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STRESS EVALUATION AT THE ROOT FILLET IN CYLINDRICAL GEAR: A SEMI-ANALYTICAL APPROACH COMPARED WITH STANDARD AND NUMERICAL METHODS

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ABSTRACT

The evaluation of the fatigue life in gears due to bending loads is still an important task due to the necessity to increment the power density of the gearboxes but reducing the weight. The usage of Finite Element Method allows to properly evaluate the stress distribution in the root fillet, also taking into account the actual contact ratio and the load sharing factor. However, such procedure is time consuming because it requires a final geometry of the gear pair and long time for preparing and solving the model. On the other hand, the standards provide a quick and precautionary insight of gears, but loosing info regarding the actual force sharing trend among teeth. The aim of this paper is suggesting a strategy for evaluating stress and fatigue behavior of the gear during the predesign phase, where the geometry is not yet finalized, but the choices are easier and less costly. The paper analyzes the ISO 6336 standard in the 2019 version, compares it to the FEM solution and proposes a semi-analytical method that maintains a good accuracy and a low computational cost. The proposed strategy also offers the possibility to correctly estimate the gear performance, because the trend of stress can be easily considered along the meshing cycle.

Keywords: Root Fillet Stress, Contact Ratio, Finite Element Method

1. INTRODUCTION

1.1 Literature review

Gears play a fundamental role in countless mechanical systems, serving as the primary means for transmitting power and motion with precision and efficiency. From automotive transmissions to industrial machinery and beyond, gears enable the smooth operation of a wide array of applications. Given their ubiquitous presence in various engineering fields, understanding the behavior and stress distribution within gears is of paramount

importance. Usually their study focuses on their dynamic behavior and its optimization for example in the reduction of the static transmission error which is one of the main sources of dynamic excitation [1]. But analyzing gear stress is also crucial for ensuring the reliability, durability, and optimal performance of geared systems. Stress analysis aids in identifying potential failure points, optimizing gear designs, and enhancing overall system efficiency. As such, delving into the intricacies of gear stress not only ensures the longevity of machinery but also contributes to advancements in engineering design and technology. In recent research, Ajmi and Vexlex [2] introduced a novel model for simulating solid wide-faced gears, emphasizing the integration of elastic couplings to address gear body distortions and tooth deflections. Their simultaneous solution approach for equations of motion and contact problems highlighted the significance of tooth shape deviations and alignment errors, revealing gear body distortions as critical factors. Moreover, Litvin et al. [3] conducted research on modified involute helical gears, providing a detailed study involving computerized design, meshing simulation, and stress analysis. Their work enhances the understanding of helical gear behavior under various conditions. In their study, Guilabalt et al. [4] emphasize the critical role of manufacturing and assembly quality in determining gear set performance. To address these influences early in the design stage, they introduce an Express model designed for rapid yet precise simulations. Combining the finite strip method with a pseudo-three-dimensional model, the model accurately computes load sharing, fillet stresses, and pressure distribution along meshing helical gear teeth. Validation against 3D finite element the model elucidates the impacts of edge contact and tooth stiffness reduction near corners, providing valuable insights for design optimization. The same research group [5] extended their study to gear pairs with tooth deviations and modifications using the same methodology. Hedlund and Lehtovaara [6] focused on the underexplored realm of helical gears, employing three-dimensional finite elements to dissect tooth bending, shearing, and foundation flexibility. Their findings

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emphasized the pivotal role of tooth foundation flexibility in load distribution between meshing teeth, with contact flexibility playing a minor role. Additionally, their study examined various local contact calculation methods, enriching our understanding of gear tribology. Pedrero et al. [7] proposed a novel model rooted in the minimum elastic potential energy criterion to predict bending and contact stresses in spur and helical gears. By integrating this model with Navier's equations and the Hertzian contact model, they achieved more accurate load distribution estimations. Their analytical equation for the inverse unitary potential offers engineers a precise tool for load distribution calculation, validated against Finite Element Method studies, showcasing its efficacy in bridging theoretical concepts with practical applications. Hotait and Kahraman [8] conducted an experimental parametric study to investigate the combined effects of shaft misalignment and gear lead modifications on load distribution and tooth bending stresses in helical gear pairs. Various helical gear pairs with different total lead crown amounts were tested under controlled shaft misalignment and low-speed conditions. Strain gauges were strategically placed along the face width of the gear teeth to measure root strains, particularly near the tooth fillet region. The study quantified variations of root strains for different levels of shaft misalignment and gear lead crown, revealing a direct correlation between lead crown and gear misalignment. Additionally, the effectiveness of lead crown in mitigating edge loading conditions caused by misalignment was demonstrated. These findings provide valuable insights and form a database for validating gear contact models concerning their ability to simulate misalignment. Peng et al. [9] presented an efficient model tailored for evaluating load distribution in helical gears, accounting for tooth modifications and misalignment errors. By integrating unloaded meshing simulation and numerical methods for elliptical Hertzian contact, they determine contact points and derive load distribution via potential energy minimization. Their model, implemented using a fixed-point iteration method, exhibits superior versatility and efficiency, validated through finite element analysis. Investigating factors such as profile crowning, lead modification, misalignment, and input torque, they reveal how these parameters influence contact patterns and load distribution, offering valuable insights for gear design optimization. Chen et al. [10] studied the impact of gear tooth spalling stresses in helical gears, developing an analytical model and comparing it with finite element simulations. In their paper, Sanchez et al. [11] introduce a non-uniform model for evaluating load distribution along the line of contact in spur and helical gears, derived from the minimum elastic potential criterion. By combining this model with linear elasticity equations, they assess fatigue tooth-root stress, identifying critical stress values and load conditions. Through a comprehensive analysis of tooth bending strength, they propose a simple analytic equation to describe the load per unit length at any point along the line of contact and during the meshing cycle. This enables a thorough examination of tooth-root bending stress, leading to recommendations for calculating the bending load capacity of spur and helical gears. Wang and Zhang [12] developed a model to analyze stiffness and stress in helical gear pairs, considering factors such as tooth profile errors and assembly deviations. This model assesses parameters like mesh stiffness, transmission error,

tooth contact stress, and tooth root stress. Jabbour and Asmar [13] introduced a method for calculating root and contact stresses in metal spur and helical gears, identifying parameters and gear configurations yielding maximum bending and contact stresses. The validity of this method is confirmed through finite element calculations. Notably, the method facilitates modeling of meshed gear pairs throughout a rotation interval, accounting for non-uniform load distribution along the lines of contact, particularly in helical gears.

1.2 Scope of the work

In this paper firstly the procedure defined by the current standards is described highlighting the main geometric parameters that influence the stresses in the fillets of helical gears. Next a semi-analytical methodology, modified for this kind of gears, is presented. The tooth stiffness is described as a skewed rectangle and the main parameters for the resulting Timoshenko beam elements with the shape of a parallelogram are derived to compute the root stresses in this configuration. For validation, the standard and the proposed semi-analytical approaches are tested against a detailed 3D finite element model on a literature test case and conclusions are drawn. This methodology differs from the rest of the literature for the low computational effort and, in any case, a good accuracy.

2. STRESS CALCULATION

Stress calculation can be performed by several methods: analytical or numerical. In this section, the most used methods are summarized, a semi-analytical procedure is presented and the main differences highlighted.

2.1 Analytical Method

When designing a set of gears, engineers typically follow the analytical method proposed by Lewis. This approach focuses on the single strongest force that acts on a single gear tooth. This force is located at the peak of the tooth and is inclined of the pressure angle. Figure 1 illustrates this concept. The method treats the tooth as a beam clamped at one end and free at the other, ignoring the force acting radially inwards. Additionally, it simplifies the tooth shape by assuming it is equivalent to a parabola that fits within the actual tooth profile. Finally, for simplicity's sake, the method usually assumes the gear has the fewest teeth possible that can still be manufactured using standard gear cutting techniques. The formula for computing the stress at the root fillet is

$$\sigma = \frac{2 \cdot T \cdot Y_{lewis}}{\lambda \cdot z \cdot m_n^3} \quad (1)$$

where T is the applied torque, z is the number of teeth, λ is the face width ratio equal to b/m_n , b is the face width, m_n is the normal module and Y_{lewis} is the form factor that considers the geometry of the particular tooth. This last coefficient depends on the number of teeth, the pressure angle, the shifting coefficient and the proportion between addendum and dedendum.

According to Equation 1, the Lewis approach to calculate gear stress does not take into account how much overlap (contact ratio) exists between the teeth and the actual point of application

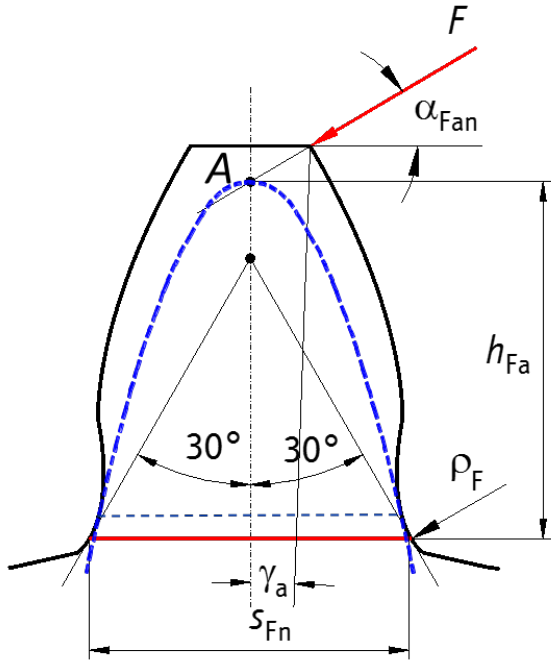


FIGURE 1: SCHEME FOR LEWIS APPROACH TO TOOTH SIZING.

of the force. In contrast, the analytical method outlined in ISO 6336-3:2019 [14] incorporates this two factors into its calculations of stress at the base of the gear tooth. This is reflected in the form coefficient defined by the ISO standard, which is expressed as follows:

$$Y_F = \frac{6h_{Fe} \cos \alpha_{Fen}}{\left(\frac{s_{Fn}}{m_n}\right)^2 \cos \alpha_n} f_\varepsilon \quad (2)$$

where h_{Fe} is the highest distance between the maximum force applied to the tooth and the root fillet (position that depends on the contact ratio), α_{Fen} is the actual inclination of the force applied to the tooth, s_{Fn} is the thickness of the tooth at the root fillet and f_ε is a coefficient that depends on the contact ratio. The values of h_{Fe} and α_{Fen} in the ISO standard differ slightly from Lewis theory h_{Fa} and α_{Fan} since in the former the load is applied at the tip of the tooth, while in the standard the High Point of Single Tooth Contact (HPSTC) is considered. Another important element for the computation of the stress at root fillet is the intensification factor due to the notch at the root. The Lewis' formula does not consider this aspect, but it incorporates that effect in the Y_{Lewis} , whereas the standard provides a specific coefficient to compute the intensification factor named Y_S . The expression of such coefficient is

$$Y_S = (1.2 + 0.13 \cdot L) q_S^{\frac{1}{1.21 + 2L^3}} \quad (3)$$

where $L = s_{Fn}/h_{Fe}$ is the ratio between the thickness of the gear at the root and the arm of the force and $q_S = s_{Fn}/2\rho_F$ is the ratio between the thickness of the gear root and the root fillet itself. So the stress at the root fillet computed according to the

ISO 6336 standard, avoiding other intensification factors due to misalignment and overloads, is:

$$\sigma_F = \frac{F_t}{b \cdot m_n} \cdot Y_F \cdot Y_S \quad (4)$$

While the ISO 6336 approach offers the advantage of incorporating the contact ratio into stress calculations, a more precise analysis requires the use of sophisticated numerical methods. The total contact ratio, denoted as $\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta$, is typically determined based on the gears' geometry. However, during operation, the applied loads cause the teeth to bend, altering their relative positions. Consequently, the contact ratio itself becomes dynamic, generally increasing as the torque applied to the gears rises, determining a sort of contact ratio improvement. The most accurate method for computing stress at the root fillet is the Finite Element Method, because the actual geometry is directly considered and no notch intensity factor is required. In addition, the actual contact ratio is mainly estimated using the Finite Element (FE) Method, but recently the authors ([15], [16]) proposed a Semi-Analytical (SA) Method for assessing the contact ratio in a very quick way. In such a procedure, it is possible to evaluate the Static Transmission Error (STE), the contact pressure on the flank and the actual load sharing factor (LSF) because all the flexibilities in the teeth in contact are considered by means of a non-linear and non-Hertzian [15] procedure. Using this method, the actual contact ratio can be easily evaluated and the effective force acting on single tooth can be computed. As a consequence, during a meshing cycle, it is possible to apply the force at its actual contact position, having the correct distance from the root fillet. In the following a semi-analytical method is presented.

2.2 Semi-analytical Method

In the proposed approach, the tooth profile, defined by the root fillet and the flank, is discretized with a certain number of non-equally spaced nodes having x and y coordinates due to the cutting process simulation. Referring to Figure 2, these nodes allow the tooth to be subdivided in the x direction into many small Timoshenko beam elements, each characterized by its own resistant section.

The reference system adopted has the y-axis arranged perpendicular to the length of the tooth so that the value of y indicates how the tooth thickness varies along its axis proceeding from the base to the tip and the z-axis is directed along the width of the gear. In such a way it is possible to compute the inertia properties of the single beam cross section. If an helical gear is considered, the cross section of the beam does not have a rectangular shape, but it can be arranged to a quadrilateral shape as visible in Figure 3. By considering this shape, it is possible to compute the principle reference frame and the principle inertia properties. Assume A as the area of the beam, β_f as the helix angle, and y_s as the thickness of the tooth on the transverse plane in the y direction. The length of the parallelogram in the z direction will be equal to the width of the strip b, while the total width $H = a + y_s$ will be given by the sum of a contribution $a = b \tan \beta_f$ (related to the angle β_f) and the thickness y_s . The helix angle β_f varies along the height of the tooth and is given by formula 5 expressed as a function of the coordinates x and y of the nodes constituting the

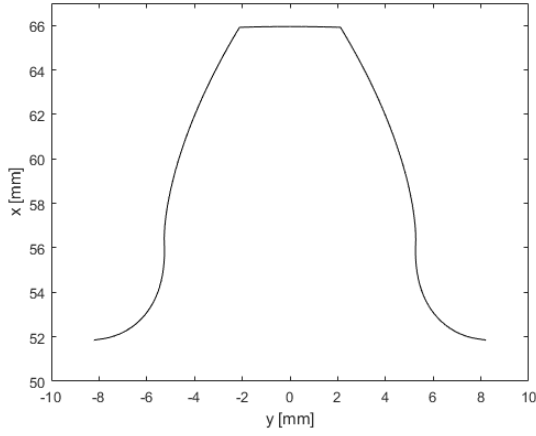


FIGURE 2: GRAPHICAL REPRESENTATION OF A TOOTH ACCORDING TO THE REFERENCE SYSTEM

tooth profile in Figure 2, the pitch radius, and the angle β .

$$\beta_f = \arctan \left(\frac{\tan(-\beta) \sqrt{x^2 + y^2}}{r_p} \right) \quad (5)$$

It is evident that the model with straight teeth is nothing more than a simplification of the helical teeth model in which the helix angle is zero, and therefore the two formulas will yield to $a = 0$ and $H = y_s$, so the rectangle will have the longer side equal to the strip width b and the shorter side equal to the tooth thickness y_s . According to that scheme, the resistant section A is determined through geometric considerations by subtracting from the area of the rectangle A_{tot} of length b and width H the two right triangles with areas A_i and A_{ii} . The following formulas indicate the steps taken to determine the area A of the resistant section of the tooth.

$$A_{\text{tot}} = b \cdot H \quad (6)$$

$$A_i = \frac{a \cdot b}{2} \quad (7)$$

$$A_{ii} = A_i \quad (8)$$

$$A = A_{\text{tot}} - A_i - A_{ii} \quad (9)$$

The coordinates of the centroid with respect to a Cartesian reference system are expressed using static moments, so the coordinates of the centroid of the rectangle with area A_{tot} with respect to the reference system (Y, Z) are given by

$$Y_g = \frac{H}{2} \quad (10)$$

$$Z_g = \frac{b}{2} \quad (11)$$

The coordinates of the centroid with respect to the Cartesian system (Y, Z) for the two triangles, respectively with areas A_i and A_{ii} , are also defined.

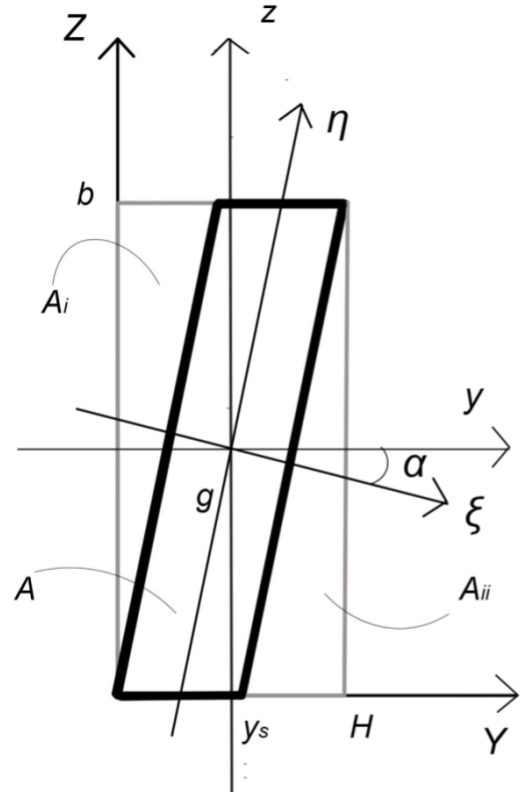


FIGURE 3: GRAPHICAL REPRESENTATION OF THE HELICAL TOOTH CROSS SECTION

$$Y_{gi} = \frac{1}{3}a \quad (12)$$

$$Z_{gi} = \frac{2}{3}b \quad (13)$$

$$Y_{gii} = \frac{2}{3}(a + y_s) \quad (14)$$

$$Z_{gii} = \frac{1}{3}b \quad (15)$$

Same approach can be applied to the moments of inertia of the rectangle with area A_{tot} with respect to the centroidal y and z axes.

$$I_{\text{tot}y} = \frac{1}{12}b^3H \quad (16)$$

$$I_{\text{tot}z} = \frac{1}{12}bH^3 \quad (17)$$

Furthermore, the moments of inertia of the two triangles with respect to their centroidal y and z axes are defined through the following known expressions. The two triangles have the same shapes and dimensions, so moments of inertia with respect to their centroidal axes are identical.

$$I_{yi} = \frac{1}{36}ab^3 \quad (18)$$

$$I_{zi} = \frac{1}{36}ba^3 \quad (19)$$

$$I_{yii} = I_{yi} \quad (20)$$

$$I_{zii} = I_{zi} \quad (21)$$

The centrifugal moment of inertia can be positive, negative, or zero as in the case of the rectangle because the mass distribution is symmetric with respect to the axes, and it can be defined as

$$I_{yzi} = -\frac{1}{72}a^2b^2 \quad (22)$$

$$I_{yzii} = I_{yzi} \quad (23)$$

To calculate the moment of inertia of the parallelogram, it is necessary to use the theorem of transposition of moments of inertia, which establishes the relationships between the second-order moments with respect to centroidal axes and the second-order moments with respect to axes parallel to those passing through the centroid of the system. The transport values need to be determined by calculating the distance in the z direction between the centroidal y axes of the two triangles and the centroidal y axis of the rectangle with area A_{tot} . Similarly, distances in the y direction between the centroidal z axes are calculated. The centroidal axes of the rectangle with area A_{tot} coincide with the centroidal axes of the characteristic parallelogram of the section, and consequently, the transport terms for the latter will be null.

$$d_{yi} = Y_{gi} - Y_g \quad (24)$$

$$d_{yii} = Y_{gii} - Y_g \quad (25)$$

$$d_{zi} = Z_{gi} - Z_g \quad (26)$$

$$d_{zii} = Z_{gii} - Z_g \quad (27)$$

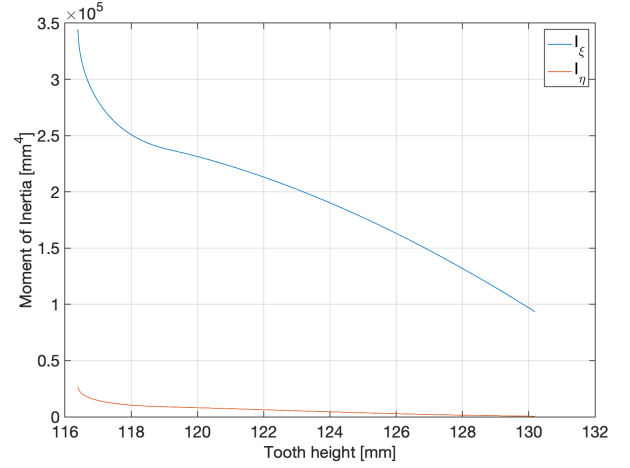


FIGURE 4: PRINCIPAL INERTIA MOMENT OF A TOOTH

Therefore, considering that the centroid of the parallelogram coincides with the centroid of the rectangle, the moments of inertia around the centroidal axes y and z and the centrifugal moment of inertia can be written as:

$$I_y = I_{\text{tot}y} - I_{yi} - I_{yii} - A_i \cdot d_{zi}^2 - A_{ii} \cdot d_{zii}^2 \quad (28)$$

$$I_z = I_{\text{tot}z} - I_{zi} - I_{zii} - A_i \cdot d_{yi}^2 - A_{ii} \cdot d_{yii}^2 \quad (29)$$

$$I_{zy} = -(I_{yzi} + I_{yzii} - (d_{yi} \cdot d_{zi}) \cdot A_i - (d_{yii} \cdot d_{zii}) \cdot A_{ii}) \quad (30)$$

Knowing how the second-order moments varies upon rotating the reference axes is necessary for the study of the principal axes of inertia and the corresponding principal moments of inertia. The axes with respect to which the moment of inertia is minimum or maximum are orthogonal and are referred to as the principal axes of inertia. It can be shown that the moments of inertia and centrifugal inertia of a planar figure constitute the coefficients of a symmetric 2×2 tensor $[I]$, constructed as follows.

$$[I] = \begin{bmatrix} I_y & I_{zy} \\ I_{zy} & I_z \end{bmatrix} \quad (31)$$

There exists a reference system having the same origin at the centroid g and rotated by an angle α with respect to the y and z axes, such that by calculating the moments with respect to its own axes, the tensor $[I]$ becomes diagonal, and the elements of the principal diagonal will be the eigenvalues of the tensor, which coincide with the principal moments of inertia I_ξ and I_η .

$$\begin{bmatrix} I_\eta & 0 \\ 0 & I_\xi \end{bmatrix} \quad (32)$$

Starting from the tensor $[I]$ from equation 31, the eigenvalues of the square matrix are calculated to obtain the principal moments of inertia of the helical teeth model. In Figure 4 the calculation, for each beam the tooth is divided in, is presented.

Given the value of the input torque C , and the load sharing factor LSF compute by the GeDy TrAss algorithm, the angle β_b , and the geometry of the gear, it is possible to calculate the total force exchanged by each tooth in contact during the meshing

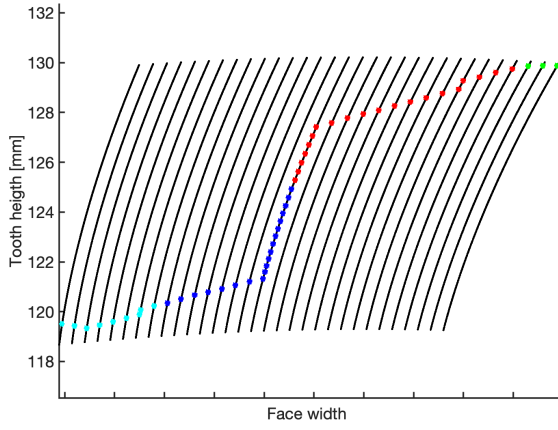


FIGURE 5: CONTACT POINTS WHERE FORCE IS APPLIED, COLORS REPRESENT THE DIFFERENT TEETH IN CONTACT ALONG THE MESHING CYCLE

according to equation 33. The total force varies depending on the tooth and angular position and is therefore defined by a matrix with a number of rows equal to the number of analyzed angular positions and columns defined by the tooth height.

$$F_n = \frac{C}{r_b \cos(\beta_b)} \cdot LSF \quad (33)$$

The total force at the contact node must then be decomposed into its components along the x , y , and z axes. For doing that it is necessary to know the actual position of the contact point and its distances with respect to the tooth axis. In Figure 5 the force application points during the mesh cycle are presented.

For force calculation, some variables are defined:

- b_y is the z coordinate of the point of application of the load (contact node)
- b_z the y coordinate with respect to a reference system with the origin at half the face width
- μ_{eff} as the angle between the direction of the total force F_n and the one perpendicular to the midpoint of the tooth
- $\mu_{\text{eff}}\beta$ the angle between the projection of the force F_n onto the plane perpendicular to the midpoint of the tooth and the frontal plane.

With these values, it is possible to proceed with the calculation of the force components along the axes of the reference system.

$$F_x = F_n \sin(\mu_{\text{eff}}) \quad (34)$$

$$F_y = F_n \cos(\mu_{\text{eff}}) \cos(\mu_{\text{eff}}\beta) \quad (35)$$

$$F_z = F_n \cos(\mu_{\text{eff}}) \sin(\mu_{\text{eff}}\beta) \quad (36)$$

$$M_x = F_n \cos(\mu_{\text{eff}}) \cos(\mu_{\text{eff}}\beta) \cdot b_y + F_n \cos(\mu_{\text{eff}}) \sin(\mu_{\text{eff}}\beta) \cdot b_z \quad (37)$$

$$M_y = F_n \sin(\mu_{\text{eff}}) \cdot b_y \quad (38)$$

$$M_z = F_n \sin(\mu_{\text{eff}}) \cdot b_z \quad (39)$$

Once the results in terms of forces and moments on the structure are obtained, even in the case of helical teeth, tensions due to normal stress and bending moment can be derived. Using a rotation matrix, the cross section coordinates are referred to the principal inertia frame (ξ, η) . Stress is indeed a combination of M_ξ and M_η , so the distribution on the section of the bending stress σ_b is obtained by superposition of effects according to equation 40

$$\sigma_b = \frac{M_\xi}{I_\xi} \eta - \frac{M_\eta}{I_\eta} \xi \quad (40)$$

The total stress σ_{tot} , is then the sum of the contribution due to compression and bending, expressed by equation 41.

$$\sigma_{\text{tot}} = \sigma_c + \sigma_b = \frac{F_x}{A_{\text{tot}}} + \frac{M_\xi}{I_\xi} \eta - \frac{M_\eta}{I_\eta} \xi \quad (41)$$

In such a way, all the stresses can be computed in every beam tooth is dividend in. A stress intensification factor must be used, and in this paper the same used in the ISO standard is implemented.

2.3 Finite Element Method

As the Finite Element Method is the most precise way to find the actual stress inside a component, in the present paper the two gear engagement is also studied with FEM. Authors developed a Finite Element Module inside GeDy TrAss with Hexahedral element with formulation that provides the same result of MSC.Nastran [17] and implemented a strategy for computing the stress along the meshing cycle. The elements include reduced order integration to eliminate shear locking and an Enhanced Assumed Strain (EAS) field to avoid spurious zero-energy modes. The mesh is automatically produced by the software to ensure great accuracy in the tooth flank and fillet zone, while fewer elements are placed on the gear body as visible in Figure 6. For the current test case both gears are discretized with 68400 nodes and 50400 elements since they are identical. The meshing interaction is simulated using linear contact springs. The value of the contact stiffness used is derived from the underlying nonlinear and non-Hertzian analyses of GeDy TrAss as well as their placement on the tooth flank depending on the position in the mesh cycle. An example of the stress distribution on the pinion of the gear pair is visible in Figure 7.

3. CASE STUDY

For comparing the results of the semi-analytical methodology, a case study presented in [13] is used. In table 1 the main parameters are summarized. In the following, the ISO standard approach will be applied, then the actual LSF will be used for computing the trend of stress along the meshing cycle. The semi-analytical method is then applied and compared to ISO and Finite Element methodology.

3.1 Application of the ISO 6336

The ISO standard approach in the edition of 2019 provided the following stress calculation for the gears in table 1.

$$\sigma_F = \frac{F_t}{b \cdot m_n} \cdot Y_F \cdot Y_S = 11 \cdot 1.014 \cdot 2.062 = 23 \text{ MPa} \quad (42)$$

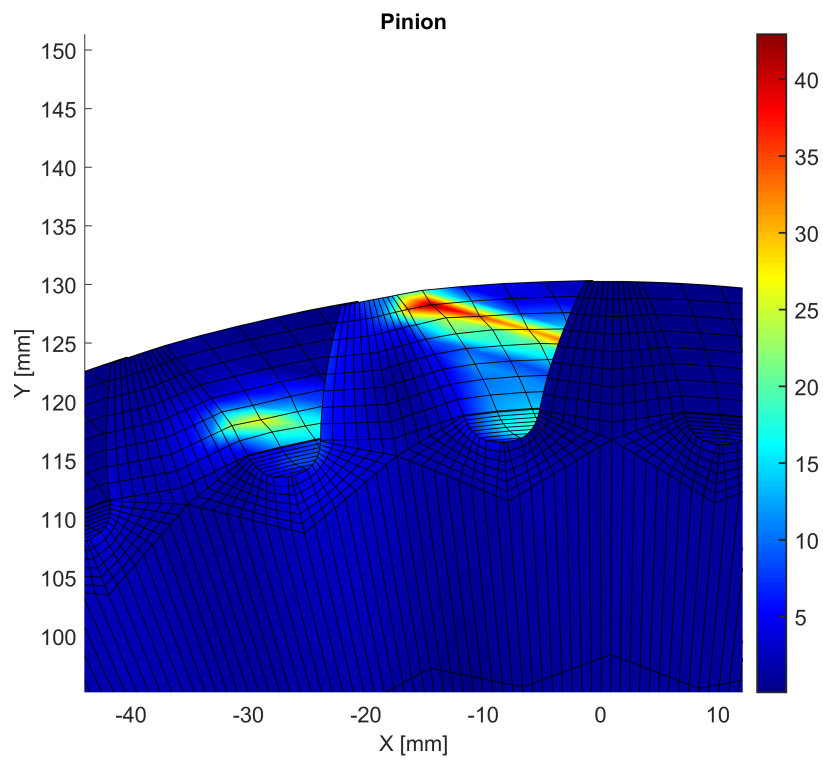


FIGURE 6: DETAIL OF THE FINITE ELEMENT MESH AND OF THE VONMISES STRESS [MPa] DISTRIBUTION ON THE PINION DURING THE ENGAGEMENT

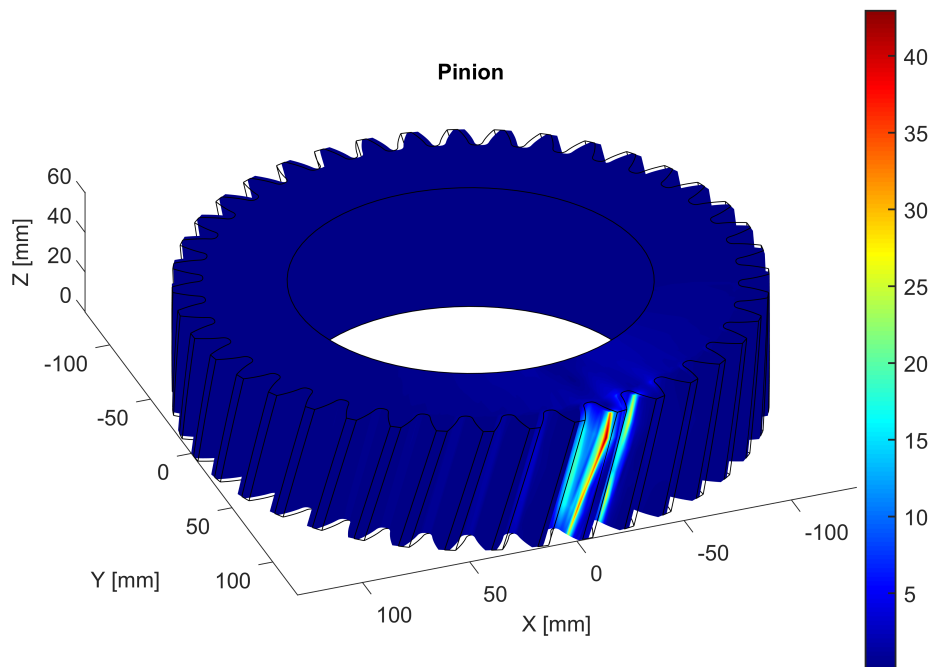


FIGURE 7: EXAMPLE OF THE VONMISES STRESS [MPa] DISTRIBUTION ON THE PINION DURING THE ENGAGEMENT

TABLE 1: DATA OF THE HELICAL GEARS CONSIDERED.

	Gear 1	Gear 2
Number of teeth z [-]	40	40
Modulus m_n [mm]	6	
Face width b [mm]	60	
Gear body width [mm]	60	
Pressure angle α [°]	20	
Helix angle β [°]	15	
Contact ratio ε_α	1.614	
Overall Contact ratio ε_γ	2.438	
Rack	ISO 53/A	
Maximum Input torque [N mm]	500000	
Material	AISI 1040	
Young's modulus [GPa]	210	
Density [$\frac{kg}{m^3}$]	7800	

The thickness of the tooth s_{Fn} at the fillet where stress is computed is 12.89 mm, the radial position from the gear axis of the cross section where stress is computed is 118.35 mm, the distance between such section and applied force $h_{Fe} = 5.72$ mm and, so, the radial position where force is applied 124.07 mm. Considering that the pitch diameter is 124.23 mm, the point of the applied force is lower than pitch diameter due to the double pair contact (overall contact ratio higher than 2), as a matter of fact $f_e=0.81$.

3.2 Adaptation of the ISO 6336 with actual Load Sharing Factor

Total force exchanged in the gear is 4430 N, on the basis of actual LSF, force on single teeth is depicted in Figure 8. Rearranging the formula 42, by knowing the actual force on each tooth and the current contact point position,

$$\sigma_{Fi} = \frac{6 \cdot F_{ti} h_{Fei} \cdot \cos \alpha_{Feni}}{b \cdot m_n \cos \alpha_n} \cdot Y_S \quad (43)$$

the stress on each tooth according to ISO is computed and depicted in Figure 9

3.3 Semi-analytical method results

Once the LSF and the actual contact positions are computed by means of GeDy TrAss algorithm, the stress can be evaluated along the tooth height. The semi-analytical method considers the tooth as a sequence of beams where the one located at the beginning of the root fillet is the clamped end. According to general theory and ISO standard, the maximum of the stress is not located at the beginning of the root fillet, but almost in the middle. As expected, the semi-analytical method provides the location of the most stressed zone in the tooth height at 118.635 mm, very close to the value computed by ISO standard. In Figure 10 the trend of bending stress for each tooth slice and for each tooth in a single meshing point is depicted in order to highlight the maximum stress along the tooth height.

By arranging all the information computed with respect to the point where maximum stress occurs, multiplying the bending stress for the Y_S factor and summing the normal stress due to

TABLE 2: RESULTS OF DIFFERENT APPLIED TORQUE. SA IS THE SEMI-ANALYTICAL METHOD, FE IS THE FINITE ELEMENT METHOD.

Torque	Method	Tooth 3	Tooth 4	Tooth 5
Maximum stress [MPa]				
500 Nm	SA	13	23	14
	FE	11	21	13
5000 Nm	SA	188	287	290
	FE	100	270	260
10000 Nm	SA	300	575	580
	FE	200	550	520

compressive state, the trend of stress in the meshing cycle can be represented in Figure 11

In order to compare with the application of the ISO formula, in Figure 12 the values are superimposed. In that Figure, same color corresponds to the same tooth in the meshing process and the dashed-dotted line depicts the trend of the semi-analytical model while the continuous line represents the ISO approach.

It is evident that the trend is influenced by the LSF used and that is the same in both approaches, but the semi-analytical method depicts a higher dependence of the stress with respect to the distance of the application point.

3.4 FE method results

By applying the FE method to the case study, the trend of the most stressed point in the fillet is evaluated and shown in Figure 13. In Figure 14 the comparison between semi-analytical procedure and FE is performed. It is evident that the trend on the two main mating teeth is the same. The difference can be ascribed to the value of the stress intensification factor Y_S that, probably, has to be evaluated with a different approach with respect to ISO standard. In any case the results are in a sufficient agreement, better than the actualization of the ISO standard. In the images reporting FE results, a little amount of stress is also reported for the other teeth interested in the engagement, tooth 1 and 5. This is due to the constraints applied to the hub that creates a stress effect in all the gear, but the effect of bending is obviously negligible.

4. REMARKS

In order to prove the capability of the code, the same gears were tested under 5000 Nm and 10000 Nm of torque. Trends will be the same between FEM and Semi-analytical method, as reported in Figure 14. In Table 2 the most important value are summarized.

The proposed semi-analytical method is based on the same philosophy of GeDy TrAss software: the sake of accuracy with low computation effort. So, beside the analytical method, that it is surely the less computational costly, this strategy provides a simplify but accurate strategy for computing stress at the root fillet particularly requested for the predesign phase. In such a way, different configurations and gear solutions can be easily tested until a good compromise is found. The semi-analytical method here proposed, compared with the FE strategy, proves to

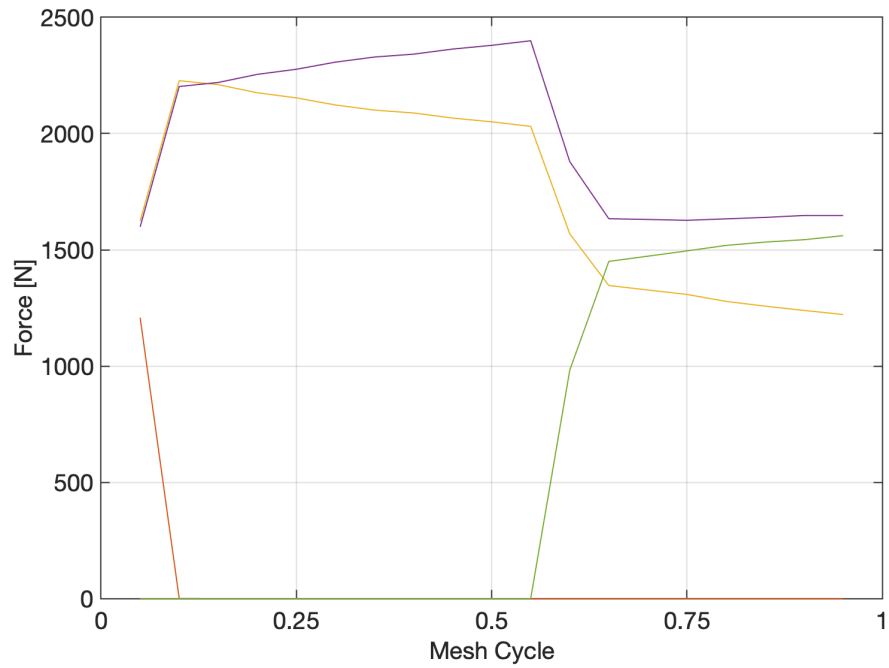


FIGURE 8: FORCE SHARING ON THE THREE TEETH ENGAGED ON A MESH CYCLE

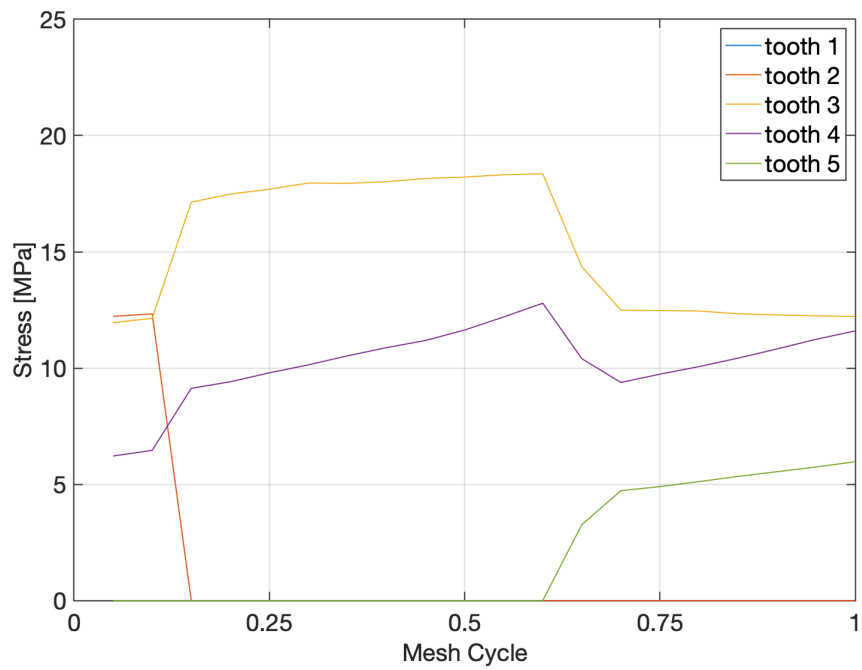


FIGURE 9: STRESS ON EACH ENGAGED TOOTH ACCORDING TO EQUATION 43

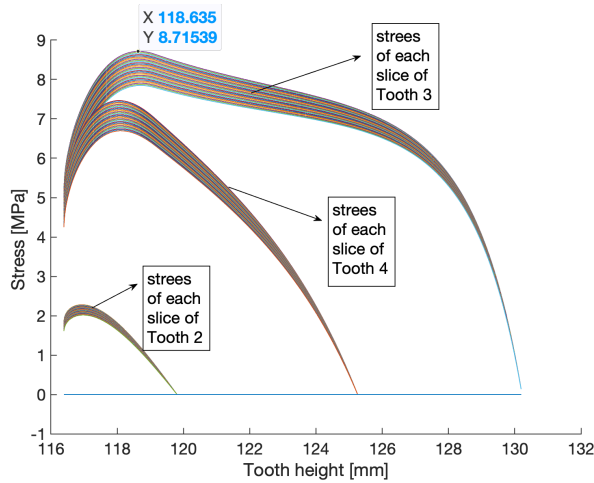


FIGURE 10: STRESS IN THE SINGLE SLICE OF THE TOOTH FOR ALL THE ENGAGED TOOTH IN A SINGLE MESHING EVENT

be order of magnitude less time consuming, not only in terms of calculation but also in terms of model preparation. Geometry is parametrically defined and the computation of the displacements and stress is almost instantaneous, whereas the Finite Element method requires to have a geometry as accurate as possible, a quasi static application of the contact between gears in many steps and the solution of each of this step requires time (some minutes). Some investigations have to be added, in particular for the different root fillet possible and the computation of the appropriate stress intensity factor. As demonstrated in table 2, the stress in tooth 2 is always higher in the prediction of Semi-analytical method than in FEM, whereas in the other teeth the stress intensity factor computed according to ISO 6336 is more accurate. In next development, the authors will take in account also the misalignment effect, both due to the compliance of gearbox components and manufacturing error. The effect of misalignment due to compliance and manufacturing error can be already computed in GeDy TrAss, so the actual LSF in misaligned condition can be computed. Hence, the effect of different load distribution and application points can be easily take into account in the Semi-analytical method for evaluating the stress in the root fillet. In addition, a good fatigue failure criterion can be combined with that strategy for properly estimate the life assessment, in particular in presence of input torque variation.

5. CONCLUSIONS

In the present work, the calculation of tooth root stress has been described in detail according to the ISO standard 6336:2019 which is widely used during the design stage of a geared transmission. In comparison with other approaches and previous versions, a few improvements have been introduced to better model the real meshing interaction. For example, the contact ratio is taken into consideration. An even more accurate approach has then been described: a semi-analytical approach discretizes the tooth using beam elements that for helical gears have the cross section of a parallelogram with variable inclination depending on the location of the slice considered. The presented approaches are compared against each other and against a detailed 3D FE model

with considerable agreement between each other. However the semi-analytical approach proves to be accurate enough to avoid FE calculations in the earlier design stages with high confidence and considerable time savings.

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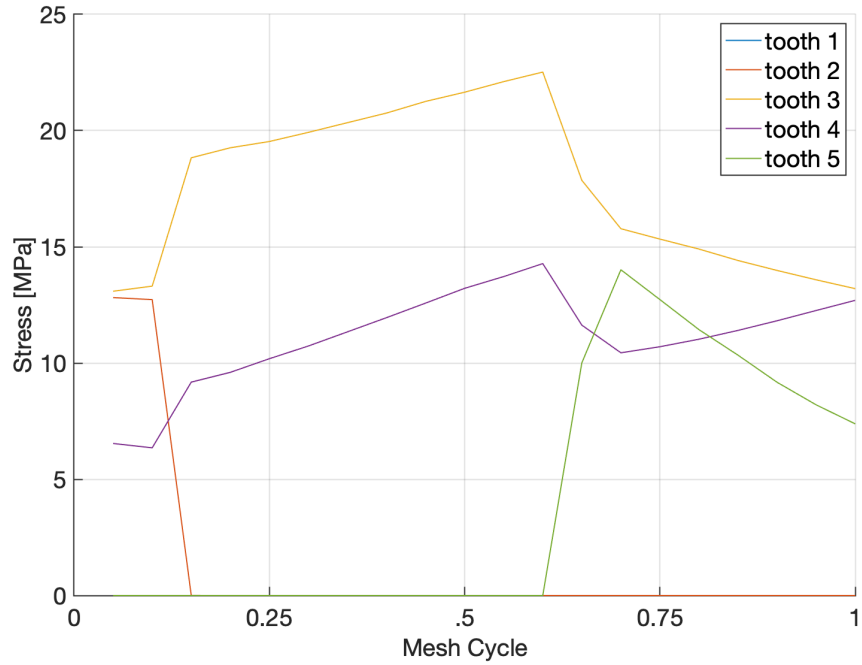


FIGURE 11: STRESS ON EACH ENGAGED TOOTH ACCORDING TO SEMI-ANALYTICAL PROCEDURE

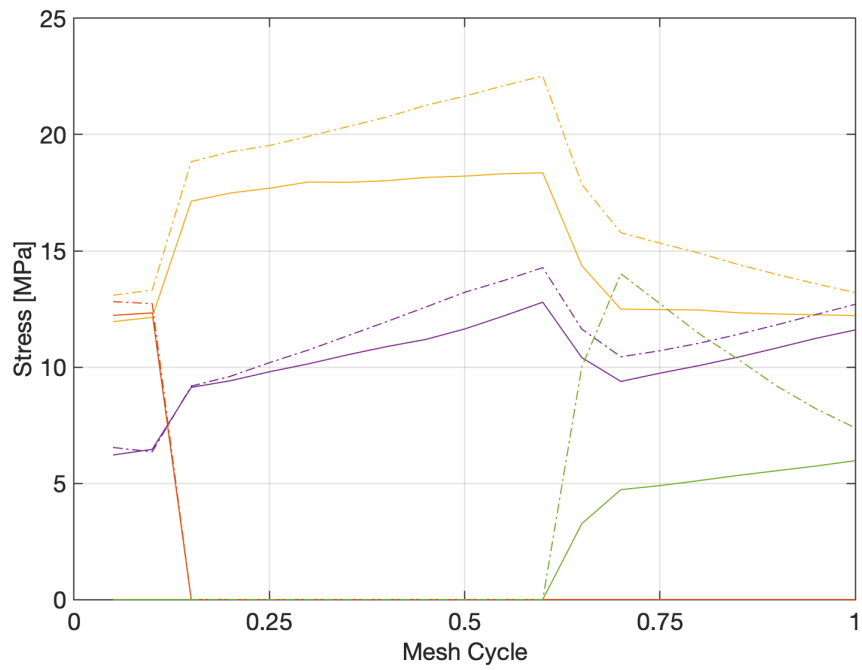


FIGURE 12: COMPARISON BETWEEN ISO WITH ACTUAL CONTACT RATIO (SOLID LINE) AND SEMI-ANALYTICAL METHOD (DASHED-DOT LINE)

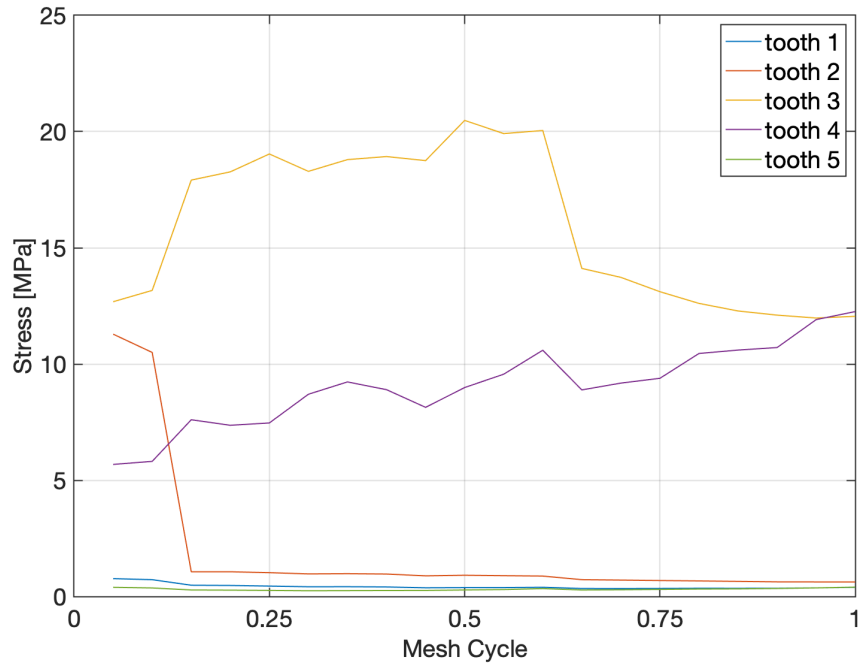


FIGURE 13: STRESS ON EACH ENGAGED TOOTH ACCORDING TO FE PROCEDURE

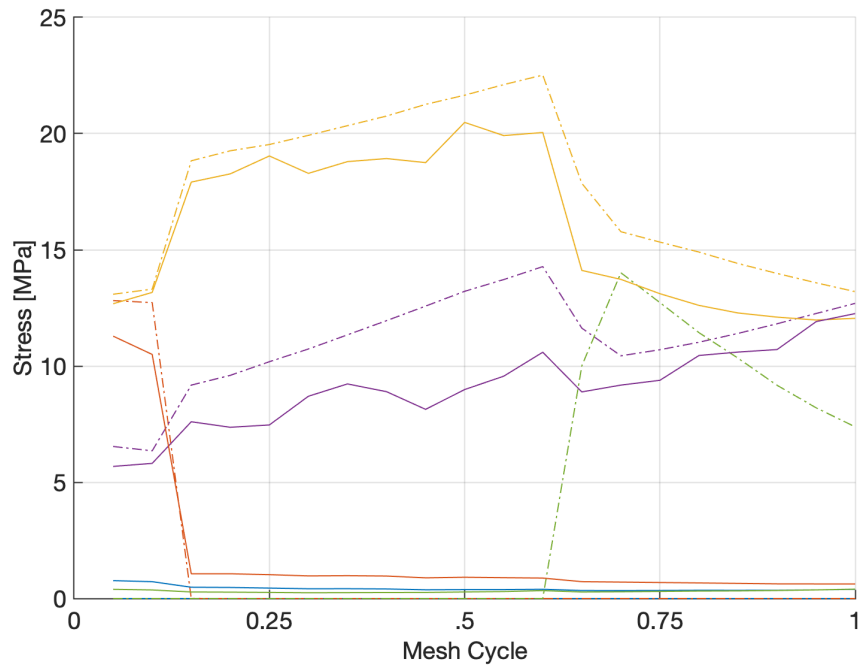


FIGURE 14: COMPARISON BETWEEN FEM (SOLID LINE) AND SEMI-ANALYTICAL METHOD (DASHED-DOT LINE)

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