Abstract tesi

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Abstract

Berger's classification of Riemannian holonomy groups restricts the possibilities to a short yet elegant list. Starting from Berger's framework, two natural generalizations arise: relaxing either the metric assumptions or the torsion-free condition.

The first direction concerns hypercomplex manifolds, where the Obata connection is the unique torsion-free connection preserving the quaternionic structure. In my thesis, I provide the earliest examples of non-flat 2-step nilpotent hypercomplex Lie algebras and prove that the holonomy algebra of the Obata connection is always abelian for such algebras. This result refines the classical theorem of Barberis–Dotti–Verbitsky.

I also studied Joyce hypercomplex manifolds, namely compact Lie groups with left-invariant hypercomplex structures. Contrary to previous expectations, I showed that their Obata holonomy is typically a proper subgroup of $\mathrm{GL}(n, \mathbb{H})$.

The second direction develops the theory of the Bismut connection, the unique Hermitian connection with totally skew-symmetric torsion. Within this framework, I examined its curvature properties, focusing on a natural analogue of the Kähler Ricci-flat condition in the non-Kähler setting, usually called the Bismut–Hermitian Einstein (BHE) condition. Using suspension techniques from K3 surfaces, I constructed new examples in dimension eight. Furthermore, I proved a structural theorem for BHE manifolds with parallel torsion, showing that their universal cover splits as a product of a Kähler Ricci-flat manifold and a simply connected Bismut-flat one. These results reveal a general holonomy reduction phenomenon, which I established in full generality.

The thesis also investigates strong hyperkähler with torsion (HKT) manifolds, establishing a rigidity result in dimension eight: such manifolds are either hyperkähler or Hopf fibrations over four-dimensional orbifolds.

Finally, I studied generalized Kähler structures on solvmanifolds. I constructed a general framework that recovers all known examples of generalized Kähler solvmanifolds while also producing new examples beyond the solvmanifold setting. In addition, I completed the classification of SKT and generalized Kähler solvmanifolds in dimension six.

In summary, the thesis advances the understanding of the holonomy of the Obata and Bismut connections, combining explicit constructions with structural theorems, and uncovering new rigidity and reduction phenomena in complex and quaternionic geometry.

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