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Making Palladio Digitally Explicit: Geometrical Parameters in Door's Ornaments

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Abstract

The Book I of Andrea Palladio's "The Four Books on Architecture", in which, after a brief discourse on the five orders and on those rules which are essential to building, private houses, streets, bridges, squares, xysti, and temples are discussed, contains the rules of Palladio's design. Over the years, many scholars have investigated how to make explicit the reasoning on the theory of architectural proportions in Palladio. A multitude of studies were done concerning five orders proportions and about the compositive rules of Villas. This ongoing research aims to make the treatise of Andrea Palladio explicit visually and mathematically by starting from his text and images. The study, at this stage, concentrates on proportional and graphical analysis for door's ornaments. The study made it possible to identify the molding elements' philological construction and the relationship between their geometric construction made up of alignments, recurring intervals, equal subdivisions, and successive partitions.

Keywords Historical treatise · Andrea Palladio · Ornament · Geometric analysis · Modeling · Modules · Proportion · Proportional analysis · Proportional systems · Ratio · Rule-based architecture · Shape grammars

Introduction

Book I of Andrea Palladio's oeuvre *I quattro libri dell'architettura* entitled, *di quelli auertimenti, che sono più necessarij nel fabricare*, contains the rules of Palladio's design. The literature on Palladio's oeuvre is significant. Some of these studies have investigated Palladio's architectural theory of proportions. Most of the research aimed to discover geometrical evidence and grammar systems on Palladio's villas and buildings (Mitrović and Djordjević 1990; Wassell 1999;

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Burns 2000). In parallel, the classical orders of architecture and their proportion rules were analyzed, e.g., by comparing various treatise solutions (Von Mauch and Normand 1998; Chitham 2005). The work of Chitham and its comparative orders offers a graphic visualization of different solutions for orders considering the most important actors of the classical architecture panorama: Vitruvius, Serlio, Vignola, Palladio, Scamozzi, Perrault, and Gibbs.

Andrea Palladio's rules of proportions for architectural orders have been analyzed and encoded to define geometrical construction rules for the three-dimensional modelling of architectural elements (Migliari 1991; Beltramini and Gaiani 2012). The digital edition of *I quattro libri dell'architettura* is also in this direction and offers the possibility to read the text and visualize the 3D model of the architectural orders using a semantic subdivision of the parts (Baldissini and Gaiani 2014: 10–14).

Andrea Palladio's construction grammar is evident in its intention to define rules for the building construction of classical orders starting from the analysis and study of other treatises (e.g., Vitruvius). The intent of other studies was to systematize the complexity of a modular system where a predefined set of solutions for orders and proportions are used to set up villa's layouts (Stiny and Mitchell 1978; Mitchell 1990; Spallone and Calvano 2022). These studies aimed to identify and recognize the design *modus-operandi* of Palladio (Mitrovic 2004; Eisenman and Roman 2015). In this direction goes the development of a shape grammar based on visual reasoning where construction rules define the assemblage of physical objects in a 3D environment (Sass 2007; Apollonio et al. 2010, 2013; Spallone and Calvano 2022). Other studies refer to Palladio's Villa layouts to analyze the visitors' path using plan analysis like JPG (Ostwald 2011; Lee et al. 2022). Following these main topics, the need to define rules for conceptualization and description of classical architecture led to a generic formalism for semantic modelling and representation of architectural elements (De Luca et al. 2007). However, no studies have yet analyzed the geometrical structure behind the moldings that are the core of Palladio's orders. In the description of the orders (Palladio 1570: 15–51), Palladio reports the proportions between elements and their sequence, but it is only in the description of door and windows ornaments (Palladio 1570: 56–59) that Palladio address the varieties of geometric construction of moldings.

The aim of this research is to analyze, understand, and represent Palladio's moldings grammar using graphical analysis and the latest three-dimensional technologies based on visual programming language (VPL) and Historical Building Information Modelling (HBIM) to make his treatise (Palladio 1570) explicit in a visual and mathematical way.

A textual and graphical analysis of pages 56 to 59 of Book I (Fig. 1) is first presented to understanding the geometrical construction of doors and windows ornaments allowed to:

- identify the geometrical construction for each ornament's solutions represented in pages 57 and 59 of Book I
- clarify and compare the geometrical construction process for molding definitions

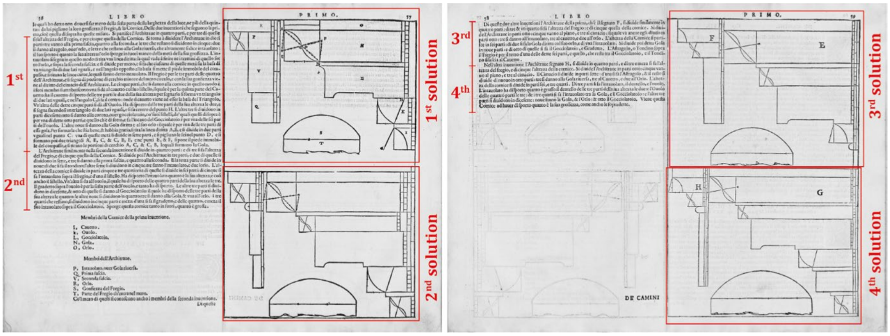


Fig. 1 Annotations of Palladio's ornaments. Image: graphic overlay by the author on Palladio 1570: 56–69

Second, after identifying the geometrical constructions for molding elements, the dimensions of each molding (height, width) were made explicit using mathematical syntax to:

- create algorithms that generate a set of molding profiles for each ornament's solutions represented in pages 57 and 59 of Book I
- generate an HBIM, parametric, 3D model of each ornament's solutions according to the best proportions of doors including minimum and maximum dimensions suggested by Palladio.

This study analyzed Palladio's drawings on pages 57 and 59 in detail by identifying their geometric construction. The text accompanying the drawings makes it easier to read the proportional relationships in height between the moldings. Within these descriptions, Palladio declares the ratio and dimensions of some moldings. For others, the dimensional value of the overhang was deducted. This study starts with a graphic and geometrical analysis.

The analysis of drawings and their descriptions allowed one to identify the sequence with which the elements were drawn, their ratio, their dimensions, and their geometrical construction genesis.

Chapters XXV-XXVI on the Dimensions of Doors and Windows and Their Ornaments

The current study was made using the anastatic edition of *I quattro libri dell'architettura* (Palladio 1570) The volume is printed by Ulrico Hoepli Editore in 2006 and offers a well detailed overview of Palladio's original drawings.

For terminological issues regarding the ornaments, the study refers to the MIT Press edition of *The Four Books on Architecture / Andrea Palladio* translated by Robert Tavernor and Richard Schofield in 1997.

In chapter XXV, Palladio describes the proportions of doors and windows. Palladio exhorts by stating that there is no precise rule for sizing doors and windows. Palladio classifies doors into:

– main doors (entrances)
– room doors

Referring back to Vitruvius, Palladio, just as he does for sizing architectural elements of ornaments, starts from the height of the rooms '*dal piano ò suolo alla superficie della travatura*' (Palladio 1570: 55). He then divides them into $3\frac{1}{2}$ parts and uses a value equal to 2 parts to size the height of the main door. The main door will have a dimensional ratio of 2:1(H:W).

Regarding room doors, Palladio defines their maximum ratio of $6\frac{1}{2}$:3 and a minimum ratio of 5:2 using the *piede* as the unit of measurement.

Windows have minimum and maximum dimensional values with respect to the size of the room to which they give light. Because there are rooms of different sizes in each house, the windows should have a breadth between $\frac{1}{4}$ and $\frac{1}{5}$ of the length of the room and their heights should be made by '*two squares and a sixt of their breadth*' (Palladio et al. 1997: 60).

All windows must be the same size for the entire floor, and Palladio suggests sizing windows according to the room that he believes has the proportionally best size: those with a ratio of 5:3 and dimensions in 30 and 18 *piedi*.

Later in the chapter, Palladio suggests a central alignment of windows, and he proposes a proportional and progressive reduction in the width of $\frac{1}{6}$ for each window for each level with respect to the window below.

Graphical Analysis

This research investigates the proportional relationship between the door's dimensions and its ornaments (Palladio 1570: 55–59) (Fig. 1).

Like the entablatures of the five orders, the ornament, or trim, around main doors and windows is divided into three main elements: the architrave, frieze, and cornice. The widths of the frieze and cornice are proportional to the width of the architrave. Proportions are also used for dimensioning all the moldings that compose each of the main elements. Palladio describes the sequence of moldings for architrave and cornice and suggests four methods. The first two methods are drawn on page 57 (Figs. 2, 3) with a graphical representation of the geometric construction for moldings and their proportional relationship (i.e., the little rectangles along the sides of the profile). A second possible option for both methods is drawn on page 59 (Figs. 4, 5).

Step-by-step, chapter XVI describes four methods considering the progressive geometric construction to design the various moldings. Each page includes two possible solutions that are represented by exploding the main architectural elements that make up the ornaments of doors and windows. Half of the page is used for each

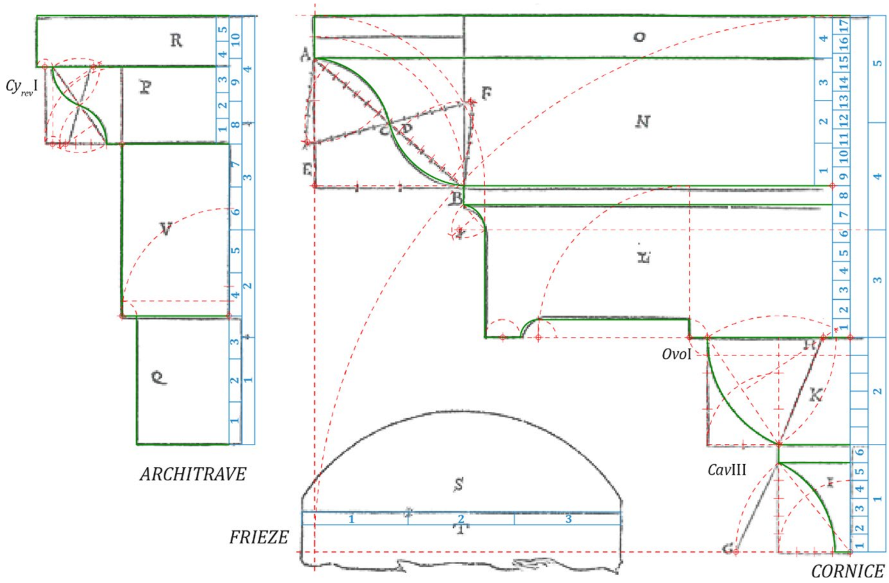


Fig. 2 Annotations of Palladio's ornaments, 1st design. Image: graphic overlay by the author on Palladio 1570: 57

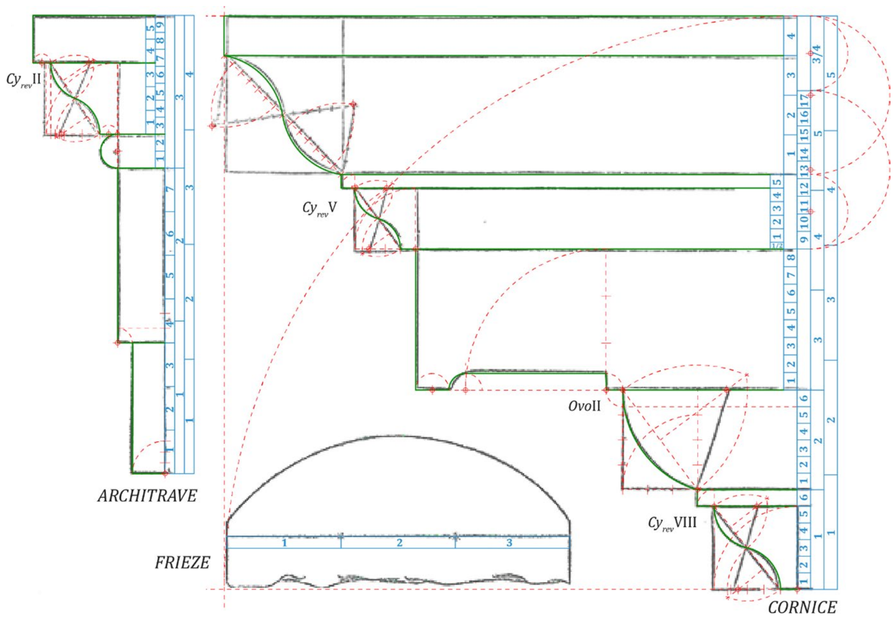


Fig. 3 Annotations of Palladio's ornaments, 2nd design. Image: graphic overlay by the author on Palladio 1570: 57

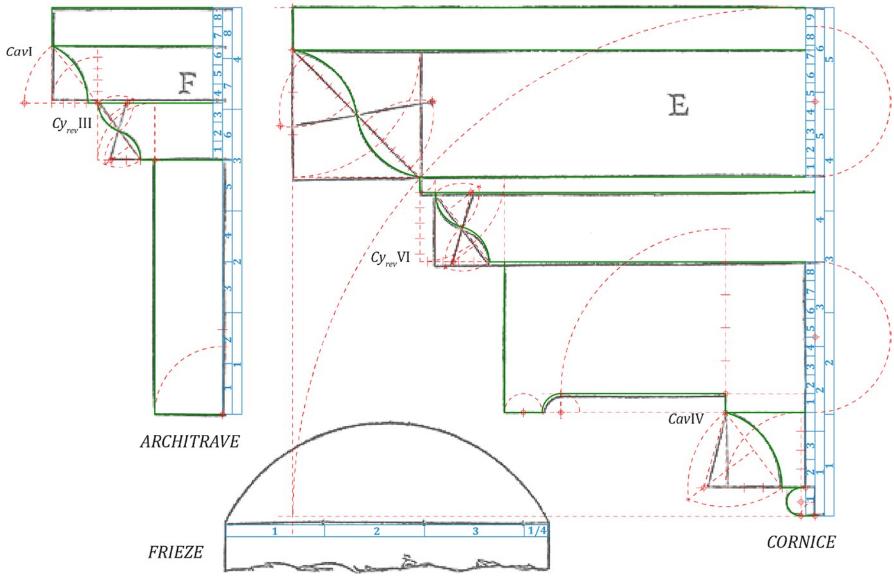


Fig. 4 Annotations of Palladio's ornaments, 3rd design. Image: graphic overlay by the author on Palladio 1570: 59

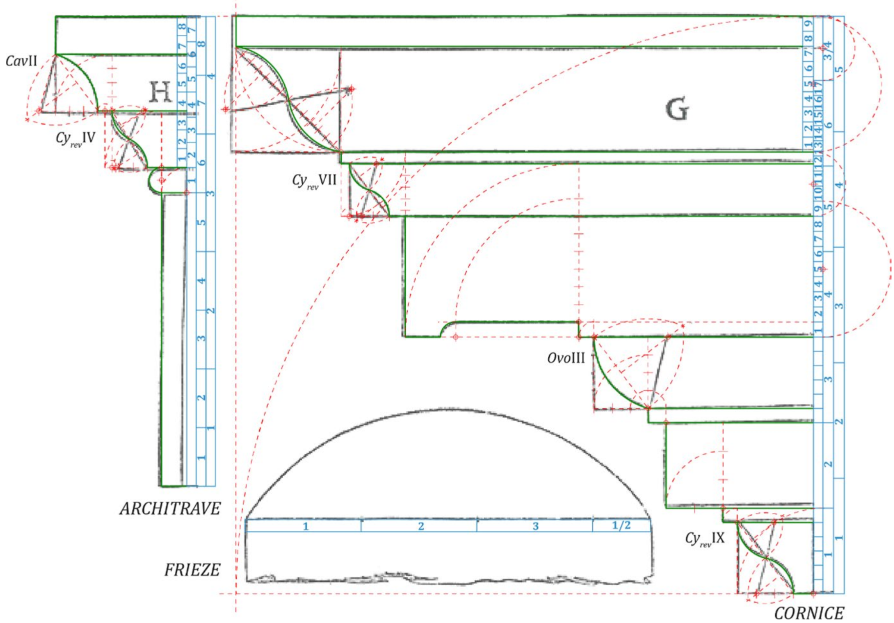


Fig. 5 Annotations of Palladio's ornaments, 4th design. Image: graphic overlay by the author on Palladio 1570: 59

solution. The *architrave* is placed on the *left*, the *frieze* is in the center and rotated 90 degrees, and the cornice is on the right.

The layout of the pages was filled to use as much space as possible giving importance to the *cornice* on the right side. The main module then drew dimensional changes for each solution. The intention of this contribution is, thus, to compare four solutions using the same dimensional module (Fig. 19).

The geometric construction of the first two ornamental solutions, the 1st and the 2nd (Palladio 1570: 57), are described punctually while concerning the second two solutions (3rd and the 4th) (Palladio 1570: 59). Many passages are omitted by referring to the previous explanations regarding their geometrical construction.

The 1st solution on the top of page 57 is comparable with the 3rd on the top of page 59. The same happens for the 2nd solution on the bottom of page 57 and the 4th on the bottom of page 59.

On both pages, moldings that characterize each main element can be identified. Their number is increased by 1 for each solution's proposal. The 1st *architrave* consists of five components: the 2nd of six (Palladio 1570: 57), the 3rd of six, and the 4th of seven (Palladio 1570: 59).

The definition of the moldings sequence and the heights of their elements are based on a main module and its successive subdivisions. The module size is in proportional relationship with the clear breadth of the door, which affects the height of *architrave*. This value is then sub-divided in four parts for each solution. The unit of the sub-division (module) is used to dimension the *frieze* and *cornice*. In solutions 1 and 3, the architrave consists of four elements. In solutions 2 and 4, the two *fasciae* on page 57's solutions are replaced by a single *piano*.

The height of the cornice always corresponds to five modules similar to its width (Ratio 1:1) while the height of the frieze varies from three modules (1st and 2nd solutions) to three-and-a-quarter modules (3rd solution) to three-and-a-half modules (4th solution) (Figs. 2, 3, 4, 5). The alignment of the frieze with the architrave is constrained between two construction lines that are in line with the second *fascia* or *piano* of the cornice on the right and with the *orlo* of the architrave on the left.

Comparative Analysis for the Geometric Construction of Moldings

Figures 2, 3, 4, 5 show the geometrical construction of each molding. These are overlayed with the original drawing of Palladio. For better readability, the original layout of elements was modified leaving enough space for the ruler used to define the proportions of the molding's elements. In this paragraph moldings are singularly analyzed. The analysis reveals that certain moldings follow different rules that can only be recognized through their graphical reconstruction in a geometrical way.

This happens for the *cyma reversa*, also known as *intavolato* in the architrave's construction and is accompanied above in the architrave with an *orlo* (Palladio 1570: 57) and *cavetto* (Palladio 1570: 59). The cornice with an upper *listel* offers multiple solutions of geometric construction (Palladio 1570: 57, 59).

In the 1st solution, after describing the proportional relationships between the three main elements, Palladio describes the parts that make up the *architrave*. The descriptive sequence follows the order of the elements starting from the bottom. However, the author usually follows the logic that would follow the punctual description of the drawing processes after introducing the first and second *fascia*. This then moves on to describing elements starting from the top: the *orlo* and then the *cyma reversa*. The order of description follows the order of geometrical construction—this is the reason why the construction sometimes starts from the bottom point on the right and sometimes starts from the point in the top left corner of the molding.

This generally happens for all solutions in the construction of a *cornice* as declared with a ratio of 1:1 in width and height. After the first construction of moldings on the bottom, Palladio directly re-starts on the top from the *orlo* and *cyma recta*.

As mentioned above, the construction of the *cyma reversa* requires particular focus because it changes depending on the solution; the description offered by Palladio is not very comprehensive. This explains why this paper reports all constructions for different solutions proposed by A. Palladio (Fig. 6). The same was done for both *cavetto* (Fig. 7) and *ovolo* (Fig. 8). The classified geometrical constructions were also annotated in Figs. 2, 3, 4, 5 to better understand where each solution was used.

Within the four solutions proposed for the definition of door ornament moldings, Palladio reports nine solutions for the *cyma reversa*. Each *architrave* has an element of this type with an overhanging *orlo* (1^{st} and 2^{nd} solutions) or combined with a *cavetto* (3^{rd} and 4^{th} solutions). In the case of the *orlo*, the relationship between the heights of both elements is a of ratio 3:2 (Cy_{rev}I in 1^{st} solution, Cy_{rev}II in 2^{nd} solution) while in the case of the *cavetto*, both elements have an equal height of 3:3 (Cy_{rev}III in 3^{rd} solution, Cy_{rev}IV in 4^{th} solution). The *cyma reversa* is part of the *architrave*, and its construction is based on the subdivision of the height into four sub-parts except for the Cy_{rev}I solution where the subdivision involves five sub-parts.

The definition of the sub-parts is used to define the relationship between the height and the extension of the element and in some cases also to define the extension of the upper *orlo* (Cy_{rev}I, Cy_{rev}II) or the beginning of the geometric construction of the *cavetto* (Cy_{rev}III, Cy_{rev}IV).

Within the *cornice*, the *cyma reversa* can be found either as first element, following the *frieze* (Cy_{rev}VIII in 2^{nd} solution, Cy_{rev}IX in 4^{th} solution) or as the element following the *gocciolatoio* (Cy_{rev}V in 2^{nd} solution, Cy_{rev}VI 3^{rd} solution, Cy_{rev}VII in 4^{th} solution). In the *cornice*, the *cyma reversa*, apart from the Cy_{rev}V and Cy_{rev}III, also includes in its construction an upper *listel* whose height is in relationship with the height of the *cyma reversa* with a ratio of 1: $4\frac{1}{2}$ (Cy_{rev}V, Cy_{rev}VI, Cy_{rev}VII) or 1:5 (Cy_{rev}VIII, Cy_{rev}IX).

In the case of Cy_{rev}V, the construction starts from the bottom upwards as in the Cy_{rev}VIII and Cy_{rev}IX solutions. As in the case of the *architrave*, the construction of the molding is based on the subdivision of the height into $5\frac{1}{2}$ (Cy_{rev}VI), 5 (Cy_{rev}VIII), $4\frac{1}{2}$ (Cy_{rev}IX), or 4 (Cy_{rev}V, Cy_{rev}VII) sub-parts. The sub-parts determine the extension of both the *cyma reversa* and the upper *listel*.

The geometric construction of the *cyma reversa* follows the steps as drawn in Fig. 6 (from left to right).

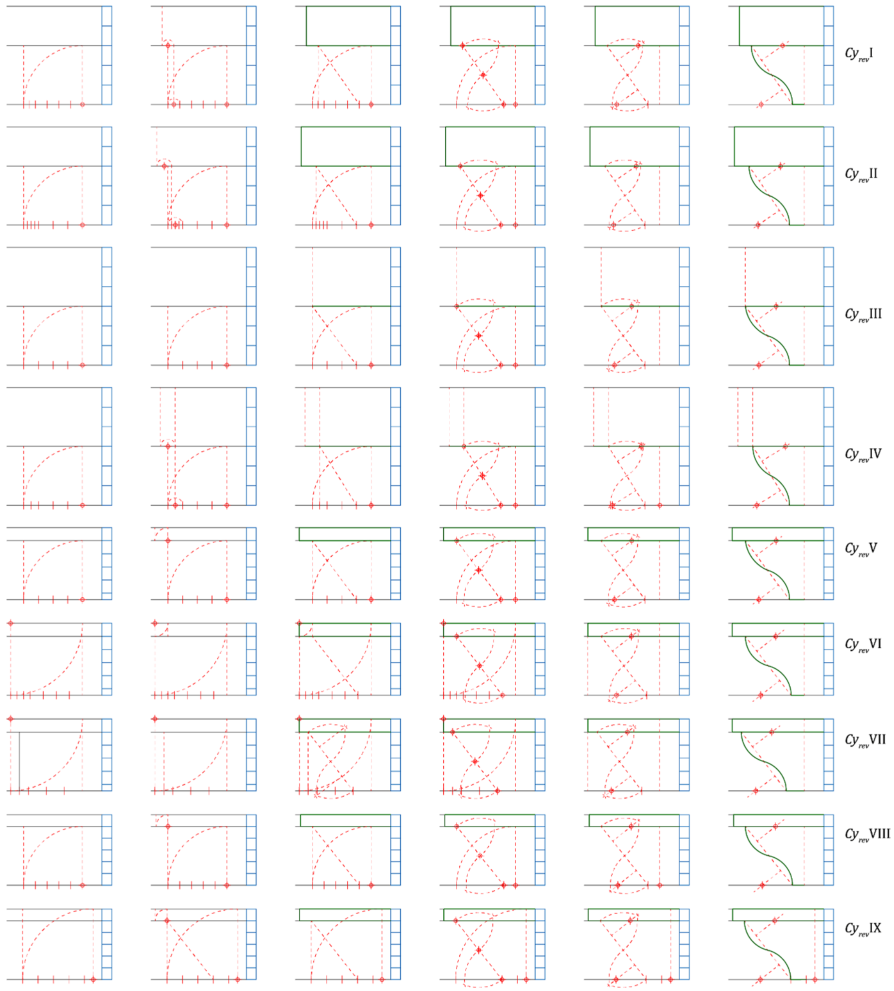


Fig. 6 Solutions for the geometrical construction of *cyma reversa*. Image: author

Similar to the *cyma reversa*, the *cavetto* also has different construction solutions (Fig. 7) and is found in both the *architrave* and the *cornice*. In the case of the *architrave*, the *cavetto's* height is in a proportional relationship with a ratio of 3:2 to the overlying *orlo* (*Cav*I in 3rd solution, *Cav*II in 4th solution).

In the case of the *cornice*, the element is in relationship with an upper *listel* with a ratio, in heights, of 1:5 (*Cav*III in 1st solution) and 1:4 (*Cav*IV in 3rd solution).

The geometric construction of the *cavetto* is based on a subdivision of the height excluding the *orlo* and the *listel* based on 4 or 5 sub-parts. The relationship between height and extension is, for most solutions, based on a ratio of 5:4 (*Cav*I, *Cav*III, *Cav*IV) and 1:1 (*Cav*II). In all the proposed solutions, the construction of the

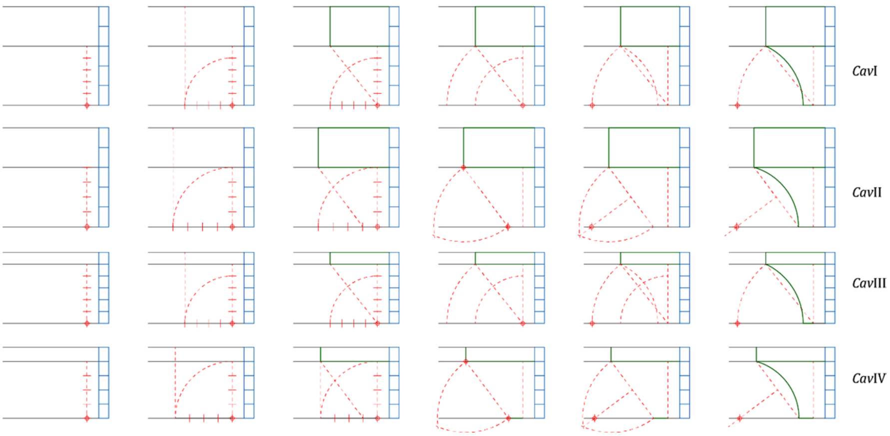


Fig. 7 Solutions for the geometrical construction of *cavetto*. Image: author

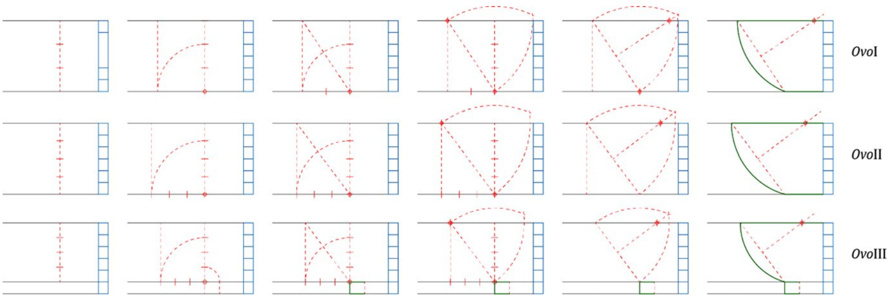


Fig. 8 Solutions for the geometrical construction of *ovolo*. Image: author

cavetto determines the extension and the construction of the *orlo* and the *listel* above it. The geometric construction of the *cavetto* follows the steps drawn in Fig. 7.

Finally, Palladio proposes three solutions for the geometric construction of the *ovolo* (Fig. 8). This element is only found within the elements of the *cornice* and below the *gocciolatoio*. In contrast with the previous solutions, the construction of the *ovolo* does not involve a direct relationship with an underlying *listel* because the element is located above a *cyma reversa* (*Ovo*I in 1st solution) or above a *cavetto* (*Ovo*II in 2nd solution).

In contrast, the *ovolo* is located above a *dentello* in the 4th solution. Its geometric construction also includes an underlying *listel* (*Ovo*III). In contrast to previously-described molding elements where the geometric construction defines the ratio between height and extension, the *ovolo* has a ratio stated directly by Palladio within the text. *Ovo*I has a ratio of 3:2; *Ovo*II and *Ovo*III have a ratio of 4:3. The geometric construction of the *ovolo* follows the steps drawn in Fig. 8.

Using Algorithms to Transcribe Palladio's Rules

This study bases itself on previously published works that analyze proportions (Mitrović and Djordjević 1990; Sass 2007) and then introduces recent computer technologies to develop algorithms that create complex three-dimensional geometries using different inputs. The algorithmic approach is similar to Palladio's way of thinking of Palladio on proportional rules (algorithms) and determined sets of values (input values).

The use of algorithms for the geometric modelling of architectural elements is nothing new, and the use of algorithms managed by software, based on a visual programming language (VPL), has rapidly grown in recent years. The VPL popularity is based on the simplicity of the interface (e.g., Grasshopper and Dynamo) that enables complex functions hidden within visual nodes to be solved. However, one of the most common drawbacks lies in losing control of what happens between the input and output of an algorithm—this is often composed of a succession of different visual nodes.

Rather, the methodological approach of this study focuses on reducing visual nodes in favor of making the mathematical formulae and functions they mask explicit. Within the VPL plugin for Revit and Dynamo used for this research, functions are made manifest according to the *DesignScript* -Dynamo textual language. Such functions can be grouped into a single node (*code block node*). This approach is intermediate between proper visual programming and the development of IronPhyton nodes generally used in the VPL environment and requires advanced programming skills.

In recent years, VPL tools have aligned themselves with the needs of various sectors including the architecture, engineering, and construction (AEC) sector where the use of building information modelling (BIM) has long been linked to managing the building construction process.

The BIM methodology can also be applied to built heritage and is defined as a new field of research related to HBIM aimed at defining workflows capable of managing the architectural heritage both from a geometric and informative point of view, using BIM platforms.

These systems and their capacity to handle heterogeneous data, including historical/archival resources, are the new frontiers for managing heritage especially in Italy where the presence of historical buildings with classical ornaments is pervasive. The VPL approach can be used for both geometrical and information modelling to systematize heterogeneous data regarding architectural heritage (Lo Turco et al. 2021; Calvano et al. 2022; Martinelli et al. 2022).

Hence, there is a need to recreate complex and parametric geometries within BIM software that are as faithful to reality as possible. VPL tools are commonly used to generate these shapes because they allow standard geometric modelling rules to be brought into BIM platforms that are still inflexible for ad-hoc three-dimensional modelling.

The creation of 3D objects modelled according to parametric rules can make them adaptive and thus reusable in multiple BIM projects. This has seen increasing use of VPL for generating 3D elements for HBIM (Paris and Wahbeh 2016; Capone and Lanzara, 2022).

In the context of HBIM, reusing certain architectural elements modelled for a given building is considered a *unicum*. Therefore it is possible to avoid constructing complex parametric rules and limits the three-dimensional modelling processes to source-based or image-based references that adequately represent the modelled object in its complexity (Lo Turco et al. 2022).

A Generic Formalism for the Moldings' Grammar Using a VPL Approach

This study follows the assumption that moldings are a combination of linear segments and arcs that can be ideally inscribed in a deformable 'bounding box' (De Luca et al. 2007: 187).

The methodology applied here is the best that suits the need of a Source-to-HBIM process. The algorithms require both a height and a width of a molding. This is a type of information that is generally available in the treatises and that is partially readable from drawings, survey drawings, and textual descriptions. This approach differs from the De Luca one that was primarily developed for a scan-to-model process where the input values for the definition of geometrical atoms are exported by data from photographs. The De Luca approach needs points instead of dimensional values.

The 'Palladio moldings set' algorithms developed here are based on the dimensions extrapolated from the geometrical construction used by Palladio to draw molding's profiles. The proportions of moldings, translated from Palladio's guidelines, generate a sequence of primitive elements (atoms) that follow numerical and geometrical proportions starting from the width of the door. In a digital environment, the sequence becomes the profile to sweep on a path (the door frame) to create a three-dimensional digital object.

This study's novelty with respect to the literature (Chitham 2005; De Luca et al. 2007) is to create a digital shape grammar for A. Palladio using VPL inside an HBIM platform to develop the first HBIM Palladio parametric library of elements.

This study uses some algorithms already created for similar purposes and entitled *VRIMnodes*: a set of 32 geometrical atoms generated using Dynamo textual programming and encoded with an alphanumerical code (Giovannini 2017a, b). Each geometrical atom (GA) is built on a deformable nine-point grid with a specified coordinate system orientation. The grid could be considered as a deformable '*bounding box*' where the height and width are mutable (Fig. 9). The profile and path of a solid 3D model of molding can be made by a single atom or by a sequence of atoms connected.

The necessity of working in different plans for profiles and paths explains why *VRIMnodes* are designed for both geometric definitions (profile/path) considering diverse coordinate systems: XZ and XY (Fig. 10). The coordinate system for each GA also defines the anchor point called *StartPt*. The *EndPt* is the last point of the curve or Polycurve that represents the geometry. The profile is a succession of these atoms anchored by their start points and end points.

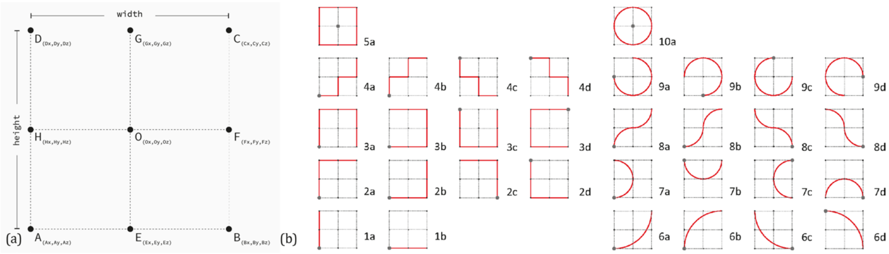


Fig. 9 **a** Structure of the nine grid-points of a geometrical atom (GA) according with De Luca et. al. 2007. **b** List of GA and their encoding in *VRIMnodes* algorithms. Image: author

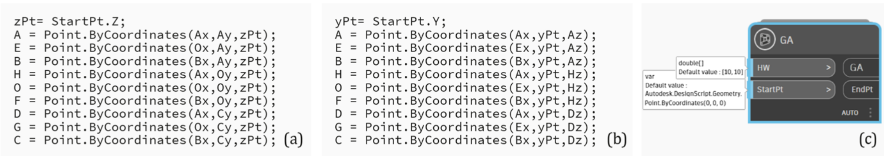


Fig. 10 The x,y,z coordinates for the nine grid-points of profiles (**a**) and paths (**b**). GA standard node appearance (**c**). Image: author

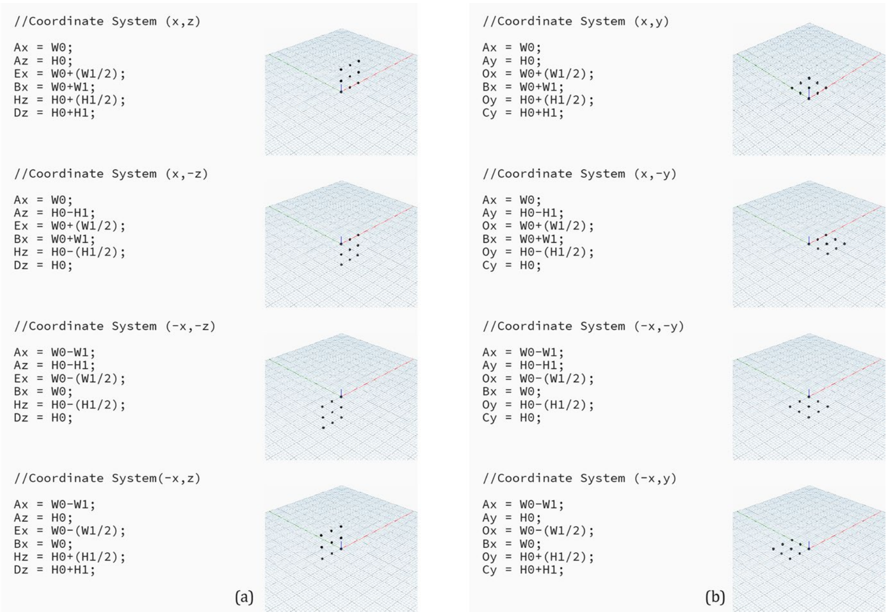


Fig. 11 Coordinate systems for the nine grid-points for profiles (**a**) and paths (**b**) according with height (H) and width (W) of StartPt and EndPt of the '*bounding box*' as input data. Image: author

The deformability of the points grid is defined by the coordinate system and how it modifies according to input data (Fig. 11).

AE	Molding	Ord.	R [H:W]	GA	Profiles of 1st design
Cornice	Orlo	12	5:6	1b	
	Cyma recta [*gola*]	13	*geom.*	8b	
	Listel (corona)	10	*geom.*	1a	
	Corona [*gocciolatoio*]	9	*geom.*	1b, 6d, 2b, 6b	
	Gradetto (corona)	11	1:1	2b	
	Ovolo	8	3:2	6a	
	Listel (cavetto)	7	3:2	1a	
	Cavetto	6		6b	
Frieze	-	5	*geom.*	7a	
Architrave	Orlo	3	*geom.*	1b, 2c	
	Cyma reversa [*intavolato*]	4	*geom.*	1b, 8a	
	Second Fascia	2	*geom.*	2b	
	First Fascia	1	*geom.*	2b	

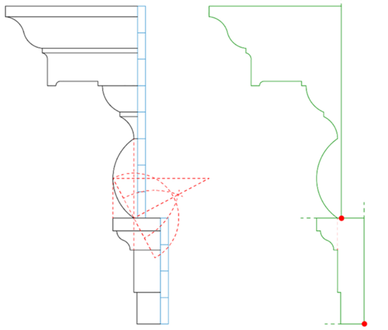


Fig. 12 First design for the ornaments of doors and windows based on Palladio 1570: 57. Image: author

To generate the solid of the required molding, a sweeping operation using path and profile is applied. The sweep requires a close profile; thus, the script includes a code that closes a series of GA into a unique profile or path.

The input data correspond to available information concerning the ‘*bounding box*’ measures. In this way, each Atom GA is placed in the workspace with the initial coordinate (x, z or x, y) called H0 and W0 and the other two measures of height and width of the ‘*bounding box*’ are called H1 and W1.

To define the geometry of each GA, one must deform the nine grid points expressed by each three-dimensional point and their specified x, y, z Cartesian coordinates using input values for height (H0, H1) and width (W0, W1) and declaring the coordinate system to set up the GA orientation. The solid could then be converted into an HBIM object using the DynamoRevit software interface.

Dimensional and Proportional Analysis for the Parametric Construction of Moldings

According to Palladio’s description of molding elements, Figs. 12, 13, 14, 15 enlightened the necessity to discretize some elements. The *Cyma reversa*, *cavetto*, and *tondino* have a predetermined overhang (width). The dimensional ratio was used to inscribe the geometrical construction of elements.

In the case of the *corona* profiles, Palladio does not mention the part that composes it but rather refers to different parts. Here, I decided to encode its elements according to the molding’s description from the 1728 ‘*Tab. Architecture*’ in the *Cyclopaedia* identifying the *gradetto*, the *drip*, the arch of the *drip*, the *larnier*, the *mouchette*, and its *listel* (Fig. 16).

Each Figure, from 12 to 15, has a schema for both profiles used for the creation of the ornament including the geometrical construction used to determine the connection between *architrave*, *frieze*, and *cornice*; otherwise, these were not perceivable in the Palladio layouts. ‘AE’ corresponds to the architectural element (architrave, frieze, cornice) to which the individual molding belongs.

AE	Molding	Ord.	R [H:W]	GA	Profiles of 2nd design
Cornice	Orlo	12	*geom.*	1b	
	Cyma recta [*gola*]	13	1:1	8b	
	Listel (cyma reversa)	14	1:1	2b	
	Cyma reversa [*intavolato*]	15	1:1	1b, 8a	
	Corona [*gocciolatoio*]	11	*geom.*	1b, 6d, 2b	
	Gradetto (corona)	10	1.1	2b	
	Ovolo	9	4:3	6a	
	Listel (cyma reversa)	8	1:1	2b	
	Cyma reversa [*intavolato*]	7	1:1	8a	
Frieze	-	6	*geom.*	7a	
Architrave	Orlo	5	*geom.*	1b, 2c	
	Cyma reversa [*intavolato*]	4	*geom.*	1b, 8a	
	Tondino	3	2:1	7a	
	Second Fascia	2	*geom.*	2b	
	First Fascia	1	*geom.*	2b	

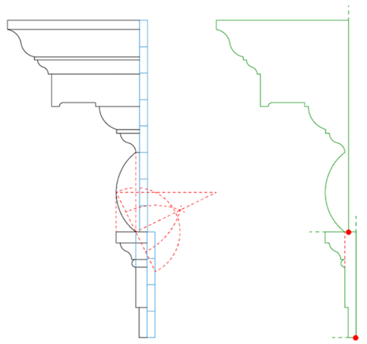


Fig. 13 Second design for the ornaments of doors and windows based on Palladio 1570: 57. Image: author

AE		Molding	Ord.	R [H:W]	GA	Profiles 3rd design
Cornice		Orlo	6	*geom.*	1b	
		Cyma recta [*gola*]	7	1:1	8b	
		Listel (cyma reversa)	8	1:1	2b	
		Cyma reversa [*intavolato*]	9		1b, 8a	
		Corona [*gocciolatoio*]	10	*geom.*	6d, 2b	
		Gradetto (corona)	11	*geom.*	2a	
		Cavetto	13	1:1	1b, 6b	
		Tondino	12	2:1	1b, 7a	
Frieze		-	5	*geom.*	7a	
Architrave	*Cimacio*	Orlo	4	*geom.*	2c	
		Cavetto	3	5:4	1b, 6b	
		Cyma reversa [*intavolato*]	2	1:1	1b, 8a	
	Fascia [*piano*]		1	*geom.*	2b	

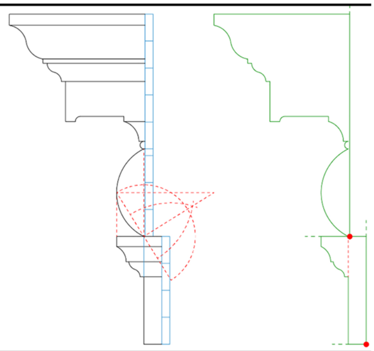


Fig. 14 Third design for the ornaments of doors and windows based on Palladio 1570: 59. Image: author

Part of the dimensional and proportional analysis was the identification of the GA associated with each molding. Here, the GA corresponds to the sequence of geometrical atoms identified for the algorithmic generation of the molding profile (cf. Fig. 9). Their recognition is necessary for the generation of a molding profile that uses the correct value for height and width for each.

'Ord.' corresponds to the order in which the moldings are constructed geometrically following the order in which Palladio describes them in the text. They sometimes state their dimensional relationships between height (H) and depth (W). The ratios made explicit in the text have been transcribed under 'R [H:W].' Otherwise, the textual *geom.* means implies that the ratio of the molding were deducted by graphical analysis. After the phase of geometrical construction

AE	Molding		Ord.	R [H:W]	GA
Cornice	Orlo		14	*geom.*	1b
	Cyma recta [*gola*]		15	1:1	8b
	Listel (cyma reversa)		16	1:1	2b
	Cyma reversa [*intavolato*]		17		1b,8a
	Corona [*gocciolatoio*]		13	*geom.*	1b, 6d, 2b
	Gradetto (corona)		12	1:1	2b
	Ovolo		11	4:3	6a
	Listel (ovolo)		10	*geom.*	2b
	Dentello		9	3:2	2b
	Listel (cyma reversa)		8	1:1	2b
	Cyma reversa [*intavolato*]		7		8a
Frieze	-		6	*geom.*	7a
Architrave	*Cimacio*	Orlo	5	*geom.*	2c
		Cavetto	4	1:1	1b, 6b
		Cyma reversa [*intavolato*]	3	*geom.*	1b, 8a
		Tondino[*astragalo*]	1	2:1	7a
	Fascia [*piano*]		2	*geom.*	2b

Profiles 4th design

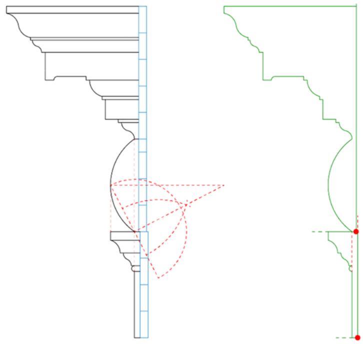


Fig. 15 Fourth design for the ornaments of doors and windows based on Palladio 1570: 59. Image: author

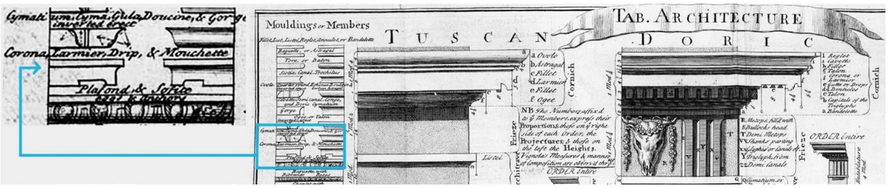


Fig. 16 Reference for encoding Corona elements. Image: Table of architecture, volume I, *Cyclopaedia* (1728) retrievable on Wikimedia

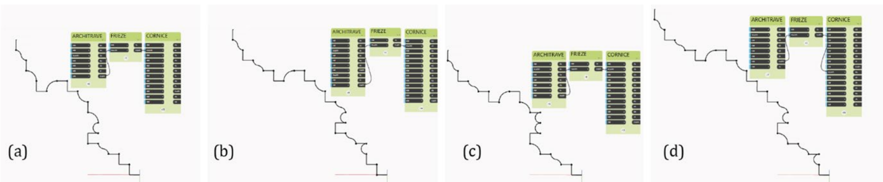


Fig. 17 Sequence of GA with a default value of 10 in height and width (H,W) for each design solution for door ornaments. **a** 1st solution, **b** 2nd solution, **c** 3rd solution, and **d** 4th solution. Image: author

of the elements, the discovered values were transcribed using mathematical syntax (Fig. 19) and then used as input values for the GAs.

Algorithmic Modelling of Moldings

The geometrical analysis was used for the first definition of the geometrical atoms sequence for both profiles: the architrave and the frieze with cornice (Fig. 17).

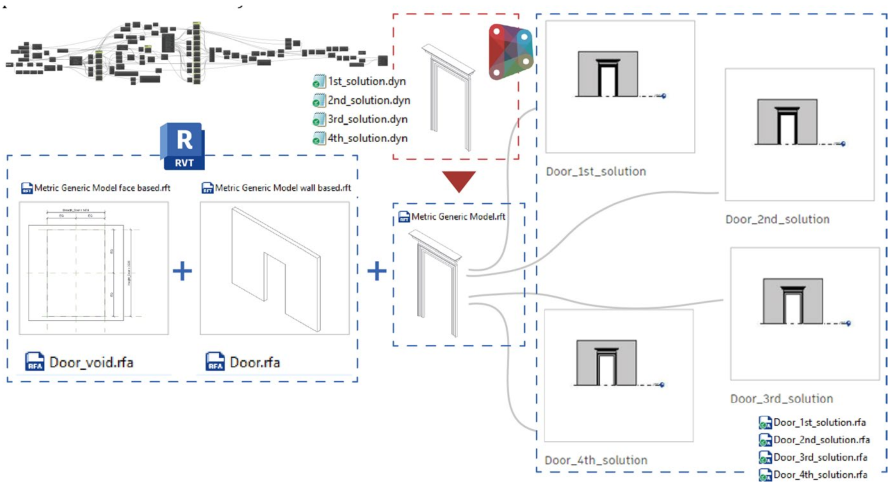


Fig. 18 Workflow for the generation of different solutions for door ornaments using the VPL approach (Dynamo) in a BIM environment (Autodesk Revit). Image: author

(a)

```
Harc_range;
Harc=Breadth*Harc_range;

// Palladio Door ARCHITRAVE
Horlo=(3/25)*Harc;
Worlo=(9/250)*Harc;
Hcymareversa=(9/50)*Harc;
Wcymareversa=(63/500)*Harc;
Ext_cymareversa=(9/250)*Harc;
H2fascia=(2/5)*Harc;
W2fascia=(1/30)*Harc;
H1fascia=(3/10)*Harc;
W1fascia=(13/60)*Harc;

// Palladio Door FRIEZE
Hfrieze=(3/4)*Harc;
Wfrieze=(99/500)*Harc;

// Palladio Door CORNICE
Hcornice=(5/4)*Harc;
Wcornice=(5/4)*Harc;

//CORNICE Elements Top
Horlo=(27/272)*Harc;
Hcymarecta=(81/272)*Harc;
Wcymarecta=(189/544)*Harc;

//CORNICE Corona
Hcorona=(21/68)*Harc;
Hlistel_mouchette=(3/68)*Harc;
Hmouchette=(1/17)*Harc;
Wmouchette=(27/544)*Harc;
Hlarnier=(1/4)*Harc;
Wlarnier=(3/34)*Harc;
Harc_drip=(3/68)*Harc;
Warc_drip=(3/68)*Harc;
Wdrip=(6/17)*Harc;
Hgradetto_corona=(3/68)*Harc;
Wgradetto_corona=(3/68)*Harc;

//CORNICE Elements Bottom
Hovolo=(1/4)*Harc;
Wovolo=(1/6)*Harc;
Hlistel_cavetto=(1/24)*Harc;
Hcavetto=(5/24)*Harc;
Wcavetto=(1/8)*Harc;
Ext_cavetto=(1/24)*Harc;
```

(b)

```
Harc_range;
Harc=Breadth*Harc_range;

// Palladio Door ARCHITRAVE
Horlo=(14/135)*Harc;
Worlo=(7/180)*Harc;
Hcyma_reversa=(7/45)*Harc;
Wcyma_reversa=(7/48)*Harc;
Ext_cyma_reversa=(7/180)*Harc;
Htondino=(2/27)*Harc;
Wtondino=(1/27)*Harc;
H2fascia=(8/21)*Harc;
W2fascia=(2/63)*Harc;
H1fascia=(2/7)*Harc;
W1fascia=(1/14)*Harc;

// Palladio Door FRIEZE
Hfrieze=(3/4)*Harc;
Wfrieze=(161/720)*Harc;

// Palladio Door CORNICE
Hcornice=(5/4)*Harc;
Wcornice=(5/4)*Harc;

//CORNICE Elements Top
Horlo=(135/1564)*Harc;
Hcymarecta=(405/1564)*Harc;
Wcymarecta=(541/1173)*Harc;
Hlistel_cymareversa2=(15/506)*Harc;
Wlistel_cymareversa2=(15/506)*Harc;
Hcymareversa2=(135/1012)*Harc;
Wcymareversa2=(405/4048)*Harc;
Ext_cymareversa2=(135/4048)*Harc;

//CORNICE Corona
Hcorona=(135/391)*Harc;
Hlarnier=(135/391)*Harc;
Wlarnier=(5/69)*Harc;
Harc_drip=(5/138)*Harc;
Warc_drip=(5/138)*Harc;
Wdrip=(120/391)*Harc;
Hgradetto_corona=(5/138)*Harc;
Wgradetto_corona=(5/138)*Harc;

//CORNICE Elements Bottom
Hovolo=(5/23)*Harc;
Wovolo=(15/92)*Harc;
Hlistel_cymareversa1=(5/138)*Harc;
Wlistel_cymareversa1=(5/138)*Harc;
Hcymareversa1=(25/138)*Harc;
Wcymareversa1=(10/69)*Harc;
Ext_cymareversa1=(5/138)*Harc;
```

(c)

```
Harc_range;
Harc=Breadth*Harc_range;

// Palladio Door ARCHITRAVE
Horlo=(3/32)*Harc;
Hcavetto=(9/64)*Harc;
Wcavetto=(27/320)*Harc;
Ext_cavetto=(9/320)*Harc;
Hcymareversa=(9/64)*Harc;
Wcymareversa=(27/256)*Harc;
Ext_cyma_reversa=(9/256)*Harc;
Hpiano=(5/8)*Harc;
Wpiano=(1/6)*Harc;

// Palladio Door FRIEZE
Hfrieze=(13/16)*Harc;
Wfrieze=(81/320)*Harc;

// Palladio Door CORNICE
Hcornice=(5/4)*Harc;
Wcornice=(5/4)*Harc;

//CORNICE Elements Top
Horlo=(5/54)*Harc;
Hcymarecta=(35/108)*Harc;
Wcymarecta=(35/108)*Harc;
Hlistel_cymareversa=(5/132)*Harc;
Wlistel_cymareversa=(5/132)*Harc;
Hcymareversa=(15/88)*Harc;
Wcymareversa=(35/192)*Harc;
Ext_cymareversa=(5/132)*Harc;

//CORNICE Corona
Hlarnier=(10/27)*Harc;
Wlarnier=(5/54)*Harc;
Harc_drip=(5/108)*Harc;
Warc_drip=(5/108)*Harc;
Wdrip=(175/432)*Harc;
Hgradetto_corona=(5/108)*Harc;

//CORNICE Elements Bottom
Hcavetto=(5/27)*Harc;
Wcavetto=(5/36)*Harc;
Ext_cavetto=(5/108)*Harc;
Htondino=(5/72)*Harc;
Wtondino=(5/144)*Harc;
Ext_tondino=(5/144)*Harc;
```

(d)

```
Harc_range;
Harc=Breadth*Harc_range;

// Palladio Door ARCHITRAVE
Horlo=(9/112)*Harc;
Hcavetto=(27/224)*Harc;
Wcavetto=(81/896)*Harc;
Ext_cavetto=(27/896)*Harc;
Hcymareversa=(27/224)*Harc;
Wcymareversa=(135/1792)*Harc;
Ext_cymareversa=(27/896)*Harc;
Htondino=(3/56)*Harc;
Wtondino=(3/112)*Harc;
Hpiano=(5/8)*Harc;
Wpiano=(3/56)*Harc;

// Palladio Door FRIEZE
Hfrieze=(7/8)*Harc;
Wfrieze=(453/1792)*Harc;

// Palladio Door CORNICE
Hcornice=(5/4)*Harc;
Wcornice=(5/4)*Harc;

//CORNICE Elements Top
Horlo=(10/153)*Harc;
Hcymarecta=(35/153)*Harc;
Wcymarecta=(35/153)*Harc;
Hlistel_cymareversa2=(5/198)*Harc;
Wlistel_cymareversa2=(5/288)*Harc;
Hcymareversa2=(5/44)*Harc;
Wcymareversa2=(25/288)*Harc;
Ext_cymareversa2=(5/144)*Harc;

//CORNICE Corona
Hlarnier=(40/153)*Harc;
Wlarnier=(35/459)*Harc;
Harc_drip=(5/153)*Harc;
Warc_drip=(5/153)*Harc;
Wdrip=(245/918)*Harc;
Hgradetto_corona=(5/153)*Harc;
Wgradetto_corona=(5/153)*Harc;

//CORNICE Elements Bottom
Hovolo=(25/162)*Harc;
Wovolo=(25/216)*Harc;
Hlistel_ovolo=(5/162)*Harc;
Wlistel_ovolo=(25/648)*Harc;
Hdentello=(5/27)*Harc;
Wdentello=(10/81)*Harc;
Hlistel_cymareversa1=(5/162)*Harc;
Wlistel_cymareversa1=(5/162)*Harc;
Hcymareversa1=(25/162)*Harc;
Wcymareversa1=(5/36)*Harc;
Ext_cymareversa1=(5/108)*Harc;
```

Fig. 19 Source code used as INPUT in the *code-block nodes* for the definition of GA dimensions (H, W) in each design solution for door ornaments: **a** 1st solution, **b** 2nd solution, **c** 3rd solution, and **d** 4th solution. Image: author

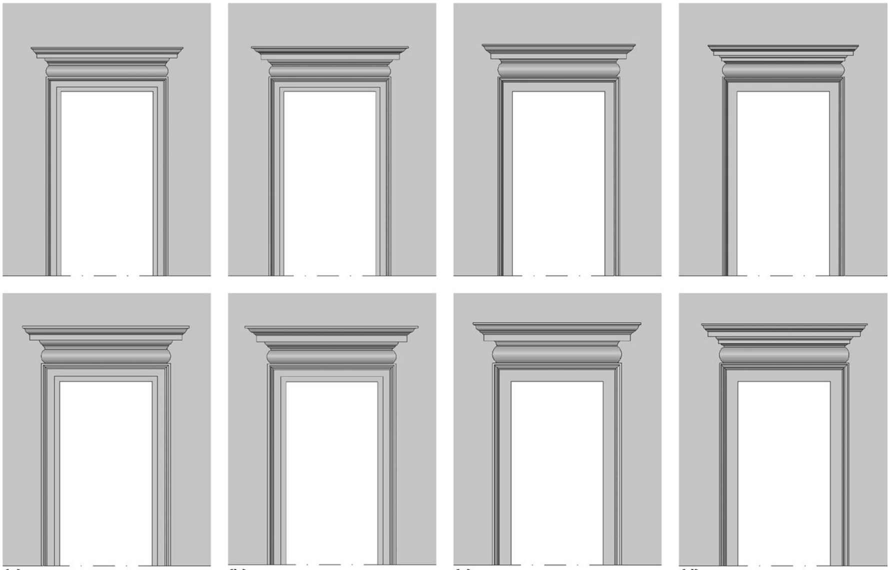


Fig. 20 Doors and their ornaments according to Andrea Palladio: **a** 1st solution, **b** 2nd solution, **c** 3rd solution, and **d** 4th solution. In the first row, the proportions considered for the height of the architrave corresponds to 1/6 of the breadth (minimum) of the door and in the second row the value corresponds to 1/5 of the breadth (maximum). Image: author

For the three-dimensional construction of the ornaments, the *architrave* can be considered as a solid: a result of a sweep extrusion of a moldings profile along the polycurve that follow the edges of the door. The *frieze* and *cornice* otherwise follow a different path, and their construction is made on the horizontal plane created by the *architrave* (see profiles in the Figs. 12, 13, 14, 15).

A 3D model was created inside a Revit Family template to generate the prototype of each solution for a parametric door and its ornaments. This used a nested family for the door void. The Door_void.rfa was based on a surface template family. The void variables were then used as dimension constraints to generate the three-dimensional models for door ornaments (Fig. 18).

In each solution, the algorithm refers to the dimensions and position of the void in a generic wall.

The mathematical formulas for each molding element were then transcribed inside a *codeblock* node to be used as the input for the height and the width of each single GA (Fig. 19).

Finally, the four solutions were collected in Fig. 20. Their comparison also comprises the two solutions according to the minimum and maximum value for the *architrave*, which is in proportional relationship with the breadth of the door: The breadth corresponds to the width of the void in the Revit family.

Conclusions

This research is part of an ongoing study with the aim of creating a PALLADIO HBIM Library of Architectural Elements. Encoding Palladio's guidelines with a parametric approach is not new (Valenti et al. 2012), but the development of a critical study about what Palladio hides in its drawings is different from previous approaches.

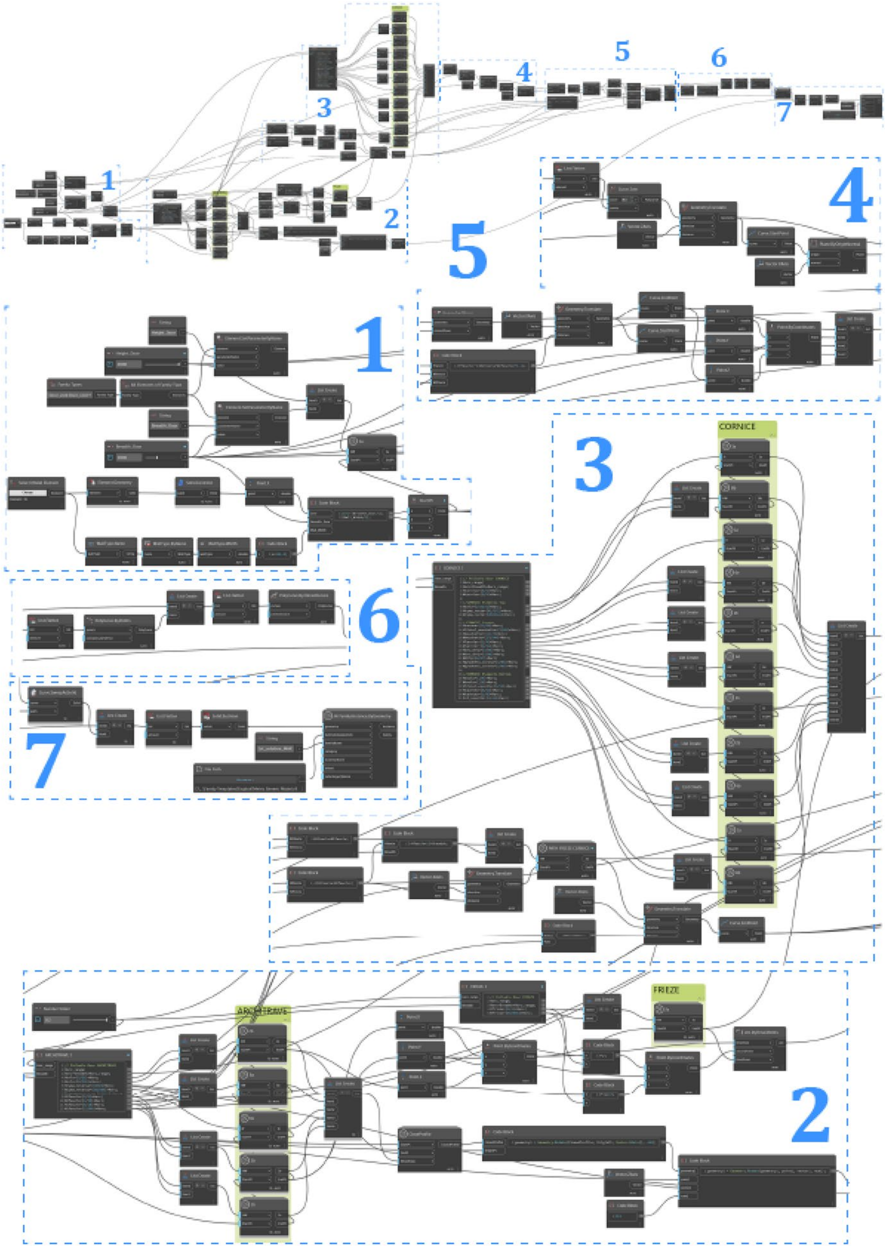
The idea of discretizing the ornaments' proportional rules described in Book I accompanied by a three-dimensional library of objects in HBIM offers the possibility of developing a 3D Palladian grammar that is findable, accessible, interoperable, and reusable (Wilkinson et al. 2016) for both didactive activities and research purposes.

The creation of a 3D Palladian grammar can be used in future work for comparisons with the planimetric layout of Palladio's design projects for villas and buildings. In addition, the 3D models of doors generated by algorithms can become part of an architectural elements library that can be reused in BIM or HBIM platforms for further semantic and information enrichment and for possible comparisons between digital replicas and original doors in Palladian buildings.

The results already obtained (Giovannini 2021) and the algorithms developed within this research offer a novel approach in the graphical analysis of Palladio's theory of proportions described in his *I quattro libri dell'architettura*.

Palladio describes the proportioning of moldings but does not identify only one way of representing them. This is seen in the description of architectural orders and in the description of the ornaments of doors and windows. Even in defining the rules of proportioning by successive partitions, space is left for subjective interpretation. The study shows that Palladio maintains a rigid geometric construction despite degrees of expressive freedom in the definition of ornaments. The graphical analysis made it possible to identify the architectural elements' philological construction and the relationship between their geometric construction made up of alignments, recurring intervals, equal subdivisions, and successive partitions.

Appendix



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Data availability Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

References

Apollonio, F.I., Corsi, C., Gaiani, M. and Baldissini S. 2010. An Integrated 3D Geodatabase for Palladio's Work. *International Journal of Architectural Computing* 8(2): 111–133. https://doi.org/10.1260/1478-0771.8.2.111

Apollonio, F.I., Baldissini, S., Clini, P., Gaiani, M., Palestini, C. and Trevisan C. 2013. the Palladiolibrary Geo-Models: an Open 3D Archive To Manage and Visualize Information-Communication Resources About Palladio. *ISPRS - International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences* XL-5/W2: 49–54. https://doi.org/10.5194/isprsarchives-XL-5-W2-49-2013

Baldissini, S. and Gaiani, M. 2014. Interacting with the Andrea Palladio Works, *Journal on Computing and Cultural Heritage (JOCCH)*, 7(2). https://doi.org/10.1145/2611374

Beltramini, G. and Gaiani, M., eds. 2012. *Palladio Lab: architetture palladiane indagate con tecnologie digitali*. Vicenza: Centro Internazionale di Studi di Architettura Andrea Palladio.

Burns, H. 2000. Andrea Palladio (1508–1580): la creazione di un'architettura sistematica e comunicabile. In *Andrea Palladio: atlante delle architetture*, eds. G. Beltramini, and A. Padoan, 35–45. Venice: Marsilio.

Calvano, M., Martinelli, L., Calcerano, F. and Gigliarelli, F. 2022. Parametric Processes for the Implementation of HBIM—Visual Programming Language for the Digitisation of the Index of Masonry Quality, *ISPRS International Journal of Geo-Information*, 11(2). https://doi.org/10.3390/ijgi11020093

Capone, M. and Lanzara, E. 2022. Parametric tools for Majolica Domes Modelling, *Nexus Network Journal*, 24(3): 673–699. https://doi.org/10.1007/s00004-022-00605-z

Chitham, R. 2005. *The Classical Orders of Architecture.* Oxford: Architectural Press.

Eisenman, P. and Roman, M. 2015. *Palladio virtuel*. New Haven, London: Yale University Press.

Giovannini, E. C. 2017a. *VRIM nodes*. Available at: https://github.com/ElisabettaCaterina/VRIMnodes.

Giovannini, E. C. 2017b. VRIM workflow: semantic H-BIM objects using parametric geometries. In *3D MODELING & BIM. Progettazione, design, proposte per la ricostruzione*, ed. Empler T., 212–229. Rome: DEI Tipografia del Genio Civile.

Giovannini, E. C. 2021. Making Palladio digitally explicit: geometrical parameters in the First Book of the Quattro Libri dell'Architettura. In *Nexus 20/21: Architecture and Mathematics Conference Book*, eds. Williams, K. and Leopold, C., 57–62. Turin: Kim Williams Books.

Lee, J.H., Ostwald, M. J. and Dawes, M. J. 2022. Examining Visitor-Inhabitant Relations in Palladian Villas, *Nexus Network Journal*, 24: 315–332. https://doi.org/10.1007/s00004-021-00589-2

De Luca, L., Véron, P. and Florenzano, M. 2007. A generic formalism for the semantic modeling and representation of architectural elements, *The Visual Computer*, 23(3): 181–205. https://doi.org/10.1007/s00371-006-0092-5

Martinelli, L., Calcerano, F. and Gigliarelli, E. 2022. Methodology for an HBIM workflow focused on the representation of construction systems of built heritage, *Journal of Cultural Heritage*, 55: 277–289. https://doi.org/10.1016/J.CULHER.2022.03.016

Von Mauch, J. M. and Normand, C. P.J. 1998. *Parallel of the Classical Orders of Architecture*, ed. Rattner Donald M. New York: Institute for the Study of Classical Architecture, Acanthus Press.

Migliari, R. 1991. Il disegno degli ordini e il rilievo dell'architettura classica: Cinque Pezzi Facili. *Disegnare idee immagini*, (2): 49–66.

Mitchell, W. J. 1990. *The logic of architecture: Design, computation, and cognition*. Cambridge: MIT Press.

Mitrovic, B. 2004. *Learning from Palladio*. New York: W.W. Norton & Company.

Mitrović, B. and Djordjević, I. 1990. Palladio ' s Theory of Proportions and the Second Book of the Quattro Libri dell ' Architettura. *Journal of the Society of Architectural Historian*, 49(3): 279–292. https://doi.org/10.2307/990519

Ostwald, M. J. 2011. The Mathematics of Spatial Configuration: Revisiting, Revising and Critiquing Justified Plan Graph Theory. *Nexus Network Journal*, 13(2): 445–470. https://doi.org/10.1007/S00004-011-0075-3

Palladio, A. 1570. *I quattro libri dell'architettura*. anastatic edition. Milan: Ulrico Hoepli.

Palladio, A., Tavernor, R. and Schofield, R. 1997. *The Four Books on Architecture*. translated edition. Cambridge: MIT Press.

Paris, L. and Wahbeh, W. 2016. Survey and representation of the parametric geometries in HBIM. *DISEGNARECON*, 9(16).

Sass, L. 2007. A Palladian construction grammar - design reasoning with shape grammars and rapid prototyping. *Environment and Planning B: Planning and Design*, 34(1): 87–106. https://doi.org/10.1068/b32071

Spallone, R. and Calvano, M. 2022. Parametric Experiments on Palladio's 5 by 3 Villas. *Nexus Network Journal*, 24(2): 287–313. https://doi.org/10.1007/S00004-022-00592-1

Stiny, G. and Mitchell, W. J. 1978. The Palladian grammar. *Environment and Planning B: Planning and Design*, 5(1): 5–18. https://doi.org/10.1068/b050005

Lo Turco, M., Calvano, M., Giovannini, E. C. and Tomalini, A. 2021. AIM! Algorithmic Information Modeling: New Strategies for a Fully Integrated Approach in the Field of Cultural Heritage. In *From Building Information Modelling to Mixed Reality*, eds. C. Bolognesi and D. Villa, 143–156. Springer International Publishing. https://doi.org/10.1007/978-3-030-49278-6_9

Lo Turco, M., Giovannini, E. C. and Tomalini, A. 2022. Parametric and Visual Programming BIM Applied to Museums, Linking Container and Content. *ISPRS International Journal of Geo-Information*, 11(7). https://doi.org/10.3390/ijgi11070411

Valenti, G. M., Casale, A., Romor, J. and Calvano, M. 2012. Un database per l'ordine architettonico: Palladio. In *Palladio Lab: architetture palladiane indagate con tecnologie digitali*, eds. G. Beltramini and M. Gaiani, 81–88. Vicenza: Centro Internazionale di Studi di Archtettura Andrea Palladio.

Wassell, S. R. 1999. The Mathematics of Palladio's Villas: Workshop '98. *Nexus Network Journal 1999*, 1(1): 121–128. https://doi.org/10.1007/S00004-998-0011-3

Wilkinson, M. D., Dumontier, M., Aalbersberg, I. *et al*. 2016. The FAIR Guiding Principles for scientific data management and stewardship. *Sci Data*, 3:160018. https://doi.org/10.1038/sdata.2016.18

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