Doctoral Dissertation
Doctoral Program in Aerospace Engineering (35^{th}cycle)

Performance Analysis of Direct and Indirect Formulations of the Legendre Pseudospectral Method for the Optimization of Spacecraft Trajectories

By

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Politecnico di Torino
2023

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2023

* This dissertation is presented in partial fulfillment of the requirements for **Ph.D. degree** in the Graduate School of Politecnico di Torino (ScuDo).

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Pseudospectral methods have gained significant popularity as a state and control parameterization technique, finding extensive use in various optimal problems due to improved CPU performance and their versatile nature. Depending on the global polynomials used for the approximation of the controls and state variables, different pseudospectral methods are presented in literature. This work focuses on Legendre Pseudospectral methods, where Lagrange polynomials, obtained from orthogonal Legendre polynomials, are employed to globally interpolate state and control variables; the collocation can then be applied using Legendre-Gauss-Lobatto, Legendre-Gauss-Radau or Legendre-Gauss points.
This thesis aims at assessing the accuracy, efficiency, and applicability of pseudospectral methods to orbital transfers and formation reconfiguration problems while using a limited number of discretization points. This feature is essential to reduce the computational effort and enhance the practical implementation of these methods.
In this thesis, both direct and indirect formulations of the pseudospectral method are considered. The direct formulation employs Legendre-Gauss-Lobatto or Legendre-Gauss-Radau points for collocation, while the indirect Legendre-Gauss-Lobatto pseudospectral method is developed to cope with the difficulty of the direct method to approximate a non-smooth function with a finite series of smooth functions and it implements a smoothing technique to improve its convergence rate.
The pseudospectral methods yield discrete-time values for the state and control variables that fulfill the discretized constraints, producing a discrete-time feasible solution. The Bellman method is employed here to validate the practical application of the solutions to continuous-time dynamics and verify optimality and accuracy. The Bellman method allows for the optimization problem to be solved recursively, moving the initial conditions toward the final ones. The trajectory is partitioned into discrete segments, and the states within each segment are integrated using discrete control interpolations. It is analyzed how the quantity of segments affects the ultimate states reached after the final integration and whether they meet the final

boundary conditions specified by the problems.
Contributions of this thesis include analysis and numerical comparison of different direct and indirect formulations of Legendre pseudospectral method. For all the approaches it has been studied the effect of the number of nodes on the control law reconstruction, highlighting their positive and negative aspects. Furthermore, the implementation of Bellman algorithm enables the conversion of discrete solutions into continuous-time solutions, thereby verifying the applicability and practicality of the discrete control law in real scenarios. These methods prove to be a viable approach for efficiently and accurately solving orbital transfer and formation reconfiguration problems with low-thrust.