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Existence of expansive solutions and Hamilton-Jacobi equations for the N-body problem

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This thesis collects recent results on the N-body problem in Celestial Mechanics, studied within the framework of the calculus of variations.

First, we consider the classical N-body problem and study the existence of action-minimizing half-entire expansive solutions with prescribed initial configurations and asymptotic directions. We treat hyperbolic, hyperbolic-parabolic, and parabolic arcs in a unified way. The variational approach we adopt consists of minimizing a suitably renormalized Lagrangian action over an appropriate functional space. Within this framework, we recover the known existence results for hyperbolic and parabolic solutions and, for the first time, establish the existence of hyperbolic-parabolic solutions for any prescribed asymptotic expansion within a suitable class.

Next, we investigate the Hamilton-Jacobi equation associated with the N-body problem in $\mathbb{R}^d$. To each expansive solution in the above classes, we associate a viscosity solution of the Hamilton-Jacobi equation, obtained as a linear correction of the value function. We then analyze the singularities of these solutions, defined as the initial configurations for which the minimizer of the corresponding variational problem is not unique. Moreover, we study the size of the closure of the singular set by proving its $\mathcal{H}^{d(N-1)-1}$-rectifiability and provide an upper bound on the Hausdorff dimension of the set of regular conjugate points.

Finally, we address the existence of periodic symmetric solutions to the N-body problem. Classical results by Ferrario and Terracini guarantee the existence of such solutions under suitable symmetry constraints on the configuration space. In addition, a numerical algorithm has been developed to generate symmetric solutions. We employ the most up-to-date version of this software, `Symorb.jl`, to compute numerical solutions and investigate their stability properties via numerical algorithms designed to compute Floquet multipliers and Morse indices. Numerical experiments are presented to illustrate our methods in both two- and three-dimensional configuration spaces and for various choices of the number of bodies.