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Keywords: CUF, Structural Theories, Neural Networks, FEM.

Abstract. This study examines the use of Convolutional Neural Networks (CNN) to determine the optimal structural theories to adopt for the modeling of composite shells, to combine accuracy and computational efficiency. The use of the Axiomatic/Asymptotic Method (AAM) on higher-order theories (HOT) based on polynomial expansions can be cumbersome due to the amount of Finite Element Models (FEM) virtually available and the problem-dependency of a theory's performance. Adopting the Carrera Unified Formulation (CUF) can mitigate this obstacle through its procedural and lean derivation of the required structural results. At the same time, the CNN can act as a surrogate model to guide the selection process. The network can inform on the convenience of a specific set of generalized variables after being trained with just a small percentage of the results typically required by the AAM. The CNN capabilities are compared to the AAM through the Best Theory Diagram (BTD) obtained using different selection criteria: errors over natural frequencies or failure indexes.

Introduction

The modeling of composite structures involves the balancing between accuracy and computational costs. Focusing on 2D models, using refined theories based on higher-order polynomial thickness expansions [1,2] is particularly useful in describing crucial aspects such as transverse anisotropy and shear deformability. However, their accuracy is strictly problem-dependent, and their variety is virtually unlimited.

The Axiomatic/Asymptotic Method [3-6] can be used to identify the best models for different levels of numerical complexity. The AAM starts with selecting a maximum order for the polynomial expansion and then gradually suppresses its terms. The resulting models are then compared to a reference using a control parameter, e.g., displacements or frequencies. Different theories can emerge as optimal for the same structural problem depending on the control parameter chosen. Implementing the AAM may be cumbersome due to the vast number of results required, becoming even less manageable for complex structures. The numerical efficiency of AAM can be augmented by Machine Learning (ML), more specifically through Convolutional Neural Networks [7]. By exploiting feature extraction capabilities, CNN can create surrogate models that identify the best theories at a fraction of the cost required by the AAM. This result is achieved by drastically reducing the FEM analyses needed to train the network successfully.

The new methodology presented in this paper is based on the Carrera Unified Formulation (CUF) [6] for deriving the finite element results used to train the network and to perform the

comparison with the outcome expected from the AAM. For the structural case presented here, the best models were selected based on the accuracy in estimating different natural frequencies and failure indexes, with the results summarized through the Best Theory Diagram (BTD).

CUF and FEM Formulation

CUF efficiently obtains the governing equations and the finite element formulations for virtually any higher-order theory. For shells, the displacement field is expressed as

$$u(\alpha, \beta, z) = F_\tau(z) u_\tau(\alpha, \beta) \qquad \tau = 1, \dots, M \tag{1}$$

$F_\tau(z)$ are the expansion functions adopted along the thickness, $u_\tau(\alpha, \beta)$ is the vector of the generalized unknown displacements, and M is the total number of expansion terms. The Einstein notation is used on τ. As an example of displacement field formulation stemming from a higher-order theory, a complete fourth-order model (E4) is reported herein extended format,

$$\begin{aligned} u_\alpha &= u_{\alpha_1} + z u_{\alpha_2} + z^2 u_{\alpha_3} + z^3 u_{\alpha_4} + z^4 u_{\alpha_5} \\ u_\beta &= u_{\beta_1} + z u_{\beta_2} + z^2 u_{\beta_3} + z^3 u_{\beta_4} + z^4 u_{\beta_5} \\ u_z &= u_{z_1} + z u_{z_2} + z^2 u_{z_3} + z^3 u_{z_4} + z^4 u_{z_5} \end{aligned} \tag{2}$$

From the geometrical and constitutive relations described in [8], and by applying the Principle of Virtual Displacements (PVD), the governing equation for the free-vibration problem can be derived for the k-th layer:

$$m^k_{\tau i s j} \ddot{u}^k_{\tau i} + k^k_{\tau s i j} u^k_{\tau i} = 0 \tag{3}$$

$k^k_{\tau s i j}$ and $m^k_{\tau i s j}$ are 3x3 matrices known as fundamental nuclei of the stiffness and mass matrices, respectively. By assembling all nodes and elements and introducing the harmonic solution, the complete formulation of the eigenvalue problem can be obtained,

$$(-\omega_n^2 M + K) U_n = 0 \tag{4}$$

Using the same approach for the static case, the governing equation reads:

$$k^k_{\tau s i j} u^k_{\tau i} = p^k_{s j} \tag{5}$$

Here, p^k_{sj} is the fundamental nucleus for the external mechanical load. Similarly , the well-known static problem formulation is derived through the assembly procedure.

$$K U_n = P \tag{6}$$

For a more in-depth description of the assembly procedure and other mathematical details, the reader can refer to [8].

Axiomatic/Asymptotic Method

AAM selects the optimal set of expansion terms, or generalized variables, to adopt for a specific problem configuration. The aim is to find the most convenient structural theory to provide the best accuracy at the lowest computational cost. Dealing with polynomial expansions, this procedure's

first step is defining the maximum order allowed. For the work presented in this paper, a maximum order of four was considered, leading to a total amount of possible theories equal to 2^{15}. This number was reduced to 2^{12} by always considering the constant terms for each of the three displacement components. The accuracy of each model can be evaluated by choosing a reference solution, e.g., the full fourth-order expansion, E4, and a control parameter. The first one considered in this paper was the percentage error over the single natural frequency, defined as follows:

$$\%E_{f_i} = 100 \times \frac{|f_i - f_i^{E4}|}{f_i^{E4}} \tag{7}$$

where f_i^{E4} is the i-th frequency evaluated using the reference full fourth-order Taylor expansion. The second indicator adopted was the percentage error over a failure index evaluated at a specific location in the structure. The 3D Hashin criterion [9] was selected for this purpose. For example, the percentage error over the index for the matrix tension (MT) mode is

$$\%E_{MT} = 100 \times \frac{|MT - MT^{E4}|}{MT^{E4}} \tag{8}$$

The outcome of this selection procedure is summarized by the BTD, a graphical representation of the accuracy achievable by varying the number of generalized variables adopted. Each point of the BTD corresponds to the best theory, given the number of active expansion terms.

Convolutional Neural Network

A CNN able to handle multi-dimensional inputs and outputs was used. The input is represented by the set of active generalized variables identifying a specific structural theory. This information is first encoded into a series of 0 and 1, corresponding to a deactivated and active term, respectively. An example of this procedure is presented here,

$$\begin{aligned} u_\alpha &= u_{\alpha_1} + z u_{\alpha_2} + z^4 u_{\alpha_5} \\ u_\beta &= u_{\beta_1} + z u_{\beta_2} + z^3 u_{\beta_4} \\ u_z &= u_{z_1} + z u_{z_2} + z^2 u_{z_3} \end{aligned} \quad \Rightarrow \quad [111001010100] \tag{9}$$

Note that the three constant terms were not included in the sequence because they were always considered active. This sequence is then re-shaped into a 3x4 matrix, constituting the actual input to the network. The output consists of the percentage errors over the first ten natural frequencies or the two evaluated failure indexes. The complete architecture is presented in Table 1. The training of the network was performed only using 10% of all possible theories and related errors.

Table 1. Parameters and architecture of the adopted CNN.

Layer Type	Filters (Size) / Neurons	Activation Function
Convolutional	128 (3x3)	ReLU
Convolutional	128 (3x3)	ReLU
Convolutional	128 (3x3)	ReLU
Flatten	-	-
Dense	128	ReLU
Dense	128	ReLU
Output	10 or 2	Sigmoid

Numerical Results

A simply-supported shell with [0°/90°/0°] stacking sequence was considered. The curvature radii were kept equal along the two curvilinear coordinates α and β, imposing R/a=5. A thickness ratio a/h=10 was used, and the following material properties were employed: $E_{11}/E_{22}=25$, $G_{12}/E_{22}=G_{13}/E_{22}=0.5$, $G_{23}/E_{22}=0.2$, $\upsilon_{12}=\upsilon_{13}=\upsilon_{23}=0.25$. Only a quarter of the shell was modeled to reduce computational costs further. This choice required the use of symmetry boundary conditions, thus allowing to consider symmetrical vibration modes exclusively. A 4x4 mesh of Q9 elements was adopted.

Starting with the free-vibration problem, Figs. 1 and 2 show the BTDs for the first and third natural frequencies, respectively. In each of them, the results obtained from the direct application of the AAM are compared to those obtained by the CNN. Table 2 shows the best theories provided by CNN for the first five frequencies having eight active degrees of freedom; an active term is indicated by a black triangle, with their order increasing from left to right, up to the fourth.

In the case of failure indexes as control parameters, a static analysis was performed on the same structure. A bi-sinusoidal pressure of unit amplitude was applied to the top surface. Failure indexes were evaluated at the center of the top and bottom edges, for the compressive and tensile modes, respectively. Figure 3 shows the resulting BTD. In each diagram, some of the resulting best models are presented.

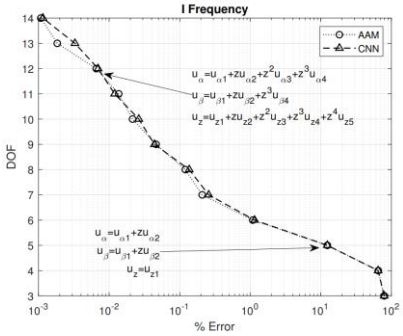

Figure 1. BTD for the 1st frequency.

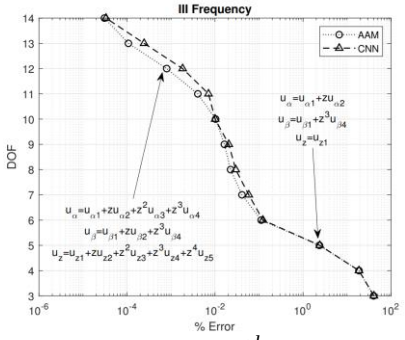

Figure 2. BTD for the 3rd frequency.

For the free-vibration case, the CNN accurately reproduced the BTD for different frequencies while also correctly providing indications regarding the relevance of specific terms. The results obtained over the failure indexes show similar levels of accuracy, with just a slight reduction when dealing with larger amounts of degrees of freedom. This behavior is related to the influence of each expansion term on the various stress components involved in the failure index estimation.

Table 2. Best models with eight active terms for the first five frequencies.

I						II						III					
u_α	▲	▲	△	▲	△	u_α	▲	▲	△	▲	△	u_α	▲	▲	△	△	△
u_β	▲	▲	△	▲	△	u_β	▲	▲	△	▲	△	u_β	▲	▲	△	▲	△
u_z	▲	△	▲	△	△	u_z	▲	△	△	△	▲	u_z	▲	▲	△	△	▲

IV						V					
u_α	▲	▲	△	▲	△	u_α	▲	▲	△	▲	△
u_β	▲	▲	△	▲	△	u_β	▲	▲	△	▲	△
u_z	▲	△	△	△	▲	u_z	▲	△	△	△	▲

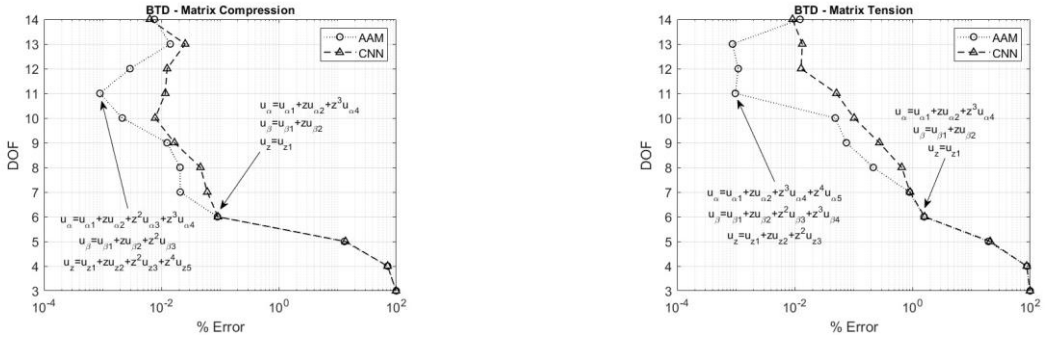

Figure 3. BTD based on the failure index

Summary

This paper explores the use of Convolutional Neural Networks in the analysis of composite shells. Focusing on higher-order theories obtained through polynomial expansions, CNN can identify the best models for various structural configurations with a fraction of the computational overhead required by the Axiomatic/Asymptotic Method. This new efficient approach can be extended to different families of problems maintaining consistent precision levels in providing the optimal modeling strategy.

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