



**Politecnico
di Torino**

ScuDo

Scuola di Dottorato ~ Doctoral School

WHAT YOU ARE, TAKES YOU FAR

Doctoral Dissertation
Doctoral Program in Energy Engineering (35th Cycle)

**Refined zigzag models for the
response of general multilayered
composite and sandwich structures:
numerical and experimental
investigations**

Matteo Sorrenti

Supervisor

Prof. Marco Gherlone

Doctoral Examination Committee:

Prof. Mehmet Dorduncu, Referee, Erciyes University

Prof. Vescovini Lorenzo, Referee, Politecnico di Milano

Politecnico di Torino

July, 2023

Summary

In recent decades, multilayered composites and sandwich structures have been widely used in various engineering fields (aerospace, automotive, naval, defence, and civil). These structures exhibit a high stiffness-to-weight ratio if compared with their metallic counterparts. Moreover, their tailoring capabilities make them interesting in design optimization. On the other hand, their intrinsic complex transverse material properties description requires an accurate structural model able to predict their structural response. Three-dimensional high-fidelity finite element models could be considered to evaluate the multilayered structural response; however, it is common to find industrial applications in which many layers are involved. Consequently, the computational cost of three-dimensional finite elements becomes prohibitive even for simple analysis. It becomes necessary to pursue another way to reach an affordable solution, even for more complex structures. In the framework of theories developed in recent years by researchers, the zigzag models represent a new class of models to investigate the structural response of multilayered composites and sandwich structures. These models, originally formulated by Prof. Di Sciuva, inspired many researchers, among them Prof. Gherlone, to put their efforts into contributing to developing accurate, efficient and computationally attractive zigzag theories.

In this context, this research aims to enhance the available refined zigzag models to investigate more general structures that exhibit a pronounced transverse anisotropy, including the effect of transverse normal deformation and non-linearities of displacement components typically present in thick multilayered structures and sandwich structures.

Chapter 1 presents a general overview of the most used structural theories available in the current literature, with particular attention to the zigzag models and variational formulations.

In Chapter 2, the enhanced Refined Zigzag Theory is formulated to analyse general multilayered structures in which the transverse shear coupling due to the material anisotropy is present. The formulation of the enhanced zigzag functions, the derivation of governing equations, and consistent boundary conditions are presented in detail.

Chapter 3 is devoted to the formulation of a new higher-order mixed model based on the en-RZT that takes into account a finer description of the transverse shear and normal stresses. The new model formulation, named $\text{en-RZT}_{\{3,2\}}^{(m)}$, implements the Hellinger-Reissner variational principle, considering a new set of independent strain variables. A penalty term guarantees the compatibility conditions on the strain quantities in the governing functional. The governing equation and consistent boundary conditions are obtained to analyse anisotropic and cross-ply multilayered composite and sandwich plates.

In Chapter 4, the new mixed model is simplified to beam analysis. The beam governing equations and consistent boundary conditions of the $\text{B-RZT}_{\{3,2\}}^{(m)}$ are derived using the previous mixed variational statement. Then, two mixed finite elements are formulated to analyse cross-ply and sandwich beam structures.

In Chapters 5 and 6, the numerical and experimental results are presented. In Chapter 5, the newly formulated models are numerically assessed to evaluate the predictivity responses for linear bending, buckling and free vibration problems. The experimental campaign on three- and four-point bending tests and free vibration analysis of sandwich structures is conducted, and the results are compared with the new beam models. The activity proposed, supported by the numerical and experimental results, wants to offer a more general and complete methodology based on the refined zigzag models to investigate multilayered composite and sandwich structures. In particular, the newly formulated models could be used to analyse structures in which the material transverse anisotropy is not negligible. Moreover, the mixed formulation of the new models could be used to investigate even thick multilayered structures. The formulated elements make these new mixed models appealing in their predictivity capabilities and affordable low computational cost compared to other available models.

During the PhD research activity, the aspects concerning model development, finite element formulations, and numerical and experimental assessment constitute the basis of this thesis work. In addition, other aspects of PhD research activity not explicitly considered in this thesis cover numerical analysis of functionally graded structures, approximate solutions using the higher-order Haar-Wavelet method and advanced aspects of finite element modelling, some of them during the period spent abroad at CIMNE, Barcelona. These interesting aspects and worthy of being mentioned for the sake of the conciseness of this dissertation, are not included in the presented models.

Here follows a list of the published articles and participations in International congresses.

Journal publications:

Di Sciuva M, Sorrenti M. A Family of C^0 Quadrilateral Plate Elements Based on the Refined Zigzag Theory for the Analysis of Thin and Thick Laminated Composite and Sandwich Plates. *Journal of Composites Science* 2019;3:1–31. <https://doi.org/10.3390/jcs3040100>.

Sorrenti M, Di Sciuva M, Tessler A. A robust four-node quadrilateral element for laminated composite and sandwich plates based on Refined Zigzag Theory. *Computers & Structures* 2021;242:1–22. <https://doi.org/10.1016/j.compstruc.2020.106369>.

Sorrenti M, Di Sciuva M, Majak J, Auriemma F. Static Response and Buckling Loads of Multilayered Composite Beams Using the Refined Zigzag Theory and Higher-Order Haar Wavelet Method. *Mech Compos Mater* 2021;57:1–18. <https://doi.org/10.1007/s11029-021-09929-2>.

Di Sciuva M, Sorrenti M. Bending and free vibration analysis of functionally graded sandwich plates: An assessment of the Refined Zigzag Theory. *Journal of Sandwich Structures & Materials* 2019:1–43. <https://doi.org/10.1177/1099636219843970>.

Di Sciuva M, Sorrenti M. Bending, free vibration and buckling of functionally graded carbon nanotube-reinforced sandwich plates, using the extended Refined Zigzag Theory. *Composite Structures* 2019;227:1–20. <https://doi.org/10.1016/j.compstruct.2019.111324>.

Sorrenti M, Di Sciuva M. An enhancement of the warping shear functions of Refined Zigzag Theory. *Journal of Applied Mechanics* 2021;88:7. <https://doi.org/10.1115/1.4050908>.

Sorrenti M, Gherlone M, Di Sciuva M. Buckling analysis of angle-ply multilayered and sandwich plates using the enhanced Refined Zigzag Theory. *PEAS* 2022;71:84. <https://doi.org/10.3176/proc.2022.1.08>.

Sorrenti M, Gherlone M. A new mixed model based on the enhanced-Refined Zigzag Theory for the analysis of thick multilayered composite plates. *Composite Structures* 2023;311:116787. <https://doi.org/10.1016/j.compstruct.2023.116787>.

International conferences presentations:

Sorrenti, M.; Di Sciuva, M.; Tessler, A.; Development of a locking-free quadrilateral element for laminated composite and sandwich plates based on refined zigzag theory. CIVIL-COMP 2019, Riva del Garda (Italy), September 16 -19, 2019.

Sorrenti, M.; Gherlone, M.; Di Sciuva, M.; Free vibration analysis of angle-ply laminated and sandwich plates using enhanced Refined Zigzag Theory. In 19th International Conference of Numerical Analysis and Applied Mechanics (ICNAAM 2021), Rhodes (Greece), September 20 – 26, 2021.

Sorrenti, M.; Gherlone, M.; Dynamic analysis of sandwich beams with adhesive layers using the mixed Refined Zigzag Theory. In 19th International Conference of Numerical Analysis and Applied Mechanics (ICNAAM 2021), Rhodes (Greece), September 20 – 26, 2021.

Sorrenti, M.; Gherlone, M.; A new mixed model based on the enhanced-Refined Zigzag Theory for the analysis of thick multilayered composite plates. In 25th International Conference on Composite Structures (ICCS25), Porto (Portugal), July 19 - 22, 2022.