# High Order Technologies in Boundary Element Methods: Analyses, Algorithms, Applications

This dissertation describes research activities aimed at improving the effectiveness of the boundary element method (BEM), with a dedicated focus on addressing mathematical and algorithmic challenges within the domains of acoustic and electromagnetic simulations. The contribution opportunities in the BEM approaches are numerous due to the challenges that that method presents in some specific application scenarios. Advancements in the BEM may include function discretization, numerical and analytical integration, or preconditioning techniques.

Currently, the most widely spread techniques involve discretization methods, which can be described as low-order since they employ low-order, typically one or two, representation functions. Nonetheless, analyses have indicated that high-order methods deliver better computational efficiency in many instances. This dissertation delves into various techniques within this field of study.

The research extended into the domain of medical imaging, specifically addressing the challenge of high-order discretization in magnetic resonance imaging (MRI) to enhance (Larmor) frequency resonance. The proposed approach yielded promising results, indicating potential improvements in resonance detection processes.

A fast direct solver for two-dimensional problems was introduced, leveraging the extraction of a circulant problem from an arbitrary structure. This method was further extended to support high-order discretization capabilities through the development of ad-hoc strategies.

A key challenge is the implied computational complexity associated with the dense matrices arising in the BEM, where a standard solver has a time complexity of up to $\mathcal{O}(N^3)$, $N$ being is the number of unknowns. Fast solvers allows to mitigate this effect, and the selected approach may be a compromise between the arising time complexity and its intrinsic adaptability. At the same time, different approaches may result in different trade-offs between computational and memory intensity. This research activity introduced a multi-kernel method designed to efficiently compress BEM matrices involving multiple operators. The presented approach effectively reduces memory costs without increasing the computational costs.

In summary, these activities contributed to the evolution of numerical methods, ranging from engineering applications to imaging techniques for medical sciences.