

# Bi-hyperbolic and tetra-hyperbolic isofrequency topologies in a gyroelectromagnetic medium

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**Abbreviated abstract:** Isofrequency topologies are studied for a gyroelectromagnetic medium obtained by stacking optically thin magnetized ferrite and semiconductor layers into a unified structure. In such a structure, both bi-hyperbolic and tetra-hyperbolic isofrequency contours appear as a simultaneous effect of both periodic arrangement of constitutive layers and external magnetic field influence. It is proposed to consider the obtained bi-hyperbolic and tetra-hyperbolic isofrequency contours as new topology classes of the wave dispersion

## **Related publications:**

- Fesenko, V. I. and V. R. Tuz, Phys. Rev. B, 99, 094404 (2019)
- Tuz V. R. and V.I. Fesenko, J. Appl. Phys., 128, 013107 (2020)

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# Motivation

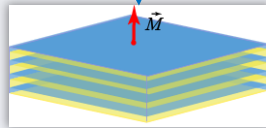
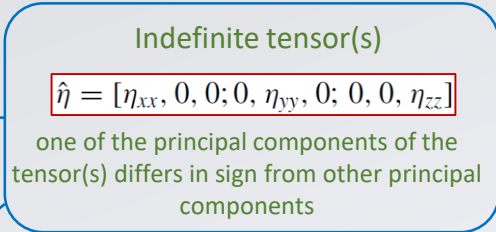
Hyperbolic (extremely anisotropic or indefinite) media



Metallic wire array  
(indefinite permittivity tensor)



Metallic split-ring resonators array  
(indefinite permeability tensor)



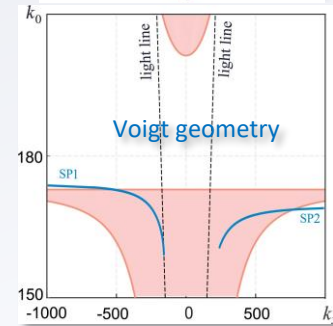
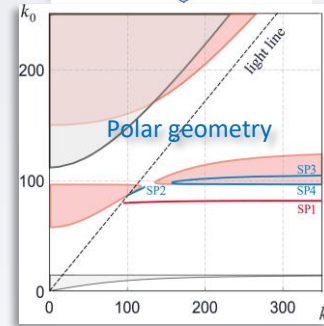
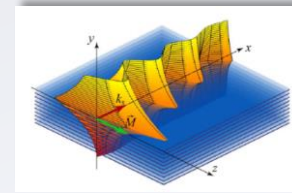
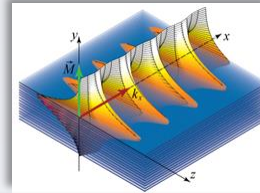
Magnetic-semiconductor superlattice  
(indefinite permittivity and permeability tensors)



Negative refraction, strong enhancement of spontaneous emission, broadband infinite density of states, light steering, sensing, the photonic spin Hall effect, super-resolution imaging, focusing, signal routing, and abnormal scattering.

## Related previous studies

The effect of coexistence of bulk and surface polaritons in a magnetic-semiconductor superlattice, which is influenced by an external static magnetic field in the polar [1] and Voigt [2] geometries, was revealed

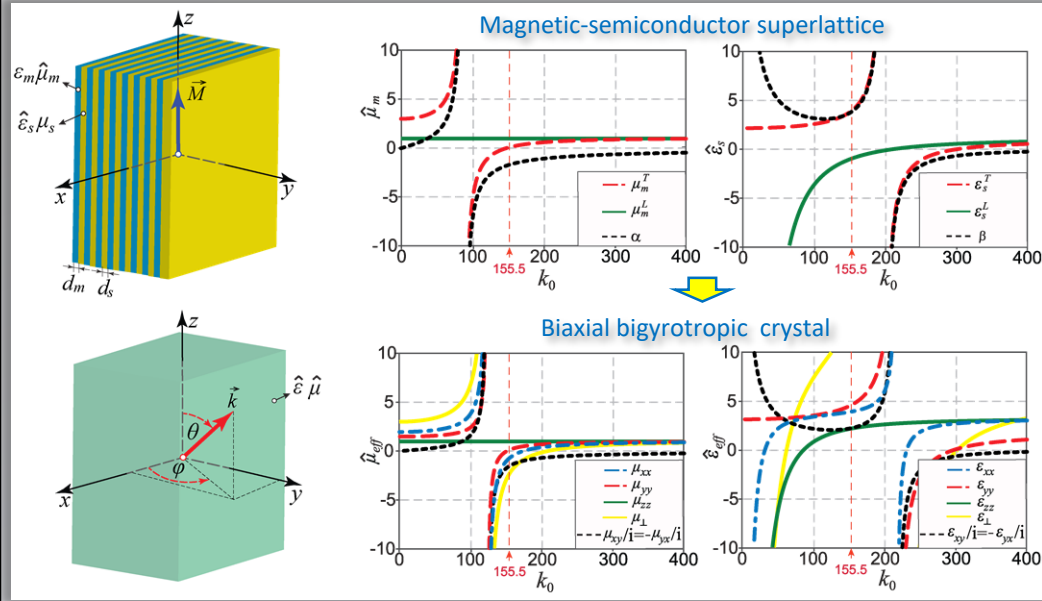


**Related publications:** 1. V.R. Tuz, et al., J. Appl. Phys. 121, 103102 (2017);  
2. V.I. Fesenko, et al., Opt. Lett., 41(9), 2093-2096 (2016);

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# Dispersion relation for bulk waves



Long-wavelength limit

$$d_m \ll \lambda, d_s \ll \lambda, L \ll \lambda$$



Homogenization procedure  
of the effective medium theory [1]



Superlattice  
BaCo/Si

Biaxial bigyrotropic  
crystal

$$\begin{Bmatrix} \epsilon_m, \hat{\mu}_m \\ \hat{\epsilon}_s, \mu_s \end{Bmatrix} \Rightarrow \hat{g}_{eff} = \begin{pmatrix} \tilde{g}_{xx} & \tilde{g}_{xy} & \tilde{g}_{xz} \\ \tilde{g}_{yx} & \tilde{g}_{yy} & \tilde{g}_{yz} \\ \tilde{g}_{zx} & \tilde{g}_{zy} & \tilde{g}_{zz} \end{pmatrix}$$

$$\tilde{g}_{xx} \neq \tilde{g}_{yy} \neq \tilde{g}_{zz} \quad \tilde{g}_{xy} = -\tilde{g}_{yx} \neq 0$$



$$\begin{aligned} & (\epsilon_{zz}\mu_{zz})^{-1} \left\{ k_x^4 \epsilon_{xx}\mu_{xx} + k_y^4 \epsilon_{yy}\mu_{yy} + k_z^4 \epsilon_{zz}\mu_{zz} + k_x^2 k_y^2 (\epsilon_{xx}\mu_{yy} + \epsilon_{yy}\mu_{xx}) + k_x^2 k_z^2 (\epsilon_{xx}\mu_{zz} + \epsilon_{zz}\mu_{xx}) + k_y^2 k_z^2 (\epsilon_{yy}\mu_{zz} + \epsilon_{zz}\mu_{yy}) \right. \\ & \left. - k_0 [k_x^2 (\epsilon_{xx}\epsilon_{zz}\mu_{\perp} + \mu_{xx}\mu_{zz}\epsilon_{\perp}) + k_y^2 (\epsilon_{yy}\epsilon_{zz}\mu_{\perp} + \mu_{yy}\mu_{zz}\epsilon_{\perp}) + k_z^2 \epsilon_{zz}\mu_{zz} (\epsilon_{xx}\mu_{yy} + \epsilon_{yy}\mu_{xx} - 2\epsilon_{xy}\mu_{xy})] + k_0^4 \epsilon_{\perp}\mu_{\perp} = 0. \right. \end{aligned}$$

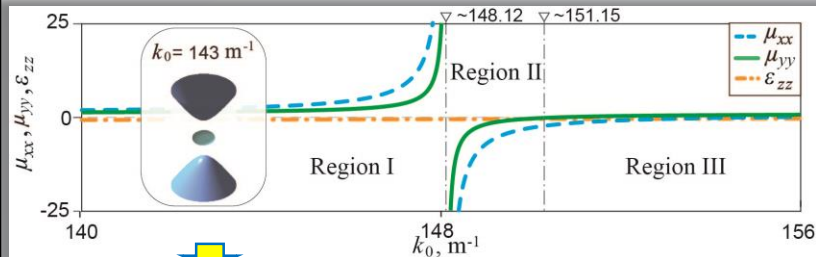


Related publication: 1. V.R. Tuz, et al., (InTech, 2018), Chapter 6, pp. 99-125. ISBN: 978-1-78923-063-5.

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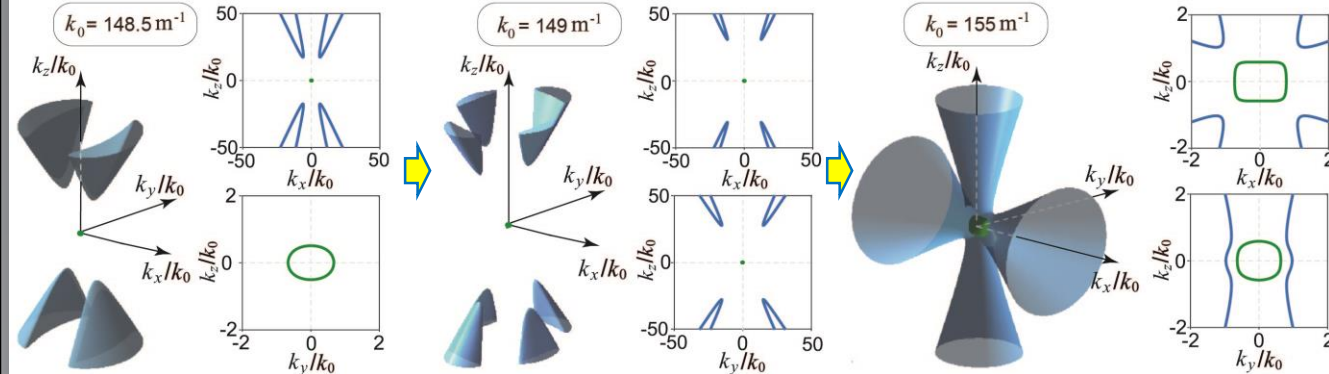
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# Topological transitions



Region I  $(\mu_{xx} > 0) \wedge (\mu_{yy} > 0)$ ,  
 Region II  $(\mu_{xx} < 0) \wedge (\mu_{yy} < 0)$ ,  
 Region III  $(\mu_{xx} < 0) \wedge (\mu_{yy} > 0)$ .  
 $\epsilon_{zz}$  is always a negative quantity.

The topological transitions of isofrequency surfaces of extraordinary high- $k$  waves from an **open type-I hyperboloid** to the form of a **cone cut into either two or four parts (tetra-hyperbolic)** [1] as well as a **bi-hyperboloid** [2,3] are revealed



- Related publications:** 1. Tuz V. R. and V.I. Fesenko, J. Appl. Phys., 128, 013107 (2020);  
 2. Fesenko, V. I. and V. R. Tuz, Phys. Rev. B, 99, 094404 (2019);  
 3. V.R. Tuz, I.V. Fedorin, V.I. Fesenko, Opt. Lett., 42(21), 4561-4564 (2017).