
445 engineering practice and to identify avenues for future research.

446 **Applicability and limitations of the formula**

447 As shown by the comparison with the best formulas available in literature, our approach
448 effectively predicts local scour depths when applied to laboratory and, with appropriate caution,
449 field data. Its predictive capability appears to be fairly independent of sediment properties (provided
450 that the sediments are cohesionless) and of flow regime, with the exception of a slight decrease in
451 performance in clear-water conditions with non-uniform sediments (see Fig. 4) and, as reported
452 by CGM20, with coarse sediments, whereby bed permeability effects are not accounted for in
453 the definition of both τ and τ_c . We recommend that future research efforts should be dedicated

454 to clarify the role of bed permeability and sediment non-uniformity in dictating shear stress and
455 critical shear stress in local scour holes.

456 The proposed formula has been developed and tested only for isolated prismatic piers with
457 a simple geometry. In principle, complex-geometry effects may be incorporated into $C_d^{2/3}$ (see
458 MB15), which serves as the equivalent of the so-called shape-parameters commonly employed in
459 the literature. Indeed, a comparison between the corrective effects of our $C_d^{2/3}$ and those reported
460 by Ettema et al. (2011) for their shape coefficients, reveals only minor differences (see Fig. S2
461 and Table S1 in Appendix SIV), suggesting that $C_d^{2/3}$ or existing shape factors could be used
462 interchangeably. This is a promising result but further studies are required to identify appropriate
463 methods for estimating drag coefficients associated with complex pier-foundation geometries and
464 relate them to effects on local scour depths.

465 Some characteristic features of the proposed approach and associated limitations can be obtained
466 by discussing the relation between the normalized scour depth y_{se}/a and the flow intensity U/U_c .
467 The experimental data reported in Fig. 6a indicate that, in presence of negligible sediment gradation
468 effects (i.e. $\sigma_g < 2$), the shift from clear-water to live-bed conditions is abrupt, with y_{se}/a reaching
469 a clear-water peak and then sharply decreasing as U/U_c exceeds 1 (Melville and Chiew 1999).
470 As the degree of sediment non-uniformity increases, the occurrence of armoring delays the onset
471 of live-bed conditions to higher flow intensities and reduces the peak scour depth, resulting in a
472 smoother transition (Melville 2008). In Fig. 6b we plot the trend of y_{se}/a predicted with our
473 formula as function of U/U_c for increasing values of σ_g . These curves were obtained by applying
474 Eqs. 13-14 to hypothetical laboratory experiments performed in a flume with adjustable slope such
475 that the water depth can be kept constant as the bulk flow velocity increases, hence U_c can be kept
476 constant while U/U_c increase progressively (see the caption of Fig. 6 for details).

477 A visual comparison of Figs. 6a and 6b clearly shows that our formula effectively replicates
478 the observed trends. As the data exhibit an evident jump in the transition between clear-water
479 and live-bed conditions (i.e., at $U/U_c \approx 1$), the formula introduces a discontinuity to reflect this
480 behavior. It should be noted that such a discontinuity, particularly evident for uniform sediments,

481 is probably unphysical and may well be an experimental artifact, partly due to the long-standing
482 difficulties in defining an effective critical condition of incipient sediment motion, here quantified
483 with U_c (see e.g. Pähtz et al. 2020). In reality, the onset of live-bed conditions and its effects
484 on scour are probably diffused in a sizable transition band (e.g. $0.9 < U/U_c < 1.5$), rather than
485 immediately after the flow intensity exceeds 1. However, this is hard to observe from experimental
486 data (Fig. 6a) and consequently to model (Fig. 6b). Classical approaches tend to avoid this issue
487 by neglecting the discontinuity present in the data and systematically overestimating scour depth
488 within the transition region (see, e.g., Melville 1997; Sheppard et al. 2014). A less conservative
489 and more realistic approach, would be to modulate Φ (Eq. 8) with e.g. a decay function forcing a
490 smooth transition to $S_e' = 1$ within a narrow range of U/U_c approaching unity. However, we believe
491 that the potential improvement in predictive accuracy would not justify the increased complexity
492 associated with the introduction of ad-hoc transition functions, also given that most engineering
493 applications involve well-developed live-bed conditions. Nevertheless, for particular applications
494 falling within this narrow transitional range, we recommend estimating y_{se} by applying both the
495 clear-water and live-bed formulations, and adopting the larger of the two estimates in order to
496 reduce the risk of underestimations.

497 For $\sigma_g \geq 2$, Eq. 14 accounts for the effect of armoring by introducing two more discontinuities
498 in the behavior of y_{se}/a versus U/U_c . The first discontinuity is associated with the onset of live-
499 bed conditions and the initiation of sediment motion ($U/U_c = 1$), as discussed above. The second
500 discontinuity coincides with the critical armoring condition ($U/U_{c,d84} = 0.8$). For $U/U_{c,d84} < 0.8$,
501 the armored layer cannot be eroded, resulting in scour estimates consistent with those for clear-
502 water conditions. Within this range, we validated Eq. 14a with the very few available experimental
503 data (see Fig. 3b). While the model performs reasonably well, due to the small size of the dataset
504 employed, it may present some limitations when applied to different datasets. Above the critical
505 armoring condition (i.e. for $U/U_{c,d84} \geq 0.8$), the high sediment transport rates mask any armoring
506 effect and the scour depth is modeled as if sediments were uniform (Eq. 14b).

507 Clearly, as per the transition between clear-water and live-bed conditions, also these two dis-

508 continuities result from the definition of sharp boundaries between different behaviors, which serve
509 well modeling purposes but do not necessarily align with the physics of sediment transport. Also
510 in these cases, when engineering applications require estimating local scour around these discon-
511 tinuities, it is important to proceed with caution and adopt the most conservative estimate derived
512 from the surrounding values.

513 **SUMMARY AND CONCLUSION**

514 This study builds upon the works of Manes and Brocchini (2015) and Coscarella et al. (2020)
515 and introduces a novel formula (or set of formulas) for predicting the equilibrium scour depth
516 at bridge piers. With respect to previous works, the proposed approach accounts for effects of
517 low relative coarseness in live-bed conditions and sediment non-uniformity. Regarding the effects
518 of low relative coarseness, we demonstrated that the dedicated corrective function introduced by
519 Coscarella et al. (2020) for clear-water conditions can be reliably applied to live-bed conditions
520 too. The influence of non-uniform sediments and, consequently, the occurrence of armoring, is
521 modeled via a sediment gradation function $f_3(\sigma_g)$ and introducing the critical armoring threshold
522 concept for live-bed conditions.

523 The proposed formula demonstrates improved accuracy over existing leading methods in re-
524 producing a large amount of experimental data from laboratory studies. It also shows enhanced
525 performance when applied to selected field datasets, particularly where sufficient information on
526 pier geometry allows for confident application. Unlike traditional approaches, which tend to provide
527 conservative scour depth estimates, this formula is tailored for risk assessment rather than design
528 scenarios. However, when including an appropriate safety margin, it can be adapted for design
529 applications as well. For example, adding the value of the pier diameter a to the predicted scour
530 depth allows for conservatively overestimating all laboratory data and more than 95% of the field
531 data.

532 At a fundamental level, future refinements to the proposed approach should focus on: (i)
533 clarifying the influence of permeability on turbulent momentum transport and the initiation of
534 sediment motion within scour holes in coarse sediment beds; (ii) improving current methods to

535 incorporate sediment non-uniformity effects and expanding experimental datasets to cover a broader
536 range of grain size distributions (higher values of σ_g); and (iii) developing reliable techniques for
537 estimating the drag coefficient (C_d) to capture the effects of complex pier and foundation geometries.

538 DATA AVAILABILITY STATEMENT

539 All data, models, and code generated or used during the study appear in the submitted article.

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546 SUPPLEMENTAL MATERIALS

547 Appendixes SI (with Fig. S1), SII, SIII and SIV (with Fig. S2 and Table S1) are reported in
548 a unique supplemental material document, available online in the ASCE Library (ascelibrary.
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TABLE 1. Range of the main experimental variables for the datasets analyzed in the present work.

Source	a (m)	U (m s ⁻¹)	U_c (m s ⁻¹)	h (m)	d_{50} (mm)	σ_g	$y_{se,m}$ (m)
Clear-water, uniform sediments							
Ettema (1980)	0.03-0.24	0.20-1.21	0.24-1.27	0.02-0.60	0.24-7.80	1.10	0.02-0.35
Dey et al. (1995)	0.06-0.08	0.17-0.26	0.22-0.27	0.04-0.05	0.26-0.58	1.30	0.05-0.10
Melville (1997)	0.02-0.77	0.21-0.33	0.31-0.39	0.05-0.20	0.80-0.90	1.40	0.02-0.44
Melville and Chiew (1999)	0.04-0.08	0.18-0.32	0.33-0.41	0.05-0.20	0.96	1.40	0.03-0.14
Sheppard et al. (2004)	0.11-0.91	0.39-0.76	0.45-1.02	0.17-1.90	0.80-2.90	1.40	0.23-1.39
Ettema et al. (2006)	0.06-0.41	0.46	0.54	1.00	1.05	1.10	0.11-0.44
Pandey et al. (2018)	0.07-0.12	0.28-1.01	0.32-1.24	0.08-0.20	0.40-11.3	1.20	0.02-0.16
Live-bed, uniform sediments							
Shen et al. (1969)	0.15	0.29-1.02	0.26-0.29	0.11-0.27	0.24	1.40	0.13-0.21
Chee (1982)	0.05-0.10	0.30-1.20	0.26-0.47	0.10	0.24-1.40	1.20-1.30	0.05-0.14
Chiew (1984)	0.03-0.05	0.31-1.84	0.27-0.73	0.17-0.34	0.24-3.20	1.18-1.33	0.03-0.10
Sheppard and Miller (2006)	0.15	0.55-2.16	0.25-0.41	0.20-0.43	0.27-0.84	1.30	0.17-0.30
Clear-water, non-uniform sediments							
Ettema (1976)	0.10	0.35-0.96	0.37-1.01	0.60	0.55-4.1	1.60-4.60	0.04-0.22
Molinas (2004)	0.02-0.22	0.20-0.50	0.45-0.57	0.19-0.40	0.55-0.75	2.24-3.40	0.01-0.21
Sheppard et al. (2004)	0.11-0.92	0.29-0.31	0.34-0.35	1.19-1.81	0.22	1.50	0.17-0.97
Raikar and Dey (2005)	0.08	0.76-1.10	0.79-1.16	0.25	4.10-14.30	1.51-2.93	0.03-0.19
Guo et al. (2012)	0.03-0.14	0.28-0.38	0.36-0.45	0.20	0.46-0.89	2.10-2.50	0.02-0.13
Live-bed, non-uniform sediments							
Shen et al. (1969)	0.15-0.92	0.38-0.66	0.30-0.35	0.18-0.67	0.46	2.20	0.17-0.67
Chiew (1984)	0.03-0.07	0.35-1.92	0.34-0.45	0.17	0.60-1.45	2.00-5.50	0.01-0.12
Field data							
Mueller and Wagner (2005)	0.30-5.49	0.21-2.41	0.35-1.75	0.21-22.50	0.40-7.50	1.60-9.00	0.31-7.19
Froelich (1988)	2.41	0.95	0.54	3.44	0.80	2.30	2.78

TABLE 2. Performance comparison for different subsets of the dataset

	Our formula	S/M	HEC-18	SM06	F88	M97
Clear-water, uniform sediments						
$\overline{E_m}$ (%)	22.7	34.5	77.0	28.7	48.9	36.0
SSE (%)	4.3	3.2	8.0	3.3	9.4	14.0
NS	0.9	0.9	0.9	0.9	0.9	0.8
P_{out} (%)	27.1	40.9	45.2	35.1	34.0	26.1
Live-bed, uniform sediments						
$\overline{E_m}$ (%)	11.3	32.1	79.1	32.1	16.2	52.4
SSE (%)	2.9	7.9	39.7	7.9	4.0	35.9
NS (%)	0.9	0.7	<0	0.7	0.9	<0
P_{out} (%)	2.4	49.2	91.8	49.2	15.0	84.1
Clear-water, non-uniform sediments						
$\overline{E_m}$ (%)	24.6	214.8	186.5	194.8	202.3	165.2
SSE (%)	8.6	103.9	67.9	95.5	88.1	109.2
NS	0.8	<0	<0	<0	<0	<0
P_{out} (%)	31.9	92.4	88.2	90.8	88.2	94.1
Live-bed, non-uniform sediments						
$\overline{E_m}$ (%)	21.2	75.0	137.8	77.7	43.6	69.6
SSE (%)	2.1	31.6	49.8	29.1	72.0	153.5
NS	1.0	0.4	0.1	0.5	<0	<0
P_{out} (%)	30.6	61.1	93.5	67.6	46.3	87.0
Full dataset						
$\overline{E_m}$ (%)	19.0	75.2	109.2	70.1	66.4	72.0
SSE (%)	4.5	18.0	22.3	16.9	22.2	37.3
NS	0.9	0.7	0.6	0.7	0.6	0.4
P_{out} (%)	20.4	57.0	77.3	56.1	40.2	69.0
Field data						
$\overline{E_m}$ (%)	92.7	236.1	135.4	232.8	315.4	327.4
SSE (%)	28.5	80.0	77.5	77.5	339.8	417.8
NS	0.4	<0	<0	<0	<0	<0
P_{out} (%)	54.4	76.5	67.6	72.0	83.8	92.6

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670 Data collected in live-bed conditions with both uniform and nonuniform sediments

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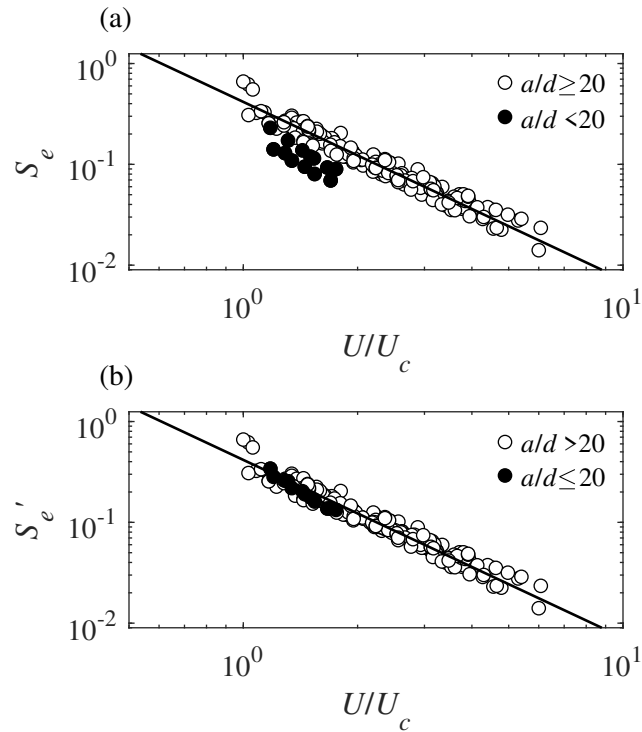


Fig. 1. (a) Live-bed scour function for uniform sediments S_e [Eq. (3)] and (b) corrected live-bed scour function for uniform sediments S'_e , versus flow intensity U/U_c [Eq. (9)]. All the data points appearing in this figure pertain to experiments carried out in live-bed conditions with uniform sediments (Table 1). The solid line represents the flow intensity function $\Phi(U/U_c)$ [Eq. 8].

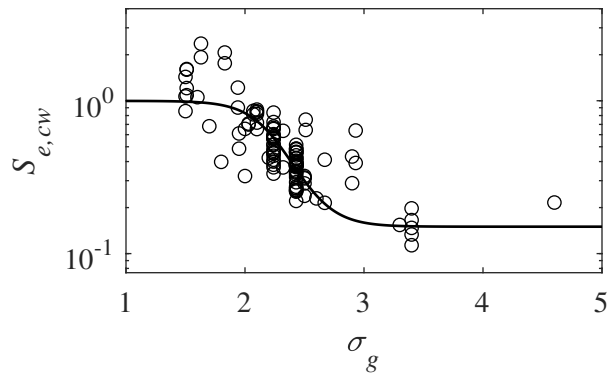


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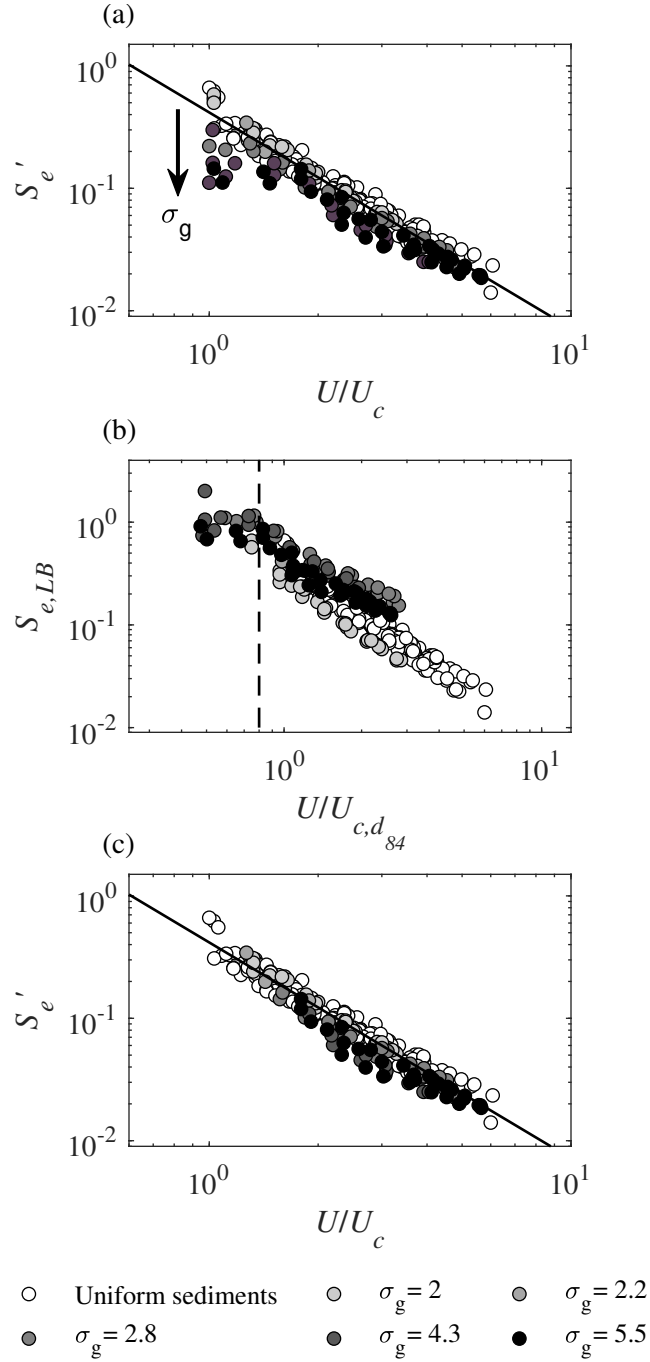
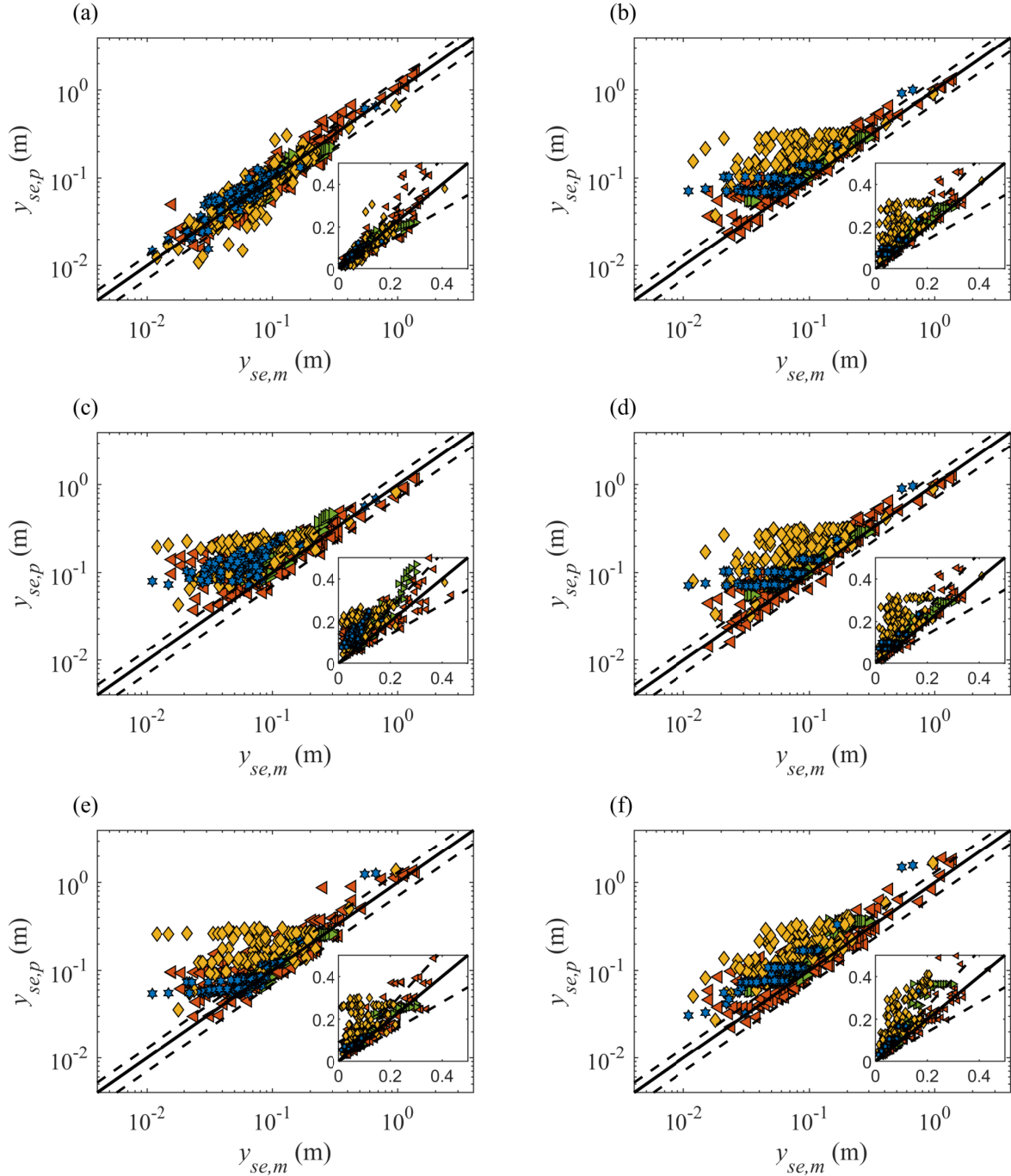


Fig. 3. (a) Corrected live-bed scour function for uniform sediments S_e' versus flow intensity U/U_c [Eq. (9)]; (b) live-bed scour function for nonuniform sediments $S_{e, LB}$ versus $U/U_{c, d_{84}}$ [Eq. (12)]; and (c) corrected live-bed scour function for uniform sediments S_e' calculated for data with $U/U_{c, d_{84}} \geq 0.8$ versus flow intensity U/U_c [Eq. (9)]. Data collected in live-bed conditions with both uniform and nonuniform sediments are used (Table 1). The dashed line in (b) represents the critical armor condition $U/U_{c, d_{84}} = 0.8$, while the solid line in (a) and (c) the flow intensity function $\Phi(U/U_c)$ [Eq. 8].



◄ CW, uniform sediment ◄ LB, uniform sediment ◆ CW, non-uniform sediment ★ LB, non-uniform sediment

Fig. 4. (a) Measured equilibrium scour depth ($y_{se,m}$) versus predictions ($y_{se,p}$) according to Eqs. (13) and (14); (b) S/M; (c) HEC-18; (d) SM06; (e) F88; and (f) M97. All the data from laboratory experiments reported in Table 1 were used. The solid lines represent a 1:1 agreement, whereas the dashed lines represent the 30% error bounds. In the insets, zoomed-in views of the same graphs are shown in linear coordinates.

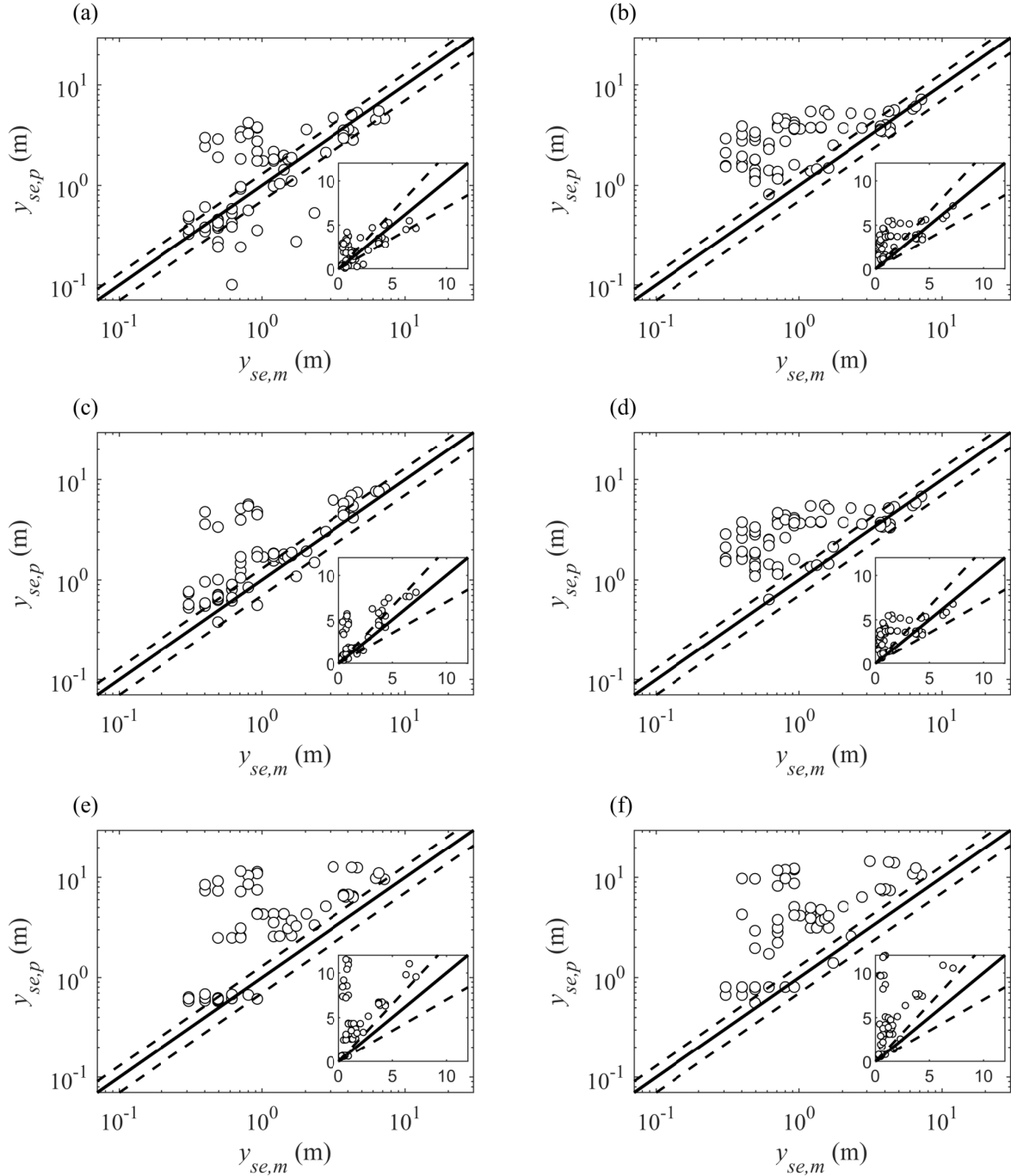


Fig. 5. (a) Measured equilibrium scour depth ($y_{se,m}$) versus predictions ($y_{se,p}$) according to Eqs. (13) and (14); (b) S/M; (c) HEC-18; (d) SM06; (e) F88; and (f) M97. All the data from field measurements reported in Table 1 were used. The solid lines represent a 1:1 agreement, whereas the dashed lines represent the 30% error bounds. In the insets, zoomed-in views of the same graphs are shown in linear coordinates.

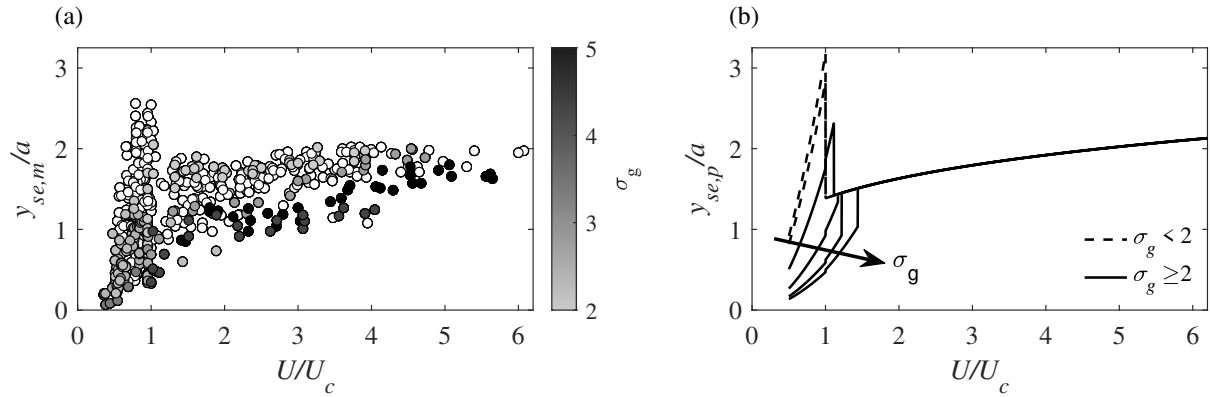


Fig. 6. (a) Normalized measured scour depth of equilibrium $y_{se,m}/a$; and (b) normalized predicted scour depth of equilibrium $y_{se,p}/a$ predicted with Eqs. (13) and (14) versus flow intensity U/U_c . In (a) all the data reported in Table 1 have been used. In (b) we apply Eqs. (13) and (14) to ideal laboratory experiments with $h = 0.25$ m, $a = 0.15$ m, $d_{50} = 2$ mm, $\sigma_g = 1, 1.8, 2.2, 2.5, 2.8, 4.5$ and $U = 0.5U_c - 7U_c$, with U_c calculated from d_{50} as explained in the main text.