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Teaching and learning eigentheory in University linear algebra: a multifaceted analysis through different theoretical lenses

ABSTRACT

The aim of this thesis is to explore how geometric representations can be effectively integrated into the teaching and learning of linear algebra to enhance understanding of the underlying algebraic formal structure, rather than just emphasizing geometric properties. Specifically, this research focuses on promoting the learning of concepts related to eigentheory: eigenvectors, eigenvalues, and eigenspaces.

The main focus is on studying the teaching and learning processes of these concepts in linear algebra courses for undergraduate programs in STEM disciplines, such as engineering, where mathematics courses such as calculus and linear algebra are usually taught in the first years.

The first part of the thesis focuses on conducting a detailed analysis of students' understanding of eigenvectors and eigenvalues, highlighting the importance of different semiotic resources and their interrelationships in supporting conceptualisation. Additionally, it explores potential causes of difficulties in understanding these concepts, in particular how certain representations may contribute to common misconceptions. This analysis involves multimodal observation of the different semiotic resources used by students during two phases of a pilot study. Further theoretical constructs are employed to address specific research questions related to the issue under investigation.

Moving on to the second part of the thesis, attention turns to the institutional aspects associated with the problem under study. The compelling results obtained in the pilot study prompted a collaboration between myself and the involved course instructor to adapt the effective aspects of the pilot study activities to the actual teaching context. This context encompasses a linear algebra course for mechanical engineering students at a public Italian university, characterized by constraints that limit inquiry activities and productive class discussions. These constraints are discussed, together with the conditions conducive to the implementation of a teaching sequence on eigentheory, aiming to leverage the positive outcomes observed in the pilot study.

The theoretical framework in the first part of the thesis predominantly focuses on learning processes related to the use of different semiotic resources. The Semiotic Bundle theory serves as a foundational construct, complemented by other specific theoretical frameworks that are suitable for addressing different issues relevant to the overarching research goal. In the second part, the adoption of the Anthropological Theory of the Didactics made it possible to shift the lens of analysis to the institutional aspects that model the actual realization of a teaching sequence on eigentheory, whose design could make use of the observations made in the first part, together with other existing literature on the subject.

The integration of these two frameworks, despite their considerable differences in terms of unit of analysis and analytical tools, seemed imperative. As a result, a significant outcome of this thesis is the proposal of a strategy to coordinate, in the sense suggested by the construct of Networking of theories, the Semiotic Bundle theory with the Anthropological Theory of Didactics, leveraging their complementary nature to deepen an important but still understudied topic.