

Walls from fluxes: An Analytic RG flow

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Walls from fluxes: an analytic RG-flow

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ABSTRACT: We construct supergravity solutions describing the near horizon limit of D1D5 systems with non-trivial boundary conditions. Upon reduction to five dimensions they define Melvin universes with NS-NS/RR fluxes, that smoothly interpolate between two different AdS_3 geometries which define fixed points for the RG-flow of the dual field theory. We discuss the decoupling limits at the two ends of the flow. We also present a systematic study of the global properties of our solution. In particular we show how, although the $AdS_3 \times S^3$ global isometry group is broken down to $SU(2)_R \times U(1)^3$ by global identifications, a full two-dimensional conformal group of isometries, with the expected central charge, is restored at infinity.

KEYWORDS: Renormalization Regularization and Renormalons, D-branes, AdS-CFT Correspondence.

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1. Introduction

The proposals [1, 2] for an holographic correspondence between String (or M-) theory on Anti-de Sitter backgrounds and suitable boundary gauge theories have provided a powerful tool for the study of the strong coupling regime of SYM theories. Amazingly enough, phenomena like confinement, chiral symmetry breaking and strong-weak Seiberg dualities in certain $\mathcal{N} = 1$ gauge theories are already described in terms of the dual classical supergravity [3]. These extremely encouraging results are however limited to a very restricted number of examples. A similar analysis in a non-conformal $\mathcal{N} = 2$ framework has been hampered so far by the presence of *enhancon singularities* and, although a number of achievements were recently made in this direction [4], a completely satisfactory picture of the gravity dual is still missing. The proposals involving maximally supersymmetric compactifications of M-theory on $AdS_4 \times S^7$ or $AdS_7 \times S^4$ are instead limited by the poor understanding of the superconformal field theories living on M-branes. Despite these immediate difficulties, the correspondence have enlightened a beautiful interplay between gauge theory and gravity physics and the impressive amount of results in the last few years justify the initial enthusiasm (for a review and references see [5]).

The low energy physics around vacua of type II (or 11-dimensional) supergravities involving AdS spaces times spheres (or more general Einstein spaces) can be efficiently described in terms of suitable gauged supergravities on AdS vacua. These effective descriptions are believed to be consistent truncations of a higher dimensional supergravity theory reduced on the internal Einstein space. A solution of the gauged supergravity defines a solution in the higher dimensional theory and vice-versa, although the details of the

lifting are often hard to determine. In a nice series of works pioneered by [6], domain wall solutions of five-dimensional $\mathcal{N} = 8$ supergravity that interpolate between Anti-de Sitter vacua with different number of supersymmetries were studied and a detailed correspondence between bulk fields and composite operators in the infrared gauge theory was constructed. Although the equations of motion of gauged supergravities which describe interesting flows are typically rather complicated to solve, many important features of the flow can already be read from the physics around the two fixed points. Moreover the complete interpolating solution, when not known analytically, can be dealt with numerically, obtaining some valuable information about the flow. In the cases where an analytic kink solution is available, a more quantitative information like correlation functions, scalar operator mixings, etc. can be determined explicitly from the flow (see [7] and references therein).

The aim of the present paper is to provide examples of analytic kink solutions of three dimensional gauged supergravity, where the details of the flow and the lift to nine or ten-dimensions can be explicitly displayed. The solutions describe the near horizon limits of D1D5 and D1D5+KK monopoles bound state systems in freely acting orbifold compactifications of type IIB. They will always contain a trivial T^4 or $K3$ part on which the D5 branes are wrapped. For simplicity we will omit this part in most of our discussion and refer to the lift as a lift to five or six dimensions. The domain wall solutions will be determined as solution of five-dimensional supergravity after reduction from more familiar six-dimensional geometries on a circle with non-trivial boundary conditions. In the case of pure D1D5 system the solution interpolates between an $AdS_3 \times S^2$ and a dilatonic $AdS_2 \times S^3$ vacuum of five dimensional supergravity. The latter can be better described in terms of a further lift to six-dimensions where it is given by the more familiar $AdS_3 \times S^3$ vacuum with constant dilaton. In the case of D1D5+KK monopole and fluxes, the five-dimensional solution can be extended all the way out of throat to a Ricci-flat asymptotic geometry with constant dilaton and therefore a sensible five-dimensional description is available at the two ends of the flow.

From the CFT point of view the walls describe the RG-evolution out of two dimensional $\mathcal{N} = (4, 0)$ conformal field theories living on the AdS_3 boundaries. Amazingly, the whole flow is generated by a non-trivial choice of boundary conditions on the familiar D1D5 systems for type IIB on M_4 with M_4 being T^4 or $K3$. More precisely the solution describes the near horizon geometry of D1D5 systems in type IIB on $M_4 \times \mathbb{R} \times (\mathbb{R}^4 \times S_1)/Z_N \sigma_{\frac{1}{N}}$, with Z_N acting as a rotation of \mathbb{R}^4 and $\sigma_{\frac{1}{N}}$ an order N shift along a longitudinal circle of radius NR . The two fixed point geometries, which we will refer to as “deep inside” and “asymptotic” regions, can be reached by sending R to zero or infinity while keeping N large but fixed. In the case $R \rightarrow 0$ the effects of the shift can be neglected and the system effectively lives in type IIB on $M_4 \times \mathbb{R} \times S_1 \times \mathbb{R}^4/Z_N$ (see [8, 9] for example). At the other end of the flow $R \rightarrow \infty$ the theory decompactifies to IIB on $M_4 \times \mathbb{R}^{1,5}$. Alternatively one can think of the orbifold as a compactification of the system on a circle of radius R , where fields on the circle are periodic only up to a Z_N rotation of the transverse S^3 . Upon reduction to five dimensions this procedure leads to Melvin solutions with non-trivial NS-NS fluxes. The Z_N will be always embedded inside an $SU(2)_L$ subgroup of the full $SO(4)$ isometry group of S^3 and therefore the solution preserves half of the original $\mathcal{N} = (4, 4)$ supersymmetries. Similar ideas have been extensively exploited in

the construction of flux-brane solutions [10, 11] (see [12] for earlier works in the subject). The case of supersymmetric flux-branes have been first discussed in [11]. Supersymmetric versions of NSNS Melvin universes and the spectra of open strings living on D-branes in these backgrounds have been discussed in [8] (see also [9] for similar results in the context of RR flux-branes).

The second part of the paper is devoted to the study of the global properties of our solutions, seen as “tilted” locally $AdS_3 \times S^3$ geometries in six dimensions. The effects of the global identifications on AdS_3 geometries have been studied in a beautiful paper by Brown and Henneaux [13]. In particular, they showed how point mass solutions carrying non-trivial momentum charges can be constructed in asymptotically AdS_3 vacua by modding the geometry over global identifications. The effect of the orbifolding in our $AdS_3 \times S^3$ geometry is in some sense milder and the solutions still define vacua carrying no charges at infinity. In addition we show how although the global isometry group is drastically reduced from $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R \times SU(2)_L \times SU(2)_R$ to an $SU(2)_R \times U(1)^3$ subgroup, the full two-dimensional conformal group is restored at infinity and is realized in terms of two copies of a Virasoro algebra with the expected central charge.

The paper is organized as follows: in section 2, we construct, via NS-NS Melvin fluxes, domain wall solutions of five-dimensional supergravity and discuss their near and asymptotic fixed point limits. Section 3 is devoted to the discussion the global properties of the kink solutions from the six-dimensional point of view. In section 4 we include some concluding remarks and comment on interesting directions of future research.

2. Melvin universe as domain walls in five-dimensional supergravity

In this section we construct supergravity solutions corresponding to geometries that look locally (but not globally) as products of AdS spaces times spheres. We follow closely the lines of [10] where similar solutions were found for M-theory on Ricci flat spaces. As in those cases the non-trivial warped geometry descends from more familiar solutions in higher dimensions upon reduction on a torus with unusual boundary conditions. More precisely we consider the case where a loop in a compact coordinate (not necessarily the eleventh coordinate) is accompanied by a non-trivial shift on the transverse sphere. Upon reduction to lower dimensions they lead to Melvin solutions with non trivial fluxes and profiles for the dilaton field. In the context of the AdS/CFT correspondence the isometries of the spheres are related to R-symmetries of the boundary conformal field theory. Turning on non-trivial fluxes will then break part of these isometries leading to less supersymmetric AdS/CFT duals. We are interested in the case where the fluxes are chosen in such a way that half of the original supersymmetries are preserved. A typical example of such a configuration is the flux 5-brane [11] of type IIA, involving a reduction on the eleventh dimensional circle accompanied by a Z_N rotation in the transverse R^4 . More general solutions involving wrapped flux-branes were studied in [14].

The general idea behind the construction of Melvin solutions in Einstein gravity is quite simple and can be described as follows [15]. Consider a given solution of the Einstein-Hilbert equation of motion in D-dimensions described by a metric G_{MN} (whose isometry group

contains the isometries of a d -dimensional torus T_d), a dilaton profile ϕ and a set of non-trivial fluxes for the RR rank n field strength H . A solution is specified also by a choice of boundary conditions along the directions of the torus. Different choices lead to inequivalent (sometimes drastically different) physics which share with the original solution only its local characteristics. After reduction to $D-d$ dimensions they give rise to a rich class of solutions of the lower dimensional gravity with various non-trivial fluxes and scalar profiles. More precisely, denoting the spacetime index by $\mu = 0, \dots, D-d-1$ and indicizing the directions of the torus T_d by $i = D-d, \dots, D-1$, a general choice of boundary conditions is given by the identifications

$$\begin{aligned} x^i &\sim x^i + 2\pi n^i R_i \\ x^\mu &\sim x^\mu + 2\pi n^i b_i^\mu R_i \end{aligned} \tag{2.1}$$

parameterized by the real parameters b_i^μ , which describe a jump of $2\pi n^i b_i^\mu R_i$ along x^μ once one goes n^i times around the cycle “ i ” of T^d . In order to perform a reduction to $D-d$ dimensions it is convenient to introduce the coordinates $\tilde{x}^\mu \sim x^\mu - b_i^\mu x^i$ with the canonical orbits

$$\begin{aligned} x^i &\sim x^i + 2\pi n^i R_i \\ \tilde{x}^\mu &\sim \tilde{x}^\mu. \end{aligned} \tag{2.2}$$

In terms of these new coordinates the D -dimensional metric can be rewritten (after reconstructing squares) as

$$\begin{aligned} ds_D^2 &= G_{MN} dx^M dx^N \\ &= g_{ij} (dx^i + A_\mu^i d\tilde{x}^\mu) (dx^j + A_\mu^j d\tilde{x}^\mu) + g_{\mu\nu} d\tilde{x}^\mu d\tilde{x}^\nu \end{aligned} \tag{2.3}$$

with

$$\begin{aligned} g_{ij} &= G_{ij} + 2 G_{\mu i} b_j^\mu + G_{\mu\nu} b_i^\mu b_j^\nu \\ A_\mu^i &= g^{ij} (G_{\mu j} + G_{\mu\nu} b_j^\nu) \\ g_{\mu\nu} &= G_{\mu\nu} - A_\mu^i g_{ij} A_\nu^j. \end{aligned} \tag{2.4}$$

After reduction on T^d we are left with a “dilaton” solution with metric $g_{\mu\nu}$, non-trivial profiles g_{ij} for the scalars coming from the metric and fluxes related to the lower dimensional gauge fields A_μ^i . In addition the six-dimensional rank n field strength, which in the new coordinates is given by

$$H_{\tilde{M}_1 \dots \tilde{M}_n} = \partial_{\tilde{M}_1} x^{M_1} \dots \partial_{\tilde{M}_n} x^{M_n} H_{M_1 \dots M_n} \tag{2.5}$$

gives rise to a rank $n-1$ and a rank n forms given by

$$\begin{aligned} H_{\tilde{\mu}_1 \dots \tilde{\mu}_{n-1}}^n &= H_{\tilde{\mu}_1 \dots \tilde{\mu}_{n-1} i} \\ H_{\tilde{\mu}_1 \dots \tilde{\mu}_n}^{n-1} &= H_{\tilde{\mu}_1 \dots \tilde{\mu}_n} - H_{\tilde{\mu}_1 \dots \tilde{\mu}_{n-1} i} A_{\tilde{\mu}_n}^i + \text{cyclic permutations}. \end{aligned} \tag{2.6}$$

We will be mainly interested in the case $d = 1$. Denoting by “ x ” the compact coordinate x_{D-1} , the D -dimensional metric can be rewritten as

$$ds_D^2 = e^{2\sigma} (dx + A_\mu d\tilde{x}^\mu) (dx + A_\nu d\tilde{x}^\nu) + g_{\mu\nu} d\tilde{x}^\mu d\tilde{x}^\nu \quad (2.7)$$

with $e^{2\sigma} \equiv \Lambda G_{xx}$ and Λ , the (D-1)-dimensional metric and the gauge field potentials given by

$$\begin{aligned} \Lambda &= 1 + 2 \frac{G_{\mu x}}{G_{xx}} b^\mu + \frac{G_{\mu\nu}}{G_{xx}} b^\mu b^\nu \\ A_\mu &= e^{-2\sigma} (G_{\mu x} + G_{\mu\nu} b^\nu) \\ g_{\mu\nu} &= G_{\mu\nu} - e^{2\sigma} A_\mu A_\nu. \end{aligned} \quad (2.8)$$

Together with the H fluxes (2.6), the field strength $F \equiv dA$, the metric g and the dilaton $e^{-2\phi} \equiv R e^{-2\Phi_D + \sigma}$, they define a solution of the (D-1)-dimensional equations of motion coming from the supergravity theory with bosonic action (in the string metric)

$$S = 2\pi \int d^{D-1}x \sqrt{-g} \left[e^{-2\phi} \left(\mathcal{R} + 4(\partial\phi)^2 - (\partial\sigma)^2 - \frac{e^{2\sigma}}{4} F^2 \right) - \frac{e^\sigma}{2n!} H_n^2 - \frac{e^{-\sigma}}{2(n-1)!} H_{n-1}^2 \right]. \quad (2.9)$$

2.1 D1D5 systems in presence of NS-NS fluxes

In this subsection we construct flux solutions of five-dimensional supergravity descending from $AdS_3 \times S^3$ vacua. The starting $AdS_3 \times S^3$ geometry have been extensively studied in the context of the AdS/CFT correspondence and are associated to two-dimensional CFTs describing the low energy excitations of bound states of D1D5 branes (or a stack of NS5 branes and fundamental strings) wrapping a four manifold M , with $M_4 = T^4$ or $K3$. It is natural to ask how different choices of boundary conditions in the $\mathcal{N} = (4, 4)$ two-dimensional CFT (after compactifying the six-dimensional black string on a circle) are realized in the dual supergravity.

The near horizon solution describing a bound state of $Q = Q_1 Q_5$ D1D5 branes is described by the $AdS_3 \times S_3$ six-dimensional metric and self-dual RR field strength

$$\begin{aligned} ds^2 &= \frac{r^2}{\ell^2} (-dt^2 + dx^2) + \frac{\ell^2}{r^2} dr^2 + \ell^2 d\Omega_3 \\ H_{txr} &= \frac{2r}{\ell^2} \\ H_{\theta\varphi_1\varphi_2} &= 2\ell^2 \sin 2\theta, \end{aligned} \quad (2.10)$$

where $r^2 = x_2^2 + \dots + x_5^2$ is the radial distance from the D-brane system, $\ell^2 = g_6 \sqrt{Q}$ the squared of the Anti-De Sitter and S^3 radius and $g_6 = g_{st}/\sqrt{v_M}$ the six-dimensional coupling constant.¹ Finally

$$d\Omega_3 = d\theta^2 + d\varphi_1^2 + d\varphi_2^2 + 2 d\varphi_1 d\varphi_2 \cos 2\theta, \quad (2.11)$$

¹We will always measure distances in units of α' .

denotes the line element along the transverse three sphere with $0 \leq \theta \leq \pi/2$, $0 \leq \varphi_1 \leq \pi$ and $0 \leq \varphi_2 \leq 2\pi$. We would like to consider the effect of introducing non-trivial boundary conditions along the “ σ ” direction on the worldvolume CFT, which is set to coincide with the “ x ” direction on the boundary of AdS_3 . In order to preserve supersymmetry we consider identifications like (2.1), where the spacetime shift is embedded on a $SU(2)_L$ subgroup of the full $SU(2)_L \times SU(2)_R$ isometry group of S^3 . More precisely we consider the case where fields are taken to be periodic on x only up to a Z_N rotation of the transverse R^4 [11]

$$\begin{aligned} x &\sim x + 2\pi n R \\ \varphi_2 &\sim \varphi_2 + 2\pi n b R \end{aligned} \tag{2.12}$$

with $bR = 1/N$. Clearly this orbifolding preserves only an $\mathcal{N} = (4, 0)$ subset of the original two-dimensional supersymmetries with R-symmetry group now reduced to $SU(2)_R \times U(1)_L$. From the five dimensional point of view (after reduction on x) our new supergravity solution can be read off from (2.7), (2.8) and is described by the metric

$$\begin{aligned} ds_5^2 &= -\frac{r^2}{\ell^2} dt^2 + \frac{\ell^2}{r^2} dr^2 + \ell^2 d\theta^2 + \\ &+ \frac{\ell^2}{\Lambda} \left[d\tilde{\varphi}_2^2 + 2 \cos \theta d\varphi_1 d\tilde{\varphi}_2 + d\varphi_1^2 \left(1 + \frac{b^2}{r^2} \ell^4 \sin^2 2\theta \right) \right] \end{aligned} \tag{2.13}$$

in terms of the new variable $\tilde{\varphi}_2 \equiv \varphi_2 - bx$. The remaining five dimensional fields are given by

$$\begin{aligned} \Lambda &= 1 + \frac{b^2 \ell^4}{r^2} \\ e^{-2(\phi - \phi_\infty)} &= R e^\sigma = R \frac{r}{\ell} \Lambda^{1/2} \\ A_{\varphi_1} &= \frac{b \ell^4 \cos 2\theta}{\Lambda r^2} \\ A_{\tilde{\varphi}_2} &= \frac{b \ell^4}{\Lambda r^2} \\ H_{rmt} &= \frac{2r}{\ell^2} \\ H_{\theta\varphi_1} &= 2b \ell^2 \sin 2\theta \\ H_{tr\mu} &= -\frac{2r}{\ell^2} A_\mu \\ H_{\theta\varphi_1\tilde{\varphi}_2} &= \frac{2\ell^2}{\Lambda} \sin 2\theta. \end{aligned} \tag{2.14}$$

Besides the background fields descending from the original RR sources we see from (2.14) that the new solution involves non-trivial NS-NS fluxes A_μ for the gauge field $g_{\mu x}$. These fluxes are responsible for the partial supersymmetry breaking and the consequent non-trivial profile of the dilaton field and the metric. In the next section we will present a detailed study of the global features of this new supergravity solution from the six dimensional point of view, but before that we would like to see how much we can learn from the

local physics described by the metric and background fields above. In particular it is interesting to notice that the flux parameter b sets a new scale in the near horizon geometry. Indeed (2.13), (2.14) can be seen as a domain wall solution interpolating between regions of small ($r \ll b\ell^2$) and large ($r \gg b\ell^2$) radial distances. Remarkably both regimes can still be accurately described inside perturbation theory and we will refer to them as the “deep inside ” and “asymptotic” regions respectively. Let us first consider the solution in the deep inside region $r \ll b\ell^2$. The limit can be achieved by turning the flux parameter b to infinity while keeping $bR = 1/N$ fixed and small, much in the same way as the near horizon geometries can be recovered from large N expansions of the exact supergravity solutions. To this end it is convenient to introduce the rescaled coordinates $\hat{\varphi}_2 \equiv \tilde{\varphi}_2/b$, $\hat{\varphi}_1 \equiv 2\varphi_1$ and $\hat{\theta} \equiv 2\theta$. The metric (2.13) may then be written (up to orders $1/b$) as

$$ds_{\text{near}}^2 = \frac{r^2}{\ell^2} (-dt^2 + d\hat{\varphi}_2^2) + \frac{\ell^2}{r^2} dr^2 + \frac{\ell^2}{4} (d\hat{\theta}^2 + \sin^2 \hat{\theta} d\hat{\varphi}_1^2) \quad (2.15)$$

which describes an $AdS_3 \times S^2$ space with radius ℓ and $\ell/2$ respectively. In a similar way one can evaluate the limit of large b for the remaining NS-NS/RR backgrounds in (2.14). The surviving components are given by

$$\begin{aligned} e^{-2(\phi-\phi_\infty)} &= R e^\sigma = b R \ell = \frac{1}{N} \ell \\ e^\sigma A_{\hat{\varphi}_1} &= \frac{1}{2} \ell \cos \hat{\theta} \\ e^{-\sigma} H_{\hat{\theta}\hat{\varphi}_1} &= \frac{1}{2} \ell \sin \hat{\theta} \\ H_{tr\hat{\varphi}_2} &= -\frac{2r}{\ell^2} \end{aligned} \quad (2.16)$$

It is worth stressing that the metric (2.15) and background fields (2.16) define a solution of the Einstein-Hilbert equations of motion by itself and can therefore be extended to any r .

Notice that in the limit $b \rightarrow \infty$, the space direction on the boundary of AdS_3 , parameterized by $\hat{\varphi}_2$, “leans” towards the x -direction in the original AdS_3 boundary. Indeed the deep inside solution (2.16), (2.15) can be alternatively derived from reduction on the φ_2 fiber inside $AdS_3 \times S^3/Z_N$.² This is in agreement with the expectations for the near horizon geometry of the D1D5 system on $M_4 \times S^1 \times \mathbb{R}^4/Z_N$. The above analysis can be extended to the whole flow by exchanging the role of x and φ_2 in (2.12), and rewriting all five dimensional quantities in terms of $\tilde{x} = x - \varphi_2/b$. We will not present here the details of this equivalent description of the flow, which follows similar lines as that presented above and leads to identical conclusions.

Turning the flux parameter b to zero one can in a similar way isolate the asymptotic geometry and background fields. Notice that in this limit the five dimensional dilaton is no longer constant and the solution is better described in six-dimensional terms where we recover our starting $AdS_3 \times S^3$ metric and background fields (2.10). The boundary of AdS_3 decompactifies in this case to Minkowski $M_{1,1}$ since R should be consistently taken to infinity in this limit in order to keep $bR = 1/N$ finite.

²Throughout the text we keep N fixed but large.

2.2 Adding KK monopoles

The D1D5 solution with fluxes (2.10) smoothly interpolates between a nearby and an asymptotic AdS_3 geometries naturally living in five and six dimensions respectively. The effective size of the sixth dimension is related to the five-dimensional dilaton and grows to infinity far away from the brane. This asymptotic behavior, although shared by most of the flux-brane configurations studied in the literature, is not generic to all known Melvin universes. In [14], the author shows how in the presence of Taub-Nut geometries, flux branes get trapped and the region where the lower dimensional picture breaks down (since dilaton diverges), is cut off. We would like now to exploit this idea to construct an interpolating solution where both ends of "the flow" admit a sensible five-dimensional interpretation.

The solution will be associated with non-trivial boundary conditions for a bound state system of D1D5 branes and KK monopoles in type IIB. In the absence of fluxes the near horizon geometry of this system can be obtained from the $AdS_3 \times S^3$ metric describing the pure D1D5 system by replacing the R^4 cone over S^3 by a Taub-Nut space. For Q_k coinciding KK monopoles this recipe yields a supergravity background with $AdS_3 \times S^3/Z_{Q_k}$ geometry, which is believed to be holographically dual to the $\mathcal{N} = (4, 0)$ boundary CFT describing the excitations of a bound state system of D1D5 branes and KK monopoles [16]. A dual version of this correspondence has been extensively studied in [17].

In the presence of fluxes the analysis of the near horizon geometry follows closely our previous results but the two geometries differ drastically in the asymptotically far regime. Another important difference with our former example is that the fluxes do not break additional supersymmetries among those already preserved by the D1D5KK system (this holds true already in the absence of D-branes, see [14]).

The starting metric (see for instance [18] and references therein) reads:

$$ds^2 = H^{-1}(-dt^2 + dx^2) + H [H_k^{-1} (d\tau + Qk (1 - \cos \theta) d\phi)^2 + H_k (dr^2 + r^2 d\theta^2 + r^2 d\phi^2 \sin^2 \theta)] \quad (2.17)$$

with

$$H = 1 + \frac{\ell^2}{r} \quad H_k = 1 + \frac{Q_k}{r} \quad (2.18)$$

the harmonic functions associated with the $\ell^2 = g_6 \sqrt{Q_1 Q_5}$ branes and KK monopole charges. In addition the D1D5 background include the self-dual RR field strength

$$\begin{aligned} H_{txr} &= \partial_r H^{-1} \\ H_{\theta\phi\tau} &= \ell^2 \sin \theta. \end{aligned} \quad (2.19)$$

After the identifications

$$\begin{aligned} x &\sim x + 2\pi n R \\ \tau &\sim \tau + 4\pi n b R \end{aligned} \quad (2.20)$$

and reduction on x we are left with the five dimensional metric (in terms of the new variable $\tilde{\tau} = \tau - 2bx$).

$$ds_5^2 = -H^{-1} dt^2 + H H_k (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) +$$

$$+ \frac{H H_k^{-1}}{\Lambda} (d\hat{\tau} + Q_k (1 - \cos \theta) d\phi)^2 \tag{2.21}$$

and NS-NS/RR fields

$$\begin{aligned} \Lambda &\equiv 1 + 4b^2 H^2 H_k^{-1} \\ e^{-2\phi} &= R e^\sigma = R H^{-1/2} \Lambda^{1/2} \\ A_\phi &= \frac{2bH^2 Q_k (1 - \cos \theta)}{\Lambda H_k} \\ A_{\hat{\tau}} &= \frac{2bH^2}{\Lambda H_k} \\ H_{tr} &= \partial_r H^{-1} \\ H_{\theta\phi} &= 2b \ell^2 \sin \theta \\ H_{\theta\phi\hat{\tau}} &= \frac{\ell^2 \sin \theta}{\Lambda} \\ H_{tr\mu} &= -\partial_r H^{-1} A_\mu. \end{aligned} \tag{2.22}$$

The two interesting “fixed point” geometries are now recovered in the limits $r \ll b^2 \ell^4 / Q_k$ and $r \gg b^2 \ell^4 / Q_k$. The crucial difference with our previously studied example is that now, in both the limits $r \rightarrow 0$ or $r \rightarrow \infty$, the dilaton e^σ stabilizes leading to a solution with sensible five dimensional description. Indeed the solution interpolates between an asymptotically far solution with Ricci flat metric $R \times (\text{Taub} - \text{Nut})_\infty$ with trivial background fields and the deep inside $AdS_3 \times S^2$ geometry

$$ds_{\text{near}}^2 = \frac{r}{\ell^2} (-dt^2 + d\hat{\tau}^2) + \frac{\ell^2 Q_k}{r^2} dr^2 + \ell^2 Q_k (d\theta^2 + \sin^2 \theta d\phi^2) \tag{2.23}$$

with $\hat{\tau} = \tau / (2b)$ and radii $2\ell \sqrt{Q_k}$ and $\ell \sqrt{Q_k}$ for the two pieces respectively. The surviving background fields in this limit are given by

$$\begin{aligned} e^{-2\phi} &= R e^\sigma = \frac{2bR\ell}{\sqrt{Q_k}} = \frac{2\ell}{N \sqrt{Q_k}} \\ e^\sigma A_\phi &= \ell \sqrt{Q_k} (1 - \cos \theta) \\ e^{-\sigma} H_{\theta\phi} &= \ell \sqrt{Q_k} \sin \theta \\ H_{tr\hat{\tau}} &= \frac{1}{\ell^2}. \end{aligned} \tag{2.24}$$

3. Study of global properties of the solutions: charges at infinity

In the previous sections we have constructed new supergravity solutions that look locally (but not globally) like products of AdS spaces times spheres. The aim of this section is to present a systematic study of the global properties and charges characterizing these brane geometries. We will adopt the hamiltonian formalism in General Relativity [19, 20], where charges associated with asymptotic isometries of a given space-time solution are defined.

3.1 Hamiltonian formalism in General Relativity

Let us briefly review the hamiltonian formalism in General Relativity [19, 20] and introduce the basic definitions and notations that will be extensively used in this section. The first step is to formalize the concept of time evolution of a system. To this end one introduces a globally defined vector field t^a and a function $t(x)$ such that $t^a \nabla_a t = 1$. The loci of constant $t(x)$ are space-like hyper-surfaces denoted by Σ_t while the vector t^a is chosen to define the time evolution of the quantities restricted to Σ_t (at least locally one would be able to define a time coordinate t and $n - 1$ space coordinates x^i such that $t^a = (\partial/\partial t)^a$). We also choose a volume form $\epsilon_{a_1 \dots a_{n-1}} = \epsilon_{a a_1 \dots a_{n-1}} t^a$ for Σ_t which is invariant under time evolution (i.e. diffeomorphism generated by t^a): $\mathcal{L}_t(\epsilon^{(n-1)}) = 0$. Finally we define our coordinate system such that $\epsilon^{(n-1)}$ has non vanishing components ± 1 .

The definition of a space-like hyper-surface and a time direction allows to introduce canonical variables which define the phase space of the system. In this formalism the generator of the time evolution, i.e. the hamiltonian, will be denoted by $H[t^a]$. If we consider the pure gravity case, the system is totally described by the metric g_{ab} . We may express g_{ab} in terms of the induced metric (h_{ab}) on Σ_t and of its components out of Σ_t given in terms of the extrinsic curvature K_{ab} . The **induced metric** h_{ab} is defined by means of a unit time-like vector n^a (but not necessarily a geodesic) orthogonal to Σ_t :

$$h_{ab} = g_{ab} + n_a n_b \tag{3.1}$$

$h_a{}^b$ being simply the projector on $T(\Sigma_t)$. The **extrinsic curvature** K_{ab} is defined on the other hand as the gradient of n^a along Σ_t

$$K_{a b} = h_a{}^c \nabla_c n_b. \tag{3.2}$$

Finally we introduce a covariant derivative D_a which acts on tensor fields $T_{a_1 \dots a_r}{}^{b_1 \dots b_s}$ on Σ_t as:

$$D_a T_{a_1 \dots a_r}{}^{b_1 \dots b_s} = h_a{}^c h_{a_1}{}^{c_1} \dots h_{a_r}{}^{c_r} h^{b_1}{}_{d_1} \dots h^{b_s}{}_{d_s} \nabla_c T_{c_1 \dots c_r}{}^{d_1 \dots d_s}. \tag{3.3}$$

We are now ready to define the canonical variables and the hamiltonian $H \equiv H[t^a]$. The Einstein lagrangian can be rewritten in terms of quantities related to Σ_t as:

$$\mathcal{L}_G = \sqrt{-g} R[g] = t_\perp \sqrt{h} \left(\mathcal{R} + (K_a{}^a)^2 - (K_{ab} K^{ab}) \right), \tag{3.4}$$

where we denote by t_\perp, t_\parallel^a the directions of t^a parallel and orthogonal to Σ_t

$$t^a = t_\perp n^a + t_\parallel^a. \tag{3.5}$$

The momentum π^{ab} conjugate to the field h_{ab} is defined in the following way:

$$\pi^{ab} = \frac{\delta \mathcal{L}_G}{\delta h_{ab}} = \sqrt{h} (K_{ab} - h_{ab} K) \tag{3.6}$$

with $\dot{h}_{ab} = h_a{}^{a_1} h_b{}^{b_1} \mathcal{L}_t h_{a_1 b_1}$. Notice that the components of t^a do not appear in the lagrangian through time derivatives and therefore they are not dynamical variables and have no associated conjugate momentum.

The hamiltonian H is expressed as:

$$\begin{aligned}
 H &= \int_{\Sigma_t} \epsilon^{(n-1)} \left(\pi^{ab} \dot{h}_{ab} - \mathcal{L}_G \right) + J[t^a] = \int_{\Sigma_t} \epsilon^{(n-1)} \left(t_{\perp} \mathcal{H}_{\perp} + t^a \mathcal{H}_{\parallel a} \right) + J[t^a] \\
 \mathcal{H}_{\perp} &\equiv -\mathcal{R} + \frac{1}{h} \left[\pi_{ab} \pi^{ab} + \frac{\pi^2}{(2-n)} \right] \\
 \mathcal{H}_{\parallel}^a &= -2 D_b \pi^{ab}.
 \end{aligned}
 \tag{3.7}$$

The additional boundary term $J[t^a]$ was shown in [19] to be required in order for the functional derivatives of H with respect to the canonical variables to be well defined, namely in order for them to vanish on $\partial\Sigma_t$, so that the boundary conditions on h_{ab} and π^{ab} at spatial infinity are not affected by time-evolution. This quantity provides a definition of the global charge associated with t^a on the solution, which is the total energy of the configuration. From functional derivation of H with respect to t^a we deduce two phase space constraints $\mathcal{H}_{\perp} = \mathcal{H}_{\parallel a} = 0$.

In analogy with the definition of $H[t^a]$ as the generator of time evolution, given an asymptotically Killing vector $\xi \in T(M_n)$ we may define the corresponding charge $H[\xi]$ as the following generator in the phase space:

$$H[\xi] = \int_{\Sigma_t} \epsilon^{(n-1)} \left(\xi_{\perp} \mathcal{H}_{\perp} + \xi_{\parallel}^a \mathcal{H}_{\parallel a} \right) + J[\xi],
 \tag{3.8}$$

where $J[\xi]$ is the boundary contribution analogous to $J[t^a]$ for the energy. An explicit expression for the functional variation of $J[\xi]$ was derived in [13] and reads

$$\begin{aligned}
 \delta J[\xi] &= \oint_{\partial\Sigma_t} dS_d \left[G^{abcd} \left(\xi_{\perp} D_c - \partial_c \xi_{\perp} \right) \delta h_{ab} + \right. \\
 &\quad \left. + \left(2 \xi_{\parallel}^b \pi^{ad} - \xi_{\parallel}^d \pi^{ab} \right) \delta h_{ab} + 2 \xi_{\parallel a} \delta \pi^{ad} \right]
 \end{aligned}
 \tag{3.9}$$

with $G^{abcd} = (\sqrt{h}/2)(h^{ac} h^{bd} + h^{ad} h^{bc} - 2 h^{ab} h^{cd})$. For solutions of the Einstein equation of motion we have $\mathcal{H}_{\perp} = \mathcal{H}_{\parallel a} = 0$ and therefore $J[\xi]$ represents the only contribution to the charge (3.8). For asymptotically flat space-times one can easily see that this general expression reduces to the standard definitions of the energy and momentum charges:

$$\begin{aligned}
 J[\partial_t] &= \lim_{r \rightarrow \infty} \oint_{\partial\Sigma_t} dS_k \sqrt{h} \left(\partial^i h_i^k - \partial^k h_i^i \right) \quad \text{ADM mass} \\
 J[\partial_i] &= \lim_{r \rightarrow \infty} 2 \oint_{\partial\Sigma_t} dS_k \pi^{ik} \quad \text{Momentum along } x^i.
 \end{aligned}
 \tag{3.10}$$

In the next subsection we will evaluate, using the general expression (3.9), the global charges characterizing the various locally AdS geometries previously defined.

3.2 Asymptotic $AdS_3 \times S_3$ isometries and central charge.

In this section we study the global properties of the six-dimensional D1D5 geometry (2.10) with boundary conditions (2.12). The non-trivial identifications in x, φ_2 break the global

$AdS_3 \times S_3$ isometry group down to a $U(1)^3 \times SU(2)_R$ subgroup with cartan generators $\partial_x, \partial_t, \partial_{\varphi_2}, \partial_{\varphi_1}$. With each of these Killing vectors we can associate a global charge through (3.9). More generally, a conserved charge can be associated with each asymptotic (not necessary global) Killing isometry. We will see how, even in the presence of non-trivial boundary conditions ($b \neq 0$ in (2.12)) a full 2d conformal group is realized in the asymptotically far geometry in terms of two copies of the Virasoro algebra with the expected central charge.

The $AdS_3 \times S_3$ metric (2.12), after the global identifications (2.12), can be written as

$$ds^2 = \frac{r^2}{\ell^2} (-dt^2 + dx^2) + \ell^2 \left[\frac{dr^2}{r^2} + d\theta^2 + d\varphi_1^2 + (d\tilde{\varphi}_2 + b dx)^2 + 2 d\varphi_1 (d\tilde{\varphi}_2 + b dx) \cos 2\theta \right] \quad (3.11)$$

in terms of the variables $x \sim x + 2\pi R$, $\tilde{\varphi}_2 \sim \tilde{\varphi}_2 + 2\pi$ with standard orbits. The metric (3.11) is preserved by the Killing generators

$$\begin{aligned} J_R^{(1)} &= \cos(2\varphi_1) \partial_\theta - \cot(2\theta) \sin(2\varphi_1) \partial_{\varphi_1} + \csc(2\theta) \sin(2\varphi_1) \partial_{\tilde{\varphi}_2} \\ J_R^{(2)} &= -\sin(2\varphi_1) \partial_\theta - \cot(2\theta) \cos(2\varphi_1) \partial_{\varphi_1} + \csc(2\theta) \cos(2\varphi_1) \partial_{\tilde{\varphi}_2} \\ J_R^{(3)} &= \partial_{\varphi_1} \\ J_L^{(3)} &= \partial_{\tilde{\varphi}_2} \\ L_0 &= \frac{iR}{2} (\partial_t - \partial_x + b \partial_{\tilde{\varphi}_2}) \\ \bar{L}_0 &= \frac{iR}{2} (\partial_t + \partial_x - b \partial_{\tilde{\varphi}_2}) \end{aligned} \quad (3.12)$$

which correspond to the global $U(1)^3 \times SU(2)_R \subset SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R \times SU(2)_L \times SU(2)_R$ left unbroken by non-trivial identifications. A closer look into the Killing equations derived from the metric (3.11) reveals however that a richer isometry algebra is restored in the asymptotically far region $r \rightarrow \infty$. The group of asymptotic isometries can indeed be identified with the full two-dimensional conformal group like in the more familiar global $AdS_3 \times S^3$ instance. The new asymptotic Killing generators, realize two copies of the Virasoro algebra and can be written as

$$L_n = \frac{iR}{2} e^{in(t-x)/R} \left[\left(1 - \frac{n^2 \ell^4}{2R^2 r^2}\right) \partial_t - \left(1 + \frac{n^2 \ell^4}{2R^2 r^2}\right) (\partial_x - b \partial_{\tilde{\varphi}_2}) - \frac{inr}{R} \partial_r \right] \quad (3.13)$$

$$\bar{L}_n = \frac{iR}{2} e^{in(t+x)/R} \left[\left(1 - \frac{n^2 \ell^4}{2R^2 r^2}\right) \partial_t + \left(1 + \frac{n^2 \ell^4}{2R^2 r^2}\right) (\partial_x - b \partial_{\tilde{\varphi}_2}) - \frac{inr}{R} \partial_r \right]. \quad (3.14)$$

The vector fields above generate asymptotic Killing isometries in the sense that all non-trivial variations of the metric (3.11) generated by them:

$$\begin{aligned} \delta_n g_{\alpha\beta} &= \nabla_{(\alpha} L_{\beta)n} = -\nabla_{(x} L_{t)n} = -\frac{n^3 \ell^2}{2R^2} e^{in(t-x)/R} \\ \delta_{\bar{n}} g_{\alpha\beta} &= \nabla_{(\alpha} \bar{L}_{\beta)n} = -\frac{n^3 \ell^2}{2R^2} e^{in(t+x)/R}; \quad \alpha, \beta = \{t, x\} \end{aligned} \quad (3.15)$$

fall off fast enough to leave unaltered the leading behavior of (3.11) at space infinity.

As before, with each generator (3.13) of the asymptotic isometries we associate a charge through (3.8). We consider the most general linear combination of the cartan generators $\xi = a_1 \partial_t + a_2 \partial_\phi + a_3 \partial_{\varphi_1} + a_4 \partial_{\tilde{\varphi}_2}$, and read the corresponding charge from the a_i coefficients of the final answer. Charges associated with non cartan generators are vanishing since they always involve trivial integrals over sines or cosines over $\partial_t \equiv S_1$.

The computation of the charge is simplified by the fact that the extrinsic curvature of the hyper-surface Σ_t vanishes and therefore only the first two terms in (3.9), which do not involve the canonical momenta π^{ab} or its variations, contribute to the charge. Indeed, being the metric t-independent and block diagonal, the induced metric on Σ_t is simply the restriction $h_{ij} = g_{ij}$ with indices $i, j \neq t$ of g_{ab} along Σ_t . In addition the only non-trivial component of the covariant derivative of $\mathbf{n} = \frac{\ell}{r} \partial_t$ is $\nabla_r \mathbf{n}^t$ which does not have components along Σ_t and therefore yields a vanishing extrinsic curvature according to the definition (3.2). We are left with potential contributions coming only from the terms involving $\xi_\perp = -a_1 r/L$ in (3.9). These two contributions can be easily worked out and vanish identically. In order to see this, let us recall that

$$\delta h_{cd} = \frac{\partial h_{cd}}{\partial b} db \tag{3.16}$$

have components along the $(x, \varphi_1, \tilde{\varphi}_2)$ plane. On the other hand the indices “d” and “c” in eq. (3.9), referring to the direction normal to the boundary $\partial\Sigma_t$ at $r = \infty$ and to the components of the derivatives respectively, can only be “r”. Once again using the fact that the metric is block diagonal (with respect to $\partial\Sigma_t$) the only contribution to (3.9) can only come from the term proportional to $h^{ab} \delta h_{ab}$ which clearly vanishes. We conclude that our vacuum configuration carries vanishing charges with respect to all Killing isometries.

Even if all Virasoro charges are zero, once evaluated on our background one can still compute the central extension by properly evaluating the boundary contributions (3.9) to the relevant commutators $[L_n, L_{-n}]$. We will however postpone this computation to the next section, where a deformation of our solution carrying non-trivial L_0 and \bar{L}_0 charges are constructed and both terms in the Virasoro algebra (see eq. (3.29) below) can be displayed.

3.3 A point mass solution

In this subsection we follow the strategy of [13] in order to construct more general solutions carrying non-trivial mass and momentum charges. We introduce a new variable $\phi = x/\ell$ and consider the metric:

$$ds^2 = - \left(1 + \frac{r^2}{\ell^2} \right) dt^2 + r^2 d\phi^2 + \frac{dr^2}{(1 + r^2/\ell^2)} + \ell^2 \left(d\theta^2 + d\varphi_1^2 + (d\tilde{\varphi}_2 + b\ell d\phi)^2 + 2 d\varphi_1 (d\tilde{\varphi}_2 + b\ell d\phi) \cos(2\theta) \right) \tag{3.17}$$

which clearly becomes (3.11) for $r \gg \ell$. In addition one can easily verify that the Ricci tensors associated with the two metrics (3.11) and (3.17) coincide and therefore they are

solutions of the same Einstein equations.³ The periodicity of ϕ however is: $\phi \sim \phi + 2\pi\alpha$ with $\alpha = R/\ell$ and therefore whenever $\alpha \neq 1$ we encounter a conical singularity at the origin. This corresponds to a point mass particle sitting at the origin with a non-trivial contribution to the energy.

More generally, following Brown and Henneaux we can accompany the cycle around ϕ by a jump in time:

$$\begin{aligned} t &\sim t - 2\pi n A \\ \phi &\sim \phi + 2\pi n \alpha. \end{aligned} \tag{3.18}$$

This identification will generate a momentum charge. A convenient choice of coordinates is given by the replacements

$$\begin{aligned} t &\rightarrow \alpha t - A \phi \\ \phi &\rightarrow -\frac{A}{\ell^2} t + \alpha \phi \\ r &\rightarrow \frac{r}{\sqrt{\alpha^2 - A^2/\ell^2}} \end{aligned} \tag{3.19}$$

which give back to ϕ the standard period and removes the jump in time:

$$\begin{aligned} \phi &= \phi + 2\pi n \\ t &\sim t. \end{aligned} \tag{3.20}$$

The metric in the new variables reads

$$\begin{aligned} ds^2 &= -\left(\frac{r^2}{\ell^2} - A^2 b^2 + \alpha^2\right) dt^2 + (r^2 - A^2 + b^2 \ell^4 \alpha^2) d\phi^2 + A \alpha (1 - b^2 \ell^2) d\phi dt + \\ &+ \ell^2 d\Omega_3 + \frac{\ell^2 dr^2}{(r^2 - A^2 + \ell^2 \alpha^2)} + 2b\ell(\ell^2 \alpha d\phi - A dt)(d\tilde{\varphi}_2 + \cos 2\theta d\varphi_1) \end{aligned} \tag{3.21}$$

which tends to $AdS_3 \times S^3$ at infinity.

We are now ready to compute the energy-momentum charges carried by the solution (3.21). As before the charges are evaluated through the boundary integral (3.9). The orthonormal vector \mathbf{n} to the hypersurface Σ_t , the induced metric h_{ab} and the canonical momentum π^{ab} are now given by

$$\begin{aligned} \mathbf{n} &= \frac{1}{r} \left(\ell \partial_t - \frac{A \ell \alpha}{r^2} \partial_\phi + A b \partial_{\varphi_2} \right) \\ h_{ab} &= -A^2 b^2 dt^2 + d\phi^2 r^2 + 2A \alpha (-1 + b^2 \ell^2) dt d\phi + \frac{\ell^2}{r^2} dr^2 + \ell^2 d\Omega_3 + \\ &+ 2b\ell(\ell^2 \alpha d\phi - A dt)(d\tilde{\varphi}_2 + \cos 2\theta d\varphi_1) + \dots \\ \pi^{r\phi} &= \pi^{\phi r} = -\frac{A \alpha \ell^3 \sin 2\theta}{r^2} + \dots \\ \pi^{r\varphi_2} &= \pi^{\varphi_2 r} = \frac{A b \alpha^2 \ell^3 \sin 2\theta}{r^2} + \dots \end{aligned} \tag{3.22}$$

³Notice that in absence of flux ,i.e. $b = 0$, and in the limit of $R \rightarrow \ell$, the metrics (3.11) and (3.17) are related by a change of coordinates and correspond to global $AdS_3 \times S^3$. Solutions displaying asymptotic $AdS_3 \times S^3$ geometry and carrying non trivial global charges have been recently studied in [21].

As before we compute the charge associated with the general linear combination $\xi = a_1 \partial_t + a_2 \partial_\phi + a_3 \partial_{\varphi_1} + a_4 \partial_{\tilde{\varphi}_2}$, and read the corresponding charge from the a_i coefficients of the final answer. The decomposition into normal and parallel part leads to:

$$\begin{aligned} \xi_\perp &= \frac{a_1 r}{\ell} \\ \xi_{||}^a &= a_2 \partial_\phi + a_3 \partial_{\varphi_1} + \left(a_4 - \frac{A a_1 b}{\ell} \right) \partial_{\tilde{\varphi}_2}. \end{aligned} \tag{3.23}$$

Plugging (3.22), (3.23) in (3.9) one finds that only the energy and momentum charges, associated with $\partial_t, \partial_\phi$, are excited in our solution. The overall additive constant is fixed by the condition that charges vanish in the regular $\alpha = 1, A = 0$ vacuum metric. In addition an overall $\frac{1}{g_6^2 V_{S^3 \times S^1}} = \frac{1}{4\pi^3 g_6^2}$ normalization factor is included in the definition (3.8) to account for the difference between the string and Einstein metrics and the normalization to one of the volume of the $S^3 \times S^1$ boundary in the case of spheres of unit radii. The final result can be written as:

$$\begin{aligned} \mathbb{J}[\ell \partial_t] &= \frac{\hat{l}^4}{2} \left(1 - \alpha^2 - \frac{A^2}{\ell^2} \right) \\ \mathbb{J}[\ell \partial_\phi] &= 2 \hat{l}^4 A \alpha, \end{aligned} \tag{3.24}$$

where $\hat{l}^4 = \ell^4/g_6^2 = Q$ reabsorbs the $1/g_6^2$ factor in front of the charge definition. We notice that b does not enter the above expression for the charges.

As promised we now evaluate the central extension of the Virasoro algebras realized by (3.14). Following [13] we can derive this term by evaluating the Poisson brackets:

$$[\mathbb{J}[\xi], \mathbb{J}[\eta]] = \delta_\eta \mathbb{J}[\xi] \tag{3.25}$$

with $\xi = L_n$ and $\eta = L_{-n}$ given by:

$$\begin{aligned} L_n &= \frac{i\ell}{2} e^{in(\frac{t}{\ell} - \phi)} \left[\left(1 - \frac{n^2 \ell^2}{2r^2} \right) \partial_t - \frac{1}{\ell} \left(1 + \frac{n^2 \ell^2}{2r^2} \right) (\partial_\phi - \ell b \partial_{\tilde{\varphi}_2}) - \frac{inr}{\ell} \partial_r \right] \\ \bar{L}_n &= \frac{i\ell}{2} e^{in(\frac{t}{\ell} + \phi)} \left[\left(1 - \frac{n^2 \ell^2}{2r^2} \right) \partial_t + \frac{1}{\ell} \left(1 + \frac{n^2 \ell^2}{2r^2} \right) (\partial_\phi - \ell b \partial_{\tilde{\varphi}_2}) - \frac{inr}{\ell} \partial_r \right]. \end{aligned} \tag{3.26}$$

The variations of the metric $\delta_\xi h_{ab}$ and canonical momentum $\delta_\xi h_{ab}$ are now defined by the Lie derivative $\delta_\xi = \mathcal{L}_\xi$ along the vector field ξ . Plugging (3.22) in (3.9), after some algebra one is left with the final result

$$[\mathbb{J}[\xi], \mathbb{J}[\eta]] = -\frac{in}{2} \hat{l}^4 \left(\alpha^2 + \frac{A^2}{\ell^2} + \frac{4A\alpha}{\ell} - n^2 \right). \tag{3.27}$$

One can see that the result (3.27) can be written as a sum of two pieces:

$$\begin{aligned} 2n \mathbb{J}[L_0] &= i\ell n (\mathbb{J}[\partial_t] - \ell \mathbb{J}[\partial_\phi]) \\ &= -\frac{in \hat{l}^4}{2} \left(\alpha^2 + \frac{A^2}{\ell^2} + \frac{4A\alpha}{\ell} - 1 \right) \end{aligned} \tag{3.28}$$

and the central term $\frac{i}{2} \hat{\ell}^4 (n^3 - n)$. We conclude that the asymptotic Killing generators realize two copies of Virasoro algebras with commutation relations:

$$\begin{aligned}
 [\mathbb{J}[L_n], \mathbb{J}[L_m]] &= (n - m) \mathbb{J}[L_{n+m}] + \frac{ic}{12} (n^3 - n) \delta_{n+m,0} \\
 [\mathbb{J}[\bar{L}_n], \mathbb{J}[\bar{L}_m]] &= (n - m) \mathbb{J}[\bar{L}_{n+m}] + \frac{ic}{12} (n^3 - n) \delta_{n+m,0} \\
 [\mathbb{J}[L_n], \mathbb{J}[\bar{L}_m]] &= 0
 \end{aligned}
 \tag{3.29}$$

with the expect central charge $c = 6 \hat{\ell}^4 = 6Q$. Notice that the result is independent of the flux parameter b .

4. Concluding remarks

In this paper we construct solutions of five-dimensional supergravity which provide a simple setting where the physics of RG-flows out of two-dimensional $N = (4, 0)$ CFTs can be quantitatively studied. The solutions are constructed by tilting $AdS_3 \times S^3$ geometries and further reducing it to five-dimensions. The final results are warped geometries of Melvin type with various NS-NS/RR fluxes.

We identify interesting decoupling limits at the two ends of the flow. In both cases the deep inside region can be accurately described by an $AdS_3 \times S^2$ exact solution of five-dimensional supergravity. This background corresponds to the reduction from the $AdS_3 \times S^3/Z_N$ type IIB vacuum on the Hopf fiber of the compact factor. In the ultra-violet, the two dimensional theory flows to a non-conformal theory with non-trivial five dimensional dilaton, better described in terms of the familiar six-dimensional $AdS_3 \times S^3$ supergravity vacuum with constant dilaton. The S^2 and S^3 isometries realize the global part of the $\mathcal{N} = (4, 0)$ and $\mathcal{N} = (4, 4)$ conformal field theories at the two ends of the flow. We stress the fact that the two AdS_3 are relatively tilted due to the non-trivial global identifications that mix the AdS_3 and S^3 geometries, making a purely three dimensional analysis along the lines of [22] more involved. In these more conventional terms the flux solutions above correspond to the flow out of an AdS_3 vacuum of three-dimensional $SU(2)$ gauged supergravity with $\mathcal{N} = 4$ unbroken supercharges, towards an asymptotic geometry with non-trivial dilaton. It would be very interesting, to apply the techniques developed in [22], to make this correspondence more precise. Alternatively one can study the flow out of the $AdS_3 \times S^3$ fixed point geometry in the asymptotic region using the tools of $\mathcal{N} = 8$ gauged supergravities [23].

In [24], the spectrum of D1D5 BPS excitations in various freely acting orbifolds and orientifolds of type IIB theory was determined and they were shown to match the AdS/CFT predictions in terms of chiral harmonics of the corresponding $AdS_3 \times S^3$ dual supergravities (see [25] for earlier results in the more familiar D1D5 systems for type IIB on T^4 or $K3$ and [26] for a D1D5 system with $AdS_3 \times S^3 \times S^3$ dual supergravity). It would be interesting to apply these techniques to provide more quantitative tests of the correspondences proposed here.

Another interesting feature of the $AdS_3 \times S^2$ decoupled geometry is that it provides a black string solution in $D = 5$ dimensions which can be used as the starting point for the construction of four-dimensional black holes.⁴

Finally it would be nice to extend the results attained in the present work to cases involving four-dimensional gauge theories. We believe that this analysis can provide a deep insight into the nature of the lifts of locally AdS_5 geometries, associated with $\mathcal{N} = 1, 2$ gauge theories, to ten dimensions. Another possibility is the study of the effects of introducing supersymmetric RR flux-branes on $AdS_5 \times S^5$.

We plan to deal with some of these issues in the near future.

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⁴Making contact for instance with the results of [27] in which a systematic geometrical analysis of the microscopic description of BPS black holes in $N = 8$ $D = 4$ supergravity was carried out.

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