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2D finite elements for the computational analysis of crack propagation in brittle materials and the handling of double discontinuities

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Abstract

Crack growth simulations by way of the traditional Finite Element Method claim progressive remeshing to fit the geometry of the fracture, severely increasing the computational effort. Methods such as the eXtended Finite Element Method (XFEM) allow to overcome this limitation by means of nodal shape functions multiplied by Heaviside step function to enrich finite element nodes. Through the medium of a discontinuous field, the entire geometry of the discontinuity can be modelled regardless of the mesh, avoiding remeshing. In this paper two shell-type XFEM elements (a three-node triangular element and a four-node quadrangular element) to evaluate crack propagation in brittle materials are presented. These elements have been implemented into the widespread opensource framework OpenSees to evaluate crack propagation into a plane shell subjected to monotonically increasing loads. Moreover, in the perspective of fracture propagation simulations, the problem of managing multiple cracks without remeshing or operating subdivisions on the integration domain has been investigated and a four-node quadrangular finite element for the computational analysis of double crossed discontinuities by the means of equivalent polynomials is presented in this paper. Equivalent polynomials allow to overcome inaccuracies on the results when performing standard numerical integration (e.g. Gauss-Legendre quadrature rule) over the entire domain of XFEM elements, without the need of defining integration subdomains. The presented work and the computational strategy behind it may be extremely useful not only in the field of fracture mechanics, but also to solve complex geometry problems or material discontinuities.

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Keywords: Extended Finite Element Method ; Discontinuities ; Equivalent Polynomials

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1. Introduction

The eXtended Finite Element Method (XFEM) is a versatile approach for the analysis of problems characterised by discontinuities and singularities such as localised deformations, material discontinuities or cracks. It was first proposed by Belytschko et al. (1999), and later improved by Moës et al. (1999).

In XFEM formulation the discontinuous displacement field is modelled along the crack surface through additional nodal degrees of freedom and enrichment shape functions.

XFEM allows to define the finite elements mesh independently to the discontinuity position and does not require any mesh refinement close to the discontinuities: that is a major advantage with respect to the standard Finite Element Method. Moreover, when using XFEM in crack propagation problems, remeshing during the analysis to track the evolution of the crack is not needed, dramatically decreasing the computational effort.

Since enrichment shape functions are discontinuous and non-differentiable, numerical problems arise if a quadrature rule (e.g. Gauss-Legendre) is used to evaluate the stiffness matrix of elements containing discontinuities. This problem can be over-come by partitioning these elements into sub-elements, so that the integrands are continuous and differentiable into each subdomain. Alternatively, a solution by means of equivalent polynomials that does not require partitioning of the integration domain has been proposed by Ventura (2006) and by Ventura et al. (2015).

In this paper, the implementation into OpenSees of a three-node triangular and a four-node quadrangular shell XFEM elements is presented. These elements are an enhancement of the finite elements with drilling degrees of freedom recently presented by the authors (Fichera et al. (2019)). The proposed elements are able to model crack propagation in brittle materials and have been used to perform static incremental analysis on plane shells.

2. XFEM formulation overview

The Extended Finite Element Method (XFEM) is a numerical method, based on the Finite Element Method (FEM), that is especially designed for handling discontinuities (Belytschko et al. (1999), Moës et al. (1999)).

In standard FEM, the displacement field of a single element of a domain Ω can be expressed as:

$$\mathbf{u}(\mathbf{x}) = \sum_{i=1}^{n} N_i(\mathbf{x})\mathbf{u}_i = \mathbf{N}^T(\mathbf{x})\mathbf{u} \tag{1}$$

where n is the number of nodes of the element, $N_i(\mathbf{x})$ are the element shape functions and $\mathbf{u}_i$ are the nodal displacement components.

Eq. (1) cannot describe the behaviour of the displacement field when discontinuities or singularities exist within the element. To overcome this limit, one can enrich the interpolation on Eq. (1) by means of an enrichment function $\Psi(\mathbf{x})$ and a certain number of additional degrees of freedom $\mathbf{a}_i$:

$$\mathbf{u}(\mathbf{x}) = \sum_{i=1}^{n} N_i(\mathbf{x})\mathbf{u}_i + \sum_{i=1}^{n} N_i(\mathbf{x})\Psi(\mathbf{x})\mathbf{a}_i \tag{2}$$

The nature of the enrichment function $\Psi(\mathbf{x})$ depends on the nature of the discontinuity that has to be described. In the case of a strong discontinuity in the displacement field (discontinuities in the solution variable of a problem, e.g. a crack), the most appropriate enrichment function is the Heaviside step function:

$$\Psi(\mathbf{x}) = H(\varphi(\mathbf{x})) \tag{3}$$

$$H(\varphi(\mathbf{x})) = \begin{cases} -1 & \varphi(\mathbf{x}) < 0 \\ 1 & \varphi(\mathbf{x}) > 0 \end{cases} \tag{4}$$

where $\varphi(\mathbf{x})$ is the signed distance of the discontinuity from the evaluation point.

XFEM formulation allows to adequately represent discontinuities or singularities in a suitable way and with strong performance in case of a pronounced non-polynomial behaviour of the solution.

It has to be noted that if the element is crossed by a discontinuity, the standard Gauss quadrature rule cannot be used, due to the non-polynomial nature of the enrichment function. This problem can be overcome by splitting the integration domain Ω in two parts Ω^- and Ω^+ along the discontinuity, so that the enrichment function is continuous and differentiable into each integration subdomain:

$$\int_{\Omega} \Psi(\mathbf{x})\mathscr{P}_n(\mathbf{x})\, d\Omega \Longrightarrow \int_{\Omega^-} \Psi(\mathbf{x})\mathscr{P}_n(\mathbf{x})\, d\Omega + \int_{\Omega^+} \Psi(\mathbf{x})\mathscr{P}_n(\mathbf{x})\, d\Omega \tag{5}$$

($\mathscr{P}_n(\mathbf{x})$ is a generic n-degree polynomial function, e.g. a term of the element stiffness matrix). Partitioning into sub-domains introduces a sort of 'mesh' condition in the elegance of the XFEM formulation. A technique to eliminate the requirement of sub-cells generation without introducing any approximation in the quadrature by means of equivalent polynomials has been proposed by Ventura (2006) and Ventura et al. (2015). It has been demonstrated that an equivalent polynomial function exists such that its integral gives the exact values of the discontinuous/non-differentiable function integrated on sub-cells. The polynomial is defined in the entire element domain, so that it can be easily integrated by Gauss quadrature, and no quadrature sub-domains have to be defined. Eq. (5) thus becomes:

$$\int_{\Omega} \Psi(\mathbf{x})\mathscr{P}_n(\mathbf{x})\, d\Omega \Longrightarrow \int_{\Omega^-} \Psi(\mathbf{x})\mathscr{P}_n(\mathbf{x})\, d\Omega + \int_{\Omega^+} \Psi(\mathbf{x})\mathscr{P}_n(\mathbf{x})\, d\Omega = \int_{\Omega} \tilde{\Psi}(\mathbf{x})\mathscr{P}_n(\mathbf{x})\, d\Omega \tag{6}$$

where $\tilde{\Psi}(\mathbf{x})$ is the equivalent polynomial function.

In the case of strong discontinuities, Heaviside function is usually used as enrichment function. Eq. (6) thus becomes:

$$\int_{\Omega^-} H(\mathbf{x})\mathscr{P}_n(\mathbf{x})\, d\Omega + \int_{\Omega^+} H(\mathbf{x})\mathscr{P}_n(\mathbf{x})\, d\Omega = \int_{\Omega} \tilde{H}(\mathbf{x})\mathscr{P}_n(\mathbf{x})\, d\Omega \tag{7}$$

where $\tilde{H}(\mathbf{x})$ is the equivalent polynomial function for this particular case.

Equivalent polynomials avoid the quadrature domain splitting at the cost of doubling the polynomial degree of the integrand function.

3. Handling multiple discontinuities using equivalent polynomials

Analysing a body containing multiple fractures is not an uncommon problem in fracture mechanics. Such problems can be still addressed by means of XFEM formulation, but quadrature domain splitting becomes more burdensome. Moreover, the equivalent polynomials law defined in Eq. (7) is able to take into account a single discontinuity for each integration domain Ω. To overcome this problem, a new equivalent polynomials formulation to manage double discontinuities has been recently proposed by the authors.

Let us examine a body Ω and let us assume it is split in four parts by the discontinuity lines q and r, as shown in Fig. 1. Let us define Ω_A the partition obtained when the normal of both discontinuities have a positive value of their b

component, therefore both normal vectors point inwards Ω_A. Starting from Ω_A, the remaining partitions (Ω_B, Ω_C and Ω_D) are defined counterclockwise by convention.

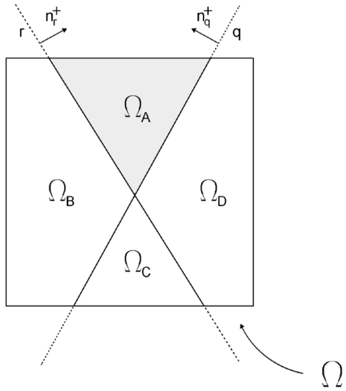


Fig. 1: 2D domain Ω crossed by two discontinuity lines: q and r.

Let us assume a n-degree polynomial $\mathscr{P}_n(\boldsymbol{x})$ to be integrated over the subdomains Ω_A, Ω_B, Ω_C or Ω_D obtained partitioning a parallelogram with two lines q and r, so that:

$$I_i = \int_{\Omega_i} \mathscr{P}_n(\boldsymbol{x})\, \mathrm{d}\Omega_i \tag{8}$$

where $i = \{A, B, C, D\}$

To solve the integral in Eq. (8) for each value of i it is required to define each domain of integration Ω_i. This is not always an easy task because integration domains may result in rather not trivial polygonal shapes. Said problem has been solved by the authors finding an equivalent polynomial $\tilde{H}_i$ that allows, for each partition Ω_i, to perform the integration over the entire domain Ω of the element with a traditional quadrature rule without partitioning Ω into subdomains. So, Eq. (8) becomes:

$$I_i = \int_{\Omega_i} \mathscr{P}_n(\boldsymbol{x})\, \mathrm{d}\Omega_i = \int_{\Omega} \tilde{H}_i(\boldsymbol{x}) \mathscr{P}_n(\boldsymbol{x})\, \mathrm{d}\Omega \tag{9}$$

The equivalent polynomial $\tilde{H}_i(\boldsymbol{x})$ depends on the equations of the two discontinuity lines q and r and has the same degree of $\mathscr{P}_n(\boldsymbol{x})$.

4. XFEM shell-type elements implementation into OpenSees

In this paper, two XFEM shell-type elements to solve fracture mechanics problems using OpenSees have been presented. The versatility of OpenSees framework made the implementation process fairly straightforward. First of all, a new 'node' Class (alternative to the original one in the code) had to be defined to dynamically change the number of degrees of freedom of each node during the analysis. This is a crucial part in order to comply the solution for the displacement field in Eq. (2).

The introduction of a new Class of nodes made possible the subsequent implementation of two new XFEM plane shell-type elements: a three-node triangular element and a four-node quadrangular element. In this first implementation, the in-plane behaviour of the proposed elements is indefinite linear elastic in the case of compression and elastic-fragile in the case of tensile stress. The out-of-plane behaviour is always linear elastic.

A pre-existing fracture can be assigned to the elements. It can be defined by means of a crossing point and the normal to the discontinuity itself. The elements can be also defined as initially undamaged; in this case they may crack during the incremental loading process.

When the principal tensile stress overcomes the material tensile resistance, a fracture will arise in the element. The fracture position is defined by means of the coordinates of the element point where this limit is exceeded.

The proposed elements can also be used to study the formation and the consequent propagation of fractures in brittle materials. Elements are modelled so that the displacement field is described by the interpolation law in Eq. (1) if they are undamaged and by the one in Eq. (2) if cracking arises. Thus, the number of degrees of freedom of the nodes will increase as the analysis progresses, since the enriched degrees of freedom $\mathbf{a}_i$ will be added to the standard degrees of freedom $\mathbf{u}_i$.

At the current state of the implementation, each element of the mesh can handle just one discontinuity. Elements able to handle multiple discontinuity has been analysed and will be introduced in the upcoming developments. Also, a non-linear compressive behaviour for the elements will be included.

5. Numerical applications

The proposed XFEM elements have been used for the analysis of plane shells containing discontinuities.

5.1. Damaged cantilever beam

In the first example the cantilever beam shown in Fig. (2) is analysed. The beam contains a pre-existing fracture and it is subject to two constant forces applied to its free end: axial force F_x and cross-sectional force F_y. Geometrical

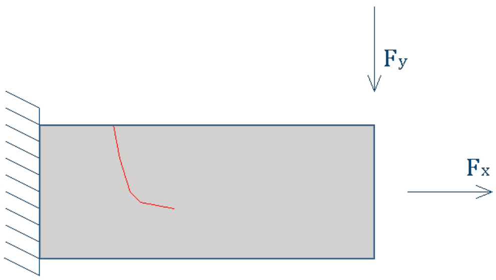


Fig. 2: Damaged cantilever beam subject to constant forces F_x and F_y. Fracture in the structural element is highlighted in red.

and mechanical properties of the beam are shown in Tab. (1, 2).

Table 1: Geometrical properties of the cantilever beam.

Geometrical properties		
$L = 50$ $[cm]$	$b = 2.5$ $[cm]$	$h = 20$ $[cm]$

The beam has been modelled both using standard FEM shell-type elements and the proposed XFEM shell-type quadrangular elements. The discretisation mesh for the structural element is shown in Fig. (3). In the case of standard FEM (Fig. (3b)), the mesh had to be refined near the fracture to follow the geometry of the discontinuity, so triangular finite elements and distorted quadrangular finite elements had to be used.

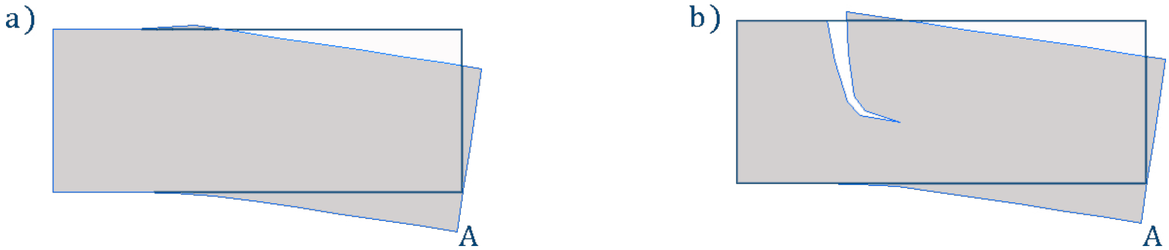
Table 2: Mechanical properties of the cantilever beam.

Mechanical properties			
$E = 2796\ [kN/cm^2]$	$\nu = 0.2$	$F_x = 20\ [kN]$	$F_y = 20\ [kN]$

(a) XFEM discretisation (b) Standard FEM discretisation.

Fig. 3: Mesh discretisation for the cantilever beam.

The deformed configurations obtained from the analysis are shown in Fig. (4). It is clear that both the standard FEM model and the XFEM one yield the exact same results in terms of displacement. In particular, the deflection of point A is the same in both models, as shown in Tab. (3).



(a) XFEM discretisation (b) Standard FEM discretisation.

Fig. 4: Results of the analysis in terms of deformed configurations.

Table 3: Deflection of point A.

Standard FEM Model	Proposed XFEM Model
$u_x = 0.00037\ [cm]$ $u_y = 0.00177\ [cm]$	$u_x = 0.00037\ [cm]$ $u_y = 0.00177\ [cm]$

These results validate the proposed elements.

5.2. Undamaged cantilever beam – progressive cracking

In the second example an undamaged cantilever beam has been analysed. The behaviour of the beam is assumed to be linear elastic in case of tensile stress and elastic-fragile in the case of compression. The material tensile resistance is defined as $f_t = 1.5\ [kN/cm^2]$.

In this case, an analysis with a monotonically increasing cross-sectional load F_y has been performed. The results of this analysis, until the collapse is reached, are shown in Fig. (5). It has to be noted that the XFEM model does not have the aim of following the exact crack propagation path, but to determine the displacement field of a structural element that is subject to progressive cracking. Thus, the fracture scheme in Fig. (5) is just indicative.

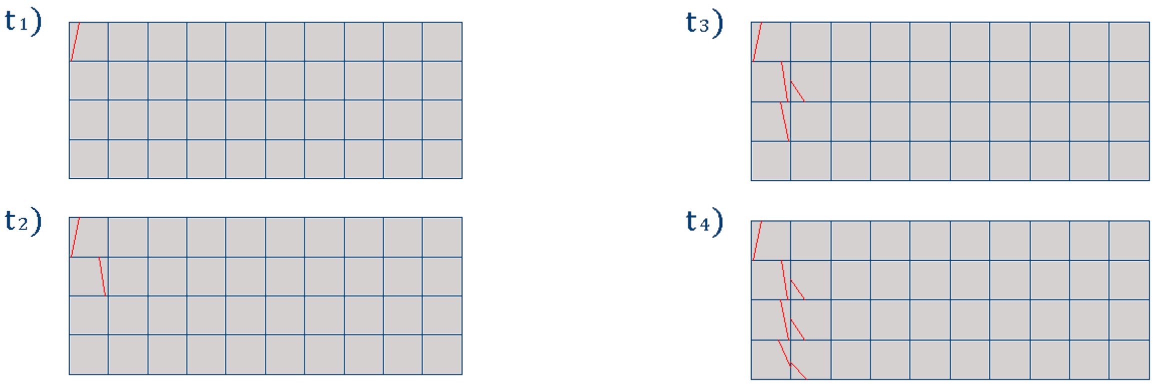


Fig. 5: Cantilever beam subject to a monotonically increasing cross-sectional load. Progressive cracking arises as the load increases with the time-step until failure is reached.

5.3. Undamaged hinged beam – progressive cracking

Finally, the undamaged hinged beam shown in Fig. (6) has been studied. As in the previous example, the beam is

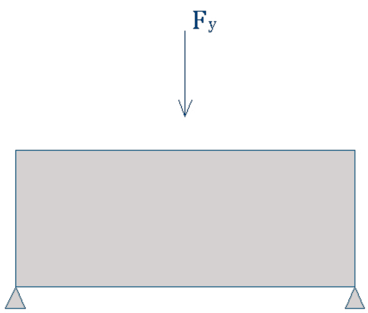


Fig. 6: Undamaged hinged beam subject to a monotonically increasing cross-sectional load F_y.

subject to a monotonically increasing force applied in the centreline. The results of the analysis are shown in Fig. (7), with the evolution of the XFEM elements under progressive cracking.

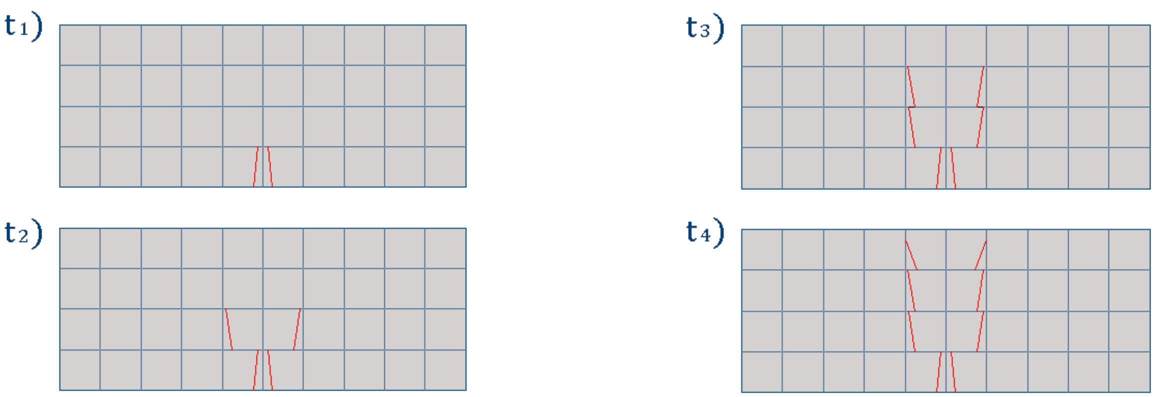


Fig. 7: Progressive cracking arises as the load increases with the time-step until failure is reached.

6. Conclusions

In this paper a method to integrate over an entire integration domain containing two discontinuities without splitting it into multiple subdomains has been presented. This method is an enhancement of the solution by means of equivalent polynomials proposed by Ventura (2006) and Ventura et al. (2015) and it can be a powerful tool in the context of XFEM, when addressing multiple fracture problems.

Also two shell-type XFEM elements have been presented: a three-node triangular element and a four-node quadrangular element. These elements have been implemented into OpenSees in order to evaluate crack propagation in brittle materials. The proposed XFEM elements are an enhancement of the finite elements with drilling degrees of freedom recently presented by the authors Fichera et al. (2019). The proposed XFEM elements have been used to evaluate crack propagation into a plane shell subject to monotonically increasing loads. The results presented in the numerical applications validate the proposed formulation and enable the use of OpenSees framework to address fracture mechanics problems.

Future developments for the proposed elements will include the handling of multiple discontinuities by a single XFEM element, as well as a non-linear compressive behaviour.

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