Periodic Integral Equation Formulation for the Numerical Analysis of Glide Structures

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***Abstract*—In this paper, we propose a technique based on integral equations, the periodic Green's function, and the method of moments discretization to analyze the dispersion diagram of non-canonical glide-symmetry unit cells. The technique is combined with a root finder approach to automatically identify the values of the wavevector that make the unit cell resonant. The proposed technique is tested on a glide fully metallic implementation of a Luneburg lens.**

***Index Terms*—Integral equation, periodic Green's function, periodic structures, glide symmetry**

I. Introduction

Higher-symmetric metasurfaces are of great interest due to their performance beyond the state-of-the-art in terms of bandwidth, beam-scanning, and losses. An accurate characterization of these structures permits a proper implementation of different technologies such as planar printed-circuit-board (PCB), 3-D printable materials for artificial lenses, slotted substrate-integrated waveguide (SIW), filters and resonators, open and closed gap waveguides. These structures are obtained from periodic unit cells, invariant under higher symmetries (glide or twist). Glide symmetry, for instance, is invariant under translation and mirroring.

The analysis of higher-symmetric structures needs accurate and fast methods to predict their behavior and improve the control in the design stage. A common technique is the mode matching [1], where the unit cell is decomposed in canonical sub-domains that are analytically solved, While this approach can speed up the computation time, it is limited to the study of canonical-like structures.

Some numerical techniques have been proposed to analyze general (non-canonical shape) unit cells. The finite-difference time-domain (FDTD) [2] and the finite integration technique (FIT) [3] have the advantage of easy implementation from differential equations. However, they require a volumetric discretization of the geometry and space, and artificial boundary conditions are applied where the domain is truncated. A three-dimensional unit cell with fine sub-wavelength details would require many unknowns, leading to a high computational cost.

Surface integral equation (SIEs) formulations have the advantage of being defined over material boundaries only in both perfect electric conductors and penetrable bodies.

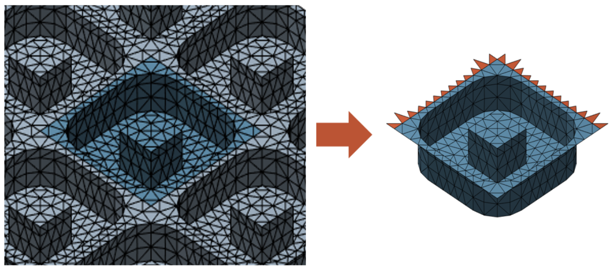

Fig. 1. The structure periodicity is included in the Green's function; to enforce the current continuity, RWG basis functions are added at the unit cell boundaries keeping the mesh conformal.

Discretizing surfaces instead of volumes strongly reduces the number of unknowns needed to describe the problem. SIEs are extensively used in radio frequency (RF) and microwave range through the method of moments (MoM) discretization, demonstrating a high accuracy and versatility.

In [4], we presented preliminary tests on a technique based on MoM-SIEs together with the periodic Green's function to properly characterize higher-symmetry structures not suitable for the mode-matching method due to their non-canonical shape. Here, we combine the MoM-SIE techinque to an automatic root finder algorithm to get a more accurate and fast information on the resonant behavior of the considered glide periodic cell.

II. MoM-SIEs to Analyze Unit Cells

The proposed technique starts from the unit cell geometry and arrives to a triangular mesh that discretizes the problem, as detailed in [4]. On the triangular mesh, Rao-Wilton-Glisson (RWG) are defined to describe the unknown surface current density [5]. To assure the continuity of the current on the boundary of the cell when the periodicity is added, we force mesh conformity along the unit cell opposite sides, adding extra triangles and RWG basis functions to the analyzed unit cell, as shown in Fig. 1.

The above mentioned discretization of the surface current using RWG, combined with the MoM [6], leads to the matrix system $[Z][I] = [V]$, where $[I]$ is the N-size vector collecting the unknown coefficients that describe the surface current, $[Z]$ is the system $N \times N$ matrix, and $[V]$ is the N-size right hand side vector.

In the case of periodic structures, the periodic Green's function (FSPGF) , defined as in [7]

$$G(\mathbf{r}, \mathbf{r}', \mathbf{k}_{t00}) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} G_{m,n} \tag{1}$$

has to be used. In (1) the spatial terms $G_{m,n}$ are equal to

$$G_{m,n} = e^{-j\mathbf{k}_{t00}\cdot\boldsymbol{\rho}_{mn}} \frac{e^{-jkR_{mn}}}{4\pi R_{mn}} \tag{2}$$

where $\mathbf{k}_{t00} = k_{x0}\hat{\mathbf{x}} + k_{y0}\hat{\mathbf{y}}$ is the transverse vector wavenumber defining the phasing between cells for the 2-D array in terms of the propagation angles of the first Floquet mode; $\boldsymbol{\rho}_{mn} = m\boldsymbol{s}_1 + n\boldsymbol{s}_2$, being $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ the lattice vectors, and $R_{mn} = \sqrt{(z-z')^2 + |\boldsymbol{\rho} - \boldsymbol{\rho}' - \boldsymbol{\rho}_{mn}|^2}$, with $\boldsymbol{\rho} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}}$ and $\boldsymbol{\rho}' = x'\hat{\boldsymbol{x}} + y'\hat{\boldsymbol{y}}$.

Among the fastest methods of calculating the periodic Green's function (1) is the the Ewald method. It expresses $G(\mathbf{r}, \mathbf{r}', \mathbf{k}_{t00})$ as the sum of a "modified spectral" and a "modified spatial" series [8] as

$$G(\mathbf{r}, \mathbf{r}', \mathbf{k}_{t00}) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} G^{E}_{m,n} + \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \widetilde{G}^{E}_{p,q} \tag{3}$$

Both series have a very rapid convergence rate. The splitting parameter E (the weight of the two sums) can be tuned to minimize the computation time of the periodic Green's function, balancing the asymptotic rate of convergence of both series, and minimizing the total number of terms needed. The "optimum" value of the Ewald splitting parameter is equal to $\sqrt{\pi/A}$, where A is the area of the unit cell [9].

The system matrix $[Z]$ depends on the wavevector $\mathbf{k}_{t00}$. To generate the unit cell dispersion diagram, we need to find, for each frequency, the value of $\mathbf{k}_{t00}$ that makes the matrix $[Z(\mathbf{k}_{t00})]$ singular. A singular matrix means the presence of a current solution in the structure with no excitation, i.e. a mode of the structure. So, for each frequency we variate $\mathbf{k}_{t00}$ in the region of interest until the determinant of the matrix $[Z(\mathbf{k}_{t00})]$ is zero.

III. Root Finder Implementation

To explain the application of the root finder algorithm, we consider the glide structure described in [10] and shown in Fig. 1. The considered frequency is 5 GHz, and we assume a propagation along $\mathbf{k}_{t00} = k_x\hat{\mathbf{x}}$ only.

The $[Z(k_x)]$ matrix is obtained numerically after the discretization of the continuous problem as described in Sect. II. It leads to a matrix with a determinant that is not exactly zero but a small number close to zero. This feature, together with the difficulties of calculating the determinant of a singular matrix leads to the use of alternative ways to check the singularity of the matrix, as the logarithm of the determinant (det-log), the reciprocal of the condition number of the matrix (Rcond), or the ratio of first and last singular values in the singular value decomposition (SVD). These different approaches are compared in Figs. 2 and 3, where, as expected, all of them depict a similar trend and minimum.

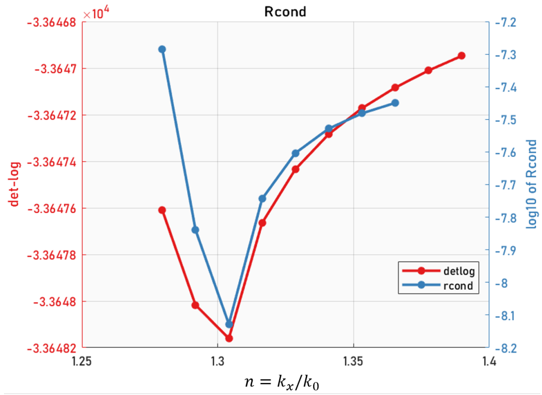

Fig. 2. System matrix singularity behaviour at 5 GHz for the unit cell defined in [10]. Comparison between det-log and reciprocal of the condition number versus the equivalent refractive index n.

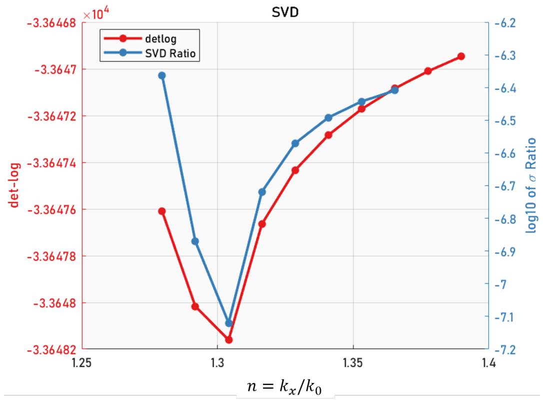

Fig. 3. System matrix singularity behaviour at 5 GHz for the unit cell defined in [10]; comparison between det-log and singular values ratio versus the equivalent refractive index n.

A uniform sampling in the equivalent refractive index $n = k_x/k_0$ could give an under-representation of what is happening close to the minimum region, possibly completly missing the minimum value completely. Moreover, although the technique requires only the generation of the unit cell $[Z(k_x)]$ matrix for each sampling point in n, it could be a heavy task if the sampling step is small (as it seems to be necessary from the stepped behavior around the minimum region). To deal with this issue, we propose to apply an automatic root finder technique applied in [11] to find the leaky-wave poles for grounded in-homogeneous dielectric slabs. The root finder algorithm starts from three initial guess points, and then look iteratively for the minimum point of the det-log (or rcond, svd) values. Hence we can run a first uniform coarse sampling in the region of interest and use then the three lowest values as initial guesses of the root-finding algorithm. Figure 4 shows the det-log behaviour versus n with a uniform sampling and a subsequent application of the root-finding algorithm. The points selected by the root-finding are highlighted in Fig. 4 by dots with a color that becomes darker iteration by iteration. The darkest dot corresponds to the det-log minumum and identifies the equivalent refractive index after 13 iterations.

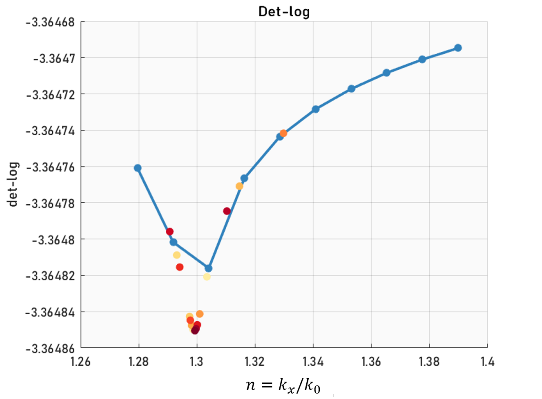

Fig. 4. Det-log of the system matrix at 5 GHz for the unit cell defined in [10]. Continuous line for uniform sampling, scattered points for root-finding approach (darker the point, higher the iteration).

IV. Conclusion and Perspectives

In this paper, we presented an alternative scheme to generate the dispersion diagram of higher-symmetry periodic structures. MoM-SIEs and periodic Green's function to analyze and design glide structures with non-canonical unit cells could substitute techniques based on mode-matching, which require decomposing each cell in canonical pieces, widening the application spectrum to non-canonical geometries. The proposed method is complemented by a root-finding algorithm that automatically finds the resonant situation, starting from the unit cell MoM matrix. The tests here presented were performed on a fully metallic implementation of a Luneburg lens.

The following research activities will involve and the efficient interpolation of Ewald-accelerated Periodic Green's function [8], and the parallelization and acceleration of the MoM matrix filling [12]. Furthermore, we expect to deal with unit cells containing finite dielectrics.

Acknowledgment

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