Informal mathematics in teacher education: The teachers' voice

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Raffaele Casi[1] and Cristina Sabena

University of Turin, Italy; raffaele.casi@unito.it

Informal Mathematics Education is an emergent field of research in which out of school spaces become protagonist of intentional learning designs. In our research we exploit cultural spaces such as art and history museums to engage teachers in a challenging teacher education programme called "InformalMath". In InformalMath teachers work in communities of practices to design informal mathematics workshops, with the support of teacher educators and museum experts. The paper presents the theoretical choices underpinning the design and implementation of InformalMath and gives voice to the enrolled teachers.

Keywords: Informal mathematics education, emergent learning, trialogical approach, museums.

Introduction

Unconventional educational settings, as seen in programs like Researchers' Nights, science festivals, and mathematical walks (see MathCityMap, Zender et al., 2020), are on the rise as platforms for math promotion. Its achievements highlight widespread interest in mathematics and science, encouraging a deeper exploration of their educational possibilities in informal contexts. Over the last few years, there has been a growing interest in learning mathematics in museums. Experiences such as (Kayhan Altay & Yetkin Özdemir, 2022; Kelton, 2021) are evidence of a new attention to this kind of activity. Such activities, pertaining to the strand of Informal Mathematics Education (IME) described by Nemirovsky, Kelton and Civil (2017), "differs from everyday mathematics because informal mathematics education environments are intentionally designed to support mathematics learning" (p. 970). It is on the intentional design of these environments that the contribution we present is focused.

As part of the first author PhD project, we designed and implemented a two-year teacher education programme for primary and lower secondary school teachers (grades 1 to 8), that we called InformalMath, focusing on the design of IME workshops. Twenty-seven teachers (13 from primary school, 14 from lower secondary school) voluntarily enrolled in InformalMath, which started in November 2020. The programme is centered on the design of *IME workshops* (Casi & Sabena, 2022). We conceive informal mathematics education workshops as a set of inquiry-based activities centered around a chosen theme. Specifically, we chose to set the workshops in art or historical museums, namely public cultural places without any apparent link with mathematics.

The design of *InformalMath* and its theoretical background

Given the novelty - at least in the Italian scenario - of informal mathematics education workshops in museums, we chose to begin our programme involving teachers first as learners. When characterising the peculiarities of IME spaces and their difference from most classroom environments, Nemirovsky and colleagues (2017) point out that the formers are characterised by:

IME-1. The learners' free choice: "for the most part, learners volunteer to participate in them or are relatively free to pursue their own interests once they are in the environment".

IME-2. The fluidity of the boundaries between disciplines: "activities may drift from mathematics to art, literature, science, games, technology, and so forth".

IME-3. The absence of traditional forms of academic assessment: "Informal mathematics education needs to be documented for the purposes of professional development and collective exchange, but learners are not individually graded with scores" (p. 970).

Given the type of involvement required to teachers, the peculiarity of the different subjects involved in their learning, and the lack of traditional forms of assessment related to the proposed activities, the type of learning that results from the workshops proposed to teachers in the InformalMath programme may be considered as an *emergent learning*. Nemirovsky proposes an intriguing contrast between teleological learning, namely "learning for the sake of passing pre-defined tests and goals" (2018, p. 401), and emergent learning, a kind of learning "elusive to anticipated aims and predicted outcomes" (p. 403). From this perspective, the learning that teachers achieve at the end of the InformalMath programme may be very different from the one expected a priori, possibly even involving disciplinary fields that had not been considered in the design. This allows the programme participants, teachers and didacticians, to place themselves asymmetrically on the "will to will" dimension, meaning the different will and motivation for participation in the programme, and symmetrically on the "intelligence to intelligence" dimension, considering instead the equality "in their autonomy for expression, recollection, conceptualization, initiative, and insight" (p. 403).

ICME13 survey on teachers working and learning through collaboration point out that

> Collaboration implies co-working (working together) and can also imply co-learning (learning together). It involves teachers in joint activity, common purpose, critical dialogue and inquiry, and mutual support in addressing issues that challenge them professionally. (Robutti et al., 2016, p. 652)

Such a collaboration can generate *communities of practice*, as defined by Wenger (1999): groups of people "informally bound by what they do together and by what they have learned through their mutual engagement in these activities". As the community created by teachers plays a fundamental role in learning, we adopted the Trialogical Learning Approach (TLA) (Paavola & Hakkarainen, 2005) for the design of our programme. This approach exploits many typical techniques of collaborative learning, focusing on strengthening them through structuring activities based on the construction of objects intended for tangible use. The adjective "trialogical" refers to the integration of the monological approach, based on the learning-as-acquisition metaphor, with the dialogical approach, referring to the learning-as-participation metaphor. In the TLA, the reference metaphor for learning is that of knowledge-creation, in which "the emphasis is not only on individuals or on community, but on the way people collaboratively develop mediating artifacts" (Paavola & Hakkarainen, 2005, p. 539). In our intent, such artifacts are precisely the informal mathematics education workshops designed by teachers in museums.

In outlining the TLA, Hakkarainen and Paavola (2009) describe six design principles that support the conception of pedagogical scenarios aimed at the collaborative construction of knowledge artefacts:

TLA-DP1. Focus on shared objects of activity which are developed collaboratively, whether they are conceptual artifacts, concrete, material products or practices taken as objects of inquiry;

TLA-DP2. Sustained and longstanding pursuit of knowledge advancement. Novelty and innovation emerge through iterative efforts taking place across extended time scales.

TLA-DP3. Knowledge-creation processes taking place in mediated interaction between individual and collective activities. Individual participants may have a key role in knowledge creation, but their efforts are embedded on a fertile ground of collective activity.

TLA-DP4. Cross-fertilization of knowledge practices between educational, professional, and research communities in terms of bringing cultures of schooling in closer contact with professional cultures and engaging students in expert-like knowledge practices from the very beginning of their studies.

TLA-DP5. Technology mediation. There must be appropriate technologies that help the participants to create and share as well as elaborate and transform knowledge artifacts.

TLA-DP6. Development through transformation and reflection. Novel ideas emerge through interaction between conceptualizations and practical explorations. Crystallization of evolving ideas in shared practices and routines plays an important role in the process. (p. 78)

We took inspiration from these principles in designing the InformalMath programme. As we perceived the objective of acquiring competences on designing mathematics education workshops as challenging for teachers and in accordance with TLA-DP2, we chose to articulate the programme over a long period of time, which indeed is fundamental in order to create a community of practice (Robutti et al., 2016, p. 665). We planned the programme along three phases. In the first phase (December 2021-March 2022), teachers have been introduced to the idea of informal mathematics education workshops and spent two entire days in two museums of history and art (Museo Nazionale del Risorgimento Italiano and Palazzo Madama, both in Torino). During each morning, teachers acted as learners and experienced an informal math workshop[1], led by a museum guide. In the afternoon, teachers were engaged in discussing the morning experience and in deepening related mathematics topics and mathematics education aspects. Museums guides and their coordinators also had the chance to meet teachers and to discuss with them out of their experience with the informal mathematics workshops. In phase 2 (April 2022 – January 2023) we engaged the teachers in exploring two new museums (Castello di Rivoli – Contemporary Art Museum and PAV – Parc of Living Art), under the guidance of museum experts, with the goal of identifying emerging themes for designing informal math workshop. In a following meeting we facilitated the reflection on the theoretical and methodological elements provided in phase 1, meant to be useful tools to design the workshops. Among them: problem-solving in mathematics education, inquiry-based learning, the role of artifacts, the informal mathematics education perspective. In this meeting, the teachers formed groups based on common interests (communities of practice). Each group chose one of the two museums and started designing an informal math workshop gauged to a hypothetical class of students. The design took five months and during this period the teachers could visit the chosen museum as they wished, meeting the museum experts and us. An important step towards the refinement of the design was the presentation of the first draft to the other groups and to museum experts, in a dedicated session. The

[1] The two workshops were designed by Raffaele Casi, Valentina Leo, Chiara Pizzarelli and Cristina Sabena, as part of the Next-Land project, running in 2019-20 (next-level.it/progetti/next-land/). The workshops were intended for 5-7 grades students. Museums guides were purposely trained by our team. See Casi and Sabena (2020).

final designs were given to the museums so that they could be included in their educational offer to schools. They are available, in the Italian language, on the InformalMath website (informalmath.unito.it). Phase 3 (February -September 2023) has just started and allows the teachers with an increased autonomy: they will choose a museum of their own town (therefore engaging in making connections with the museums experts, in autonomous way) and design a new mathematics education workshop. They will again work in small communities of practice, and we as mathematics educators we will be at the teachers' disposal to give feedback or help when they deem it is necessary. The idea is that teachers will develop the sufficient competence to be able to meet the challenge in the future, if they will decide to strengthen the links between schools and cultural highlights of territories in which schools are located, so that *meaningful cultural experiences may be offered to all students* (as renting a bus to take students to museums is a cost that some families may not afford).

Research focus and methods

Twenty-seven teachers enrolled in the first phase of the programme, both from primary and lower secondary school (grades 1-8). Twenty-two of them chose to continue the programme in phase 2, and twelve will continue in phase 3 also. Given the intelligence-to-intelligence symmetry arising from the design choices, it seemed to us natural to consider in our research and as teacher educators the voices of the teachers involved. In this paper, we focus on reporting teachers' voices after their involvement in phase 2. To do this, we set up two data collection instruments. The first tool is a personal essay, prompted by the following question:

As you are aware, the InformalMath programme was conceived following the pedagogical principles of emergent learning […]. It is thus important for us as trainers to know what learnings emerged in you as a result of your participation in the programme, particularly in phase 2. We would like to ask you to tell us in written form what learnings for your professional work as a mathematics teacher have emerged in you because of your participation in phase 2 of the programme.

Seventeen out of 22 teachers wrote the essay. The second tool was a focus group: after collecting personal papers, the four design groups participated in a focus group with the trainers, which included discussion of the following topics: *How did phase 2 of the programme go? What difficulties did you face, and what resources did you activate to overcome them? What difficulties related to mathematics did you identify? What enrichment did you gain from your participation in phase 2 of the programme? What suggestions - if any - could you give us for a possible re-edition of the programme?*

The combination of the two instruments, allows us to integrate a personal and a community viewpoint. Specifically, in this paper we address the following research questions:

RQ-1. What emergent learning did the teachers notice as a result of participating in the project?
RQ-2. What critical issues and difficulties did the teachers perceive?

We analyzed the collected data through the lens of qualitative content analysis (Mayring, 2015), using inductive category formation.

Results

To answer RQ-1, we first examined the transcripts of the focus groups. With inductive category formation we built up categories concerning the emergent learning reported by teachers.

Subsequently, we analyzed the individual essays using the same methodology. In Table 1, we list the most frequent categories, showing the occurrences found, the number of groups and of personal essays in which we encountered these categories.

Table 1: Categories of emergent learning founded in focus groups and personal essays

Emergent learning reported by teachers	Occurrences	# Groups	# Essays
L1. Dialogue between teachers: an experience of personal and professional enrichment	17 occurrences	3 groups	15 essays
L2. The museum environment: a new experience of designing educational activities outside the classroom	14 occurrences	3 groups	9 essays
L3. Studying mathematics: the need to learn more about mathematics concepts, both for planning and for teaching	12 occurrences	3 groups	5 essays
L4. Learning by doing: reflecting on the way learning is generated by design work	9 occurrences	4 groups	0 essays
L5. Getting new eyes: the InformalMath experience as an eye education to look for mathematics all around, to pick up ideas for the design of mathematical activities, in the museum and in the classroom	9 occurrences	4 groups	6 essays
L6. The meaningfulness of the mathematical experience: when designing the IME workshop, teachers are led to ask themselves whether the mathematical activity in which they will involve students is meaningful for learning	9 occurrences	3 groups	0 essays

Due to space limitation, we will only provide examples and discuss L2, L5 and L6. With respect to L2, we report Bianca's (pseudonym) voice:

Bianca: The most difficult thing for me was starting from deductive thinking. What to look for in the museum? Something to stimulate the search for mathematics in things. And to think about how children would do the reverse process. So, from our inductive thinking to their deductive thinking. So, they would go and find, while we searched: I looked for something in the piece of art that would inspire, something related to mathematics, while in our project we did the exact opposite. This was in my opinion the most beautiful thing on the one hand, but also obviously more complicated for me.

Cristina: Tell us more, I am curious about this idea

Bianca I give a practical example: when we saw Metro cubo di infinito [pièce of Michelangelo Pistoletto], we noticed that it is made by rectangles. The faces were rectangles, forming a cube. Whereas in our design we ask children to build a cube from rectangles. So, there is a totally opposite process, because we first saw the cube, and then we reasoned that with rectangles you can build a cube, by putting them in a particular position. Instead, the children must find the solution, so it's a different process. Have I explained myself a little better?

The same concept expressed is found in Elena's personal essay:

> The transition was the opposite of the usual way: I moved from the practical phase (exploration of the museum) to the theoretical phase (development of a workshop), experiencing at first hand, physically, the artifacts proposed by the museum. (Elena)

The teachers reflect on the process of designing IME workshops, which is stimulated by a discovery experience and follows a different path with respect to the typical processed in formal education context. In both cases, the starting point for the design is a mathematical topic, but in the informal setting the idea arises by the very experience of visiting the museum.

In the following, we report some excerpts regarding L5, in which we observe how teachers did notice a change, feeling that they can see the reality around them with mathematical eyes, even beyond the museum experience.

Bianca: When we went back to visit the Rivoli Museum the second time [after we had written the first draft of the workshop], we focused more on geometry, and then we saw, as if our cataract veil had been removed, geometric shapes everywhere…

Marianna The greatest asset of this course was precisely that: being able to find inspiration in any other field, anywhere else. [...] But this principle, perhaps the very goal you set for the programme – looking at reality with a mathematical eye – has been achieved.

I certainly learned to see environments, normally unrelated to a more strictly mathematical context, from a different point of view. […] I can say that I find myself more and more often wandering around places, buildings and streets, trying to find connections with mathematics... (Beatrice)

The perspective of finding mathematics in unexpected environments forces a different kind of attention and engagement, which are typical of the attitude of those who explore the unknown. As explorers, we analysed the new environment and let ourselves be surprised by the elements that inspired us. [...] Marcel Proust said that "The true voyage of discovery consists not in seeking new lands, but in having new eyes", and this summarises the most important learning I feel I have experienced. (Dario)

Dario, in particular, underlines the dimension of the unexpected, referred to the mathematics that could potentially emerge in a certain environment.

As regards L6, we report the voice of a group of secondary school teachers.

Dario I haven't studied mathematics at university, so I don't know whether the simplification of the topic that I propose to students and that I have clear in my mind is good or whether it leaves out fundamental aspects.

Lina I agree, I always have a lot of insecurities, due to my non-mathematical background. I think the biggest difficulty was, "Am I doing meaningful mathematics?"

Cristina This is a big question! Is it a question that also comes up in your usual teaching, in the classroom? Or has it come up more with this project?

Beatrice In the classroom, I think it's easier to understand if it's effective or not what you do, you have guidelines to follow: a programme, a textbook, exercises. You can manage to set it up and make it meaningful. As Lina, I wondered "This activity, does it matter? Does it teach something or is it just entertaining? Can you see the mathematics?". I've had this feeling during this programme, not in the classroom.

Contrasted to the mathematics education carried out in the classroom, the experience of designing the IME workshop appears to disrupt the teachers' self-confidence. On the other hand, this experience allows them to ask themselves fundamental questions about the meaningfulness of their mathematics education activity. The search for an answer to this kind of question may engage teachers in deeper epistemological reflections, which are crucial in the formal education process also.

To answer the RQ-2, we looked for critical issues and difficulties perceived and reported by teachers. As in the personal essays we could not identify any critical issue, we examined the focus groups transcripts only. Table 2 shows the most frequent categories, their occurrences, and the number of groups in which we encountered it.

Table 2: Categories of critical issues and difficulties perceived founded in focus groups

Critical issues and difficulties perceived reported by teachers	Occurrences	# Groups
D1. Time and space: the need for time and space (physical and/or virtual) for collaboration between teachers	21 occurrences	4 groups
D2. Primary - Secondary: difficulties (and resources) arising from the interaction of teachers of different grades	13 occurrences	3 groups
D3. Relationship with museum experts: difficulties in "speaking the same language"	12 occurrences	4 groups
D4. Ineffectiveness of work fragmentation: mode of work in which each member of the group deals with one part, and then the final work is "tailored"	9 occurrences	2 groups
D5. Doing and undoing the design: the need to revise one's own ideas and stances	8 occurrences	3 groups

We briefly comment D1 and D2. D1 relates to time and space. All the occurrences identified point to the fact that also teachers involved in InformalMath suffer from a lack of adequate time and space to dedicate to their professional development. This issue is well known in the literature (Brodie, 2020).

Regarding D2, in the three groups composed of both primary and secondary school teachers, it emerged several times how the presence of teachers from different grades, initially perceived as a difficulty, was subsequently seen as an obstacle to be overcome and finally as a resource that provided added value to the projects implemented:

Federica: When we middle schoolers were placed in the primary group, at the beginning I said "Oh dear, who knows how we will manage to work!", because I was convinced that we would be working on two very different levels. Instead, I was pleased to notice that the needs and critical points of primary school are the same as those of sixth grade. So, when we were planning the activities, it really didn't feel like we were primary school teachers and middle school maths teachers. This also helped me a lot in thinking about the design of the vertical curriculum for my school.

Emma I also at the beginning felt what you say, Federica, when we found ourselves into working together, because unfortunately maybe we have a sort of prejudice.

Elena It's not easy to create a group with people who don't know each other, and from different school levels. But it didn't scare me or discourage me, because we were all there for the same reason, no one was forcing us, and we all started with a lot of determination and ideas. A lot of desire to do all the things we wanted to do.

In Elena's voice we can recognize how being conscious of a fundamental feature of IME, namely the learners' free choice and the volunteer participation in the activities, prevented her of being scared or discouraged by the challenge of co-working with colleagues from a different school level.

Conclusion

In participating in InformalMath, teachers felt part of the learning community, exercising their intelligence to intelligence symmetry, especially in reflecting on emerging learnings and detected difficulties. It seems to us that some noticed learnings can be attributed to the specificity of the informal experience in the museum, while others refer more generally to the chosen work method (specifically, TLA and community of practice). Regarding critical issues, we point out that some of them have just been acknowledged, while others have been turned into resources. All this is a treasure for our community of practice, in view of the forthcoming phase 3. From a research perspective, it will be interesting to observe the evolution of emerged learning, as well as the emergence of new ones. A more challenging research path will be investigating if and how the learning emerged in

InformalMath will percolate in the teachers' classroom practice, where formal mathematics learning takes place.

References

Brodie, K. (2020). Resources for and from collaboration: A conceptual framework. In H. Borko & D. Potari (Eds.), *Proceedings of the Twenty-Fifth ICMI Study*, (pp. 37–48). National and Kapodistrian University of Athens.

Casi, R., & Sabena, C. (2022). Informal mathematics experiences in museums: What potential for teacher professional development? In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of European Research in Mathematics Education (CERME12)* (pp. 3065–3066). ERME/Free University of Bozen-Bolzano.

Hakkarainen, K., & Paavola, S. (2009). Toward a trialogical approach to learning. In B. Schwarz, T. Dreyfus, & R. Herskowitz (Eds.), *Transformation of knowledge through classroom interaction* (pp. 73–88). Routledge.

Kayhan Altay, M., & Yetkin Özdemir, E. (2022). The use of museum resources in mathematics education: A study with preservice middle-school mathematics teachers. *Journal of Education for Teaching*, *49*(4), 616–629. https://doi.org/10.1080/02607476.2022.2150534

Kelton, M. L. (2021). Mathematics learning pathways on a school fieldtrip: Interactional practices linking school and museum activity. *Visitor Studies*, *24*(2), 220–242. https://doi.org/10.1080/10645578.2021.1939984

Mayring, P. (2015). Qualitative content analysis: Theoretical background and procedures. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education: Examples of methodology and methods* (pp. 365–380). Springer Netherlands. https://doi.org/10.1007/978-94-017-9181-6_13

Nemirovsky, R. (2018). Pedagogies of Emergent Learning. In G. Kaiser, H. Forgasz, M. Graven, A. Kuzniak, E. Simmt, & B. Xu (Eds.), *Invited Lectures from the 13th International Congress on Mathematical Education* (pp. 401–421). Springer International Publishing. https://doi.org/10.1007/978-3-319-72170-5_23

Nemirovsky, R., Kelton, M. L., & Civil, M. (2017). Toward a vibrant and socially significant informal mathematics education. In J. Cai (Ed.), *Compendium for research in mathematics education*. ERIC.

Paavola, S., & Hakkarainen, K. (2005). The knowledge creation metaphor – An emergent epistemological approach to learning. *Science & Education*, 14, 535–557. https://doi.org/10.1007/s11191-004-5157-0

Robutti, O., Cusi, A., Clark-Wilson, A., Jaworski, B., Chapman, O., Esteley, C., Goos, M., Isoda, M., & Joubert, M. (2016). ICME international survey on teachers working and learning through collaboration: June 2016. *ZDM*, 48, 651–690. https://doi.org/10.1007/s11858-016-0797-5

Wenger, E. (1999). *Communities of practice: Learning, meaning, and identity*. Cambridge University Press.

Zender, J., Gurjanow, I., Cahyono, A. N., & Ludwig, M. (2020). New studies in mathematics trails. *International Journal of Studies in Education and Science* (IJSES), *1*(1), 1–14 https://doi.org/10.46328/ijses.8