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Review article

Empowering wave energy with control technology: Possibilities and pitfalls

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ABSTRACT

With an increasing focus on climate action and energy security, an appropriate mix of renewable energy technologies is imperative. Despite having considerable global potential, wave energy has still not reached a state of maturity or economic competitiveness to have made an impact. Challenges include the high capital and operational costs associated with deployment in the harsh ocean environment, so it is imperative that the full energy harnessing capacity of wave energy devices, and arrays of devices in farms, is realised. To this end, control technology has an important role to play in maximising power capture, while ensuring that physical system constraints are respected, and control actions do not adversely affect device lifetime. Within the gamut of control technology, a variety of tools can be brought to bear on the wave energy control problem, including various control strategies (optimal, robust, nonlinear, etc.), data-based model identification, estimation, and forecasting. However, the wave energy problem displays a number of unique features which challenge the traditional application of these techniques, while also presenting a number of control 'paradoxes'. This review articulates the important control-related characteristics of the wave energy control problem, provides a survey of currently applied control and control-related techniques, and gives some perspectives on the outstanding challenges and future possibilities. The emerging area of control co-design, which is especially relevant to the relatively immature area of wave energy system design, is also covered.

1. Introduction

Though the concept of harnessing wave energy has been around from quite some time, with the earliest patent filed in 1799 (Ross, 1995), commercial wave energy remains somewhat elusive (Guo & Ringwood, 2021b), with the level of R&D activity largely following energy concerns (oil crises, climate action, energy security, etc.). Currently, grid-connected wave energy is more expensive than many of its marine renewable energy counterparts (Ringwood, 2022), including floating offshore wind. Nevertheless, in the drive to a fossil-free energy system, the need for complementarity of renewable resources arises, unless abundant, and cheap, energy storage can be facilitated. To this end, wave has been shown to be relatively uncorrelated with the other main unpredictable renewable resource, wind (Bhattacharya et al., 2021; Fusco, Nolan, & Ringwood, 2010; Kluger, Haji, & Slocum, 2023) (with solar and tidal deemed to be relatively predictable, though also intermittent), also suggesting the opportunity for hybrid offshore wind/wave devices (McTiernan & Sharman, 2020). A final appealing characteristic of wave energy is its energy density. For example, at a latitude of 15°N (northeast trades), the solar insolation is 0.17 kW/m^2. However, the average wind generated by this solar radiation is about 20 knots (10 m/s), giving a power intensity of 0.58 kW/m^2 which, in turn, has the capability to generate waves with a power intensity of 8.42 kW/m^2 (McCormick, 2013). This suggests that, given sufficient technology maturity, wave energy devices could be considerably smaller than their solar and wind counterparts, alleviating (to some extent) environmental concerns.

The relatively slow development of wave energy technology is due to a number of factors, including the need for survivability in the hostile sea environment, oscillating nature of the energy flux, wide variation in wave climate at any single location, and the variation in wave amplitude and period for a given (stationary) sea state. In addition, there is pervasive uncertainty in the optimal wave energy device concept, since both potential and kinetic energy forms can be harnessed, with well over 200 prototypes suggested (147 are listed in Koca et al. (2013) as of 2013), with 340 patents filed in the UK alone over the period 1855 to 1973 (Clément et al., 2002).

A significant number of prominent wave energy companies have risen and fallen, many bruised by the high costs associated with full-scale prototype construction, testing, and deployment. A recently proposed WEC development protocol (Weber, 2012) suggests that the performance of a wave energy concept should be assured (though

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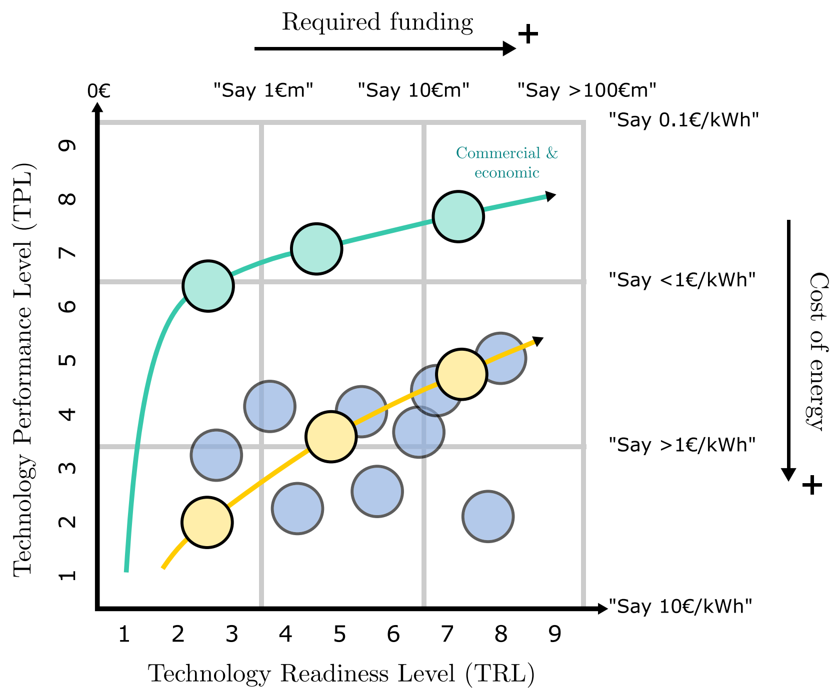

Fig. 1. WEC development protocol, favouring raising TPL (performance) before TRL (size/cost).
Source: Adapted from Weber (2012).

design iteration, simulation and small-scale testing) before progressing to large scale, as shown in Fig. 1.

To this end, control technology has an important role to play in enhancing the WEC technology performance level (TPL). Specifically, an energy maximising control system can adapt the WEC characteristics (e.g. resonance) to different waves and wave climates. In addition, and consistent with the TPL-biased protocol suggested in Fig. 1, the entire device, including power take-off (PTO) mechanism, which converts the mechanical WEC motion into a more useful form, and the controller, can be collectively optimised to maximise TPL (see Section 6 regarding control co-design, and Section 3.1 for control performance objectives).

There are various multipliers suggested by a variety of wave energy researchers and technologists regarding the performance improvement (in captured energy) of WECs due to control, varying from 2 to 4 (Babarit & Clément, 2006). However, in practice, the degree of performance enhancement will ultimately depend on the nature of the WEC itself (for example, resonators lend themselves well to control), the type of power take-off (PTO) system employed (including the physical constraints of the PTO, see Section 6.3) and the specific control algorithm employed. That said, with a relatively modest increase in capital cost associated with the addition of control technology (some sensors, actuators, computer), compared to the potential benefit (productivity multiplier) and the WEC body/PTO cost, it is likely that the judicious use of control technology with WEC systems is an economic 'no-brainer'. Nevertheless, the full impact of the use of control technology in the wave energy domain needs to be carefully evaluated, since aggressively controlled WEC systems (with enhanced motion and forces) may incur additional operational costs, in the form of extra maintenance requirements, or breakages (with consequent loss of production through downtime).

One particular challenge, in both writing this paper and addressing a control design problem for a specific WEC, is the diversity of WEC types, making it virtually impossible to propose a generic system model, or control philosophy. For the most part, the focus here will be on hydrodynamic control (Korde & Ringwood, 2016) of oscillating body WECs, which includes point absorbers (French & Bracewell, 1986), oscillating wave surge converters (Whittaker & Folley, 2012), hinge barges (Paparella & Ringwood, 2016), etc. One noteworthy control study is that by Shabara and Abdelkhalik (2023) which deals with *variable shape* oscillating WECs. Two categories of WEC not generally covered in this paper are overtopping devices, which present a more traditional (low head) hydroelectric power problem, and oscillating water columns (OWCs), where the focus is normally on speed control of the air turbine (Rosati, Henriques, & Ringwood, 2022), with a rather tenuous link between the turbine and the device hydrodynamics (Rosati & Ringwood, 2022).

A further challenge is to address the range of possible PTO systems. Fig. 2 shows the full conversion train, for a mechanically wave-actuated system (i.e. excludes OWC, for example). To put the scope of the current paper in context, consideration will be given to the nature of the wave excitation, it is force on the WEC, and the opposing (or assisting) mechanical force provided by the PTO system. For direct electromechanical PTO systems, this force is normally mediated by the generator torque/force, via the device-side power converter. For PTOs containing hydraulic components, other manipulated variables are available. However, in this paper, detailed models of hydraulic or electrical components will not be given, though some comments about potential smoothing effects, or non-ideal behaviour, will be included.

A final challenge is in the diversity of potential application uses of wave energy converters. While much R&D effort is directed towards grid-scale electricity production, a number of niche applications of wave energy also exist, including the production of potable water (Bacelli, Gilloteaux, & Ringwood, 2009), or powering of data buoys (McLeod & Ringwood, 2022), which may have some specific requirements in terms of control objectives.

The main contribution of this paper is the provision of an up-to-date overview of the issues, techniques, and potential benefits/drawbacks associated with the application of control science to wave energy systems. These includes aspects related to WEC modelling, and model uncertainty, as well as a detailed (complete) review of the various WEC control philosophies, and the opportunities for control co-design in achieving an optimally performing controlled WEC system. The remainder of the paper is laid out as follows: Section 2 describes the wave resource, which is crucial in understanding the operational context of WEC systems, and takes the reader through the various levels of WEC modelling, working from high- to low-fidelity, and

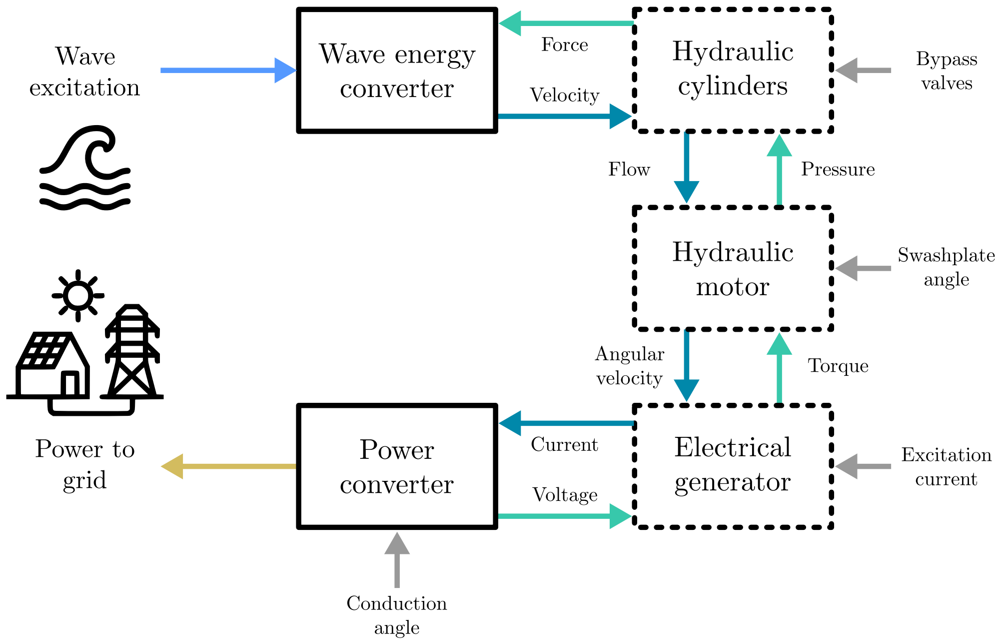

Fig. 2. Complete PTO system, with most potential energy domain changes.

indicating the various potential applications at each level, including model-based control design. Opportunities for data-based modelling are also highlighted. In Section 3, which is the main focus of the paper, various WEC control philosophies are described, against the backdrop of the control objectives articulated in Section 3.1, and particular issues that arise in WEC control, including non-causality and model uncertainty. Many of the WEC control philosophies reviewed require an estimate and/or a forecast of the wave excitation force experienced by a device, and methods to provide these quantities are described and reviewed in Section 4. Section 5 provides a specific contribution to the paper, in highlighting the departures of the characteristics of WEC models/controllers/objectives from more conventional, and well-accepted conventions, in control systems science. The various opportunities relating to optimisation of WEC geometry, array layout, and PTO configuration within a control co-design framework are outlined in Section 6 while, finally, some perspectives and conclusions are given in Section 7.

1.1. Notation

$\mathbb{R}^+$ is used to indicate the set of non-negative real numbers. The notation $\mathbb{I}_n$ is used to denote the identity element of the space $\mathbb{C}^{n\times n}$ under the standard matrix product. The Fourier transform of a function f, provided it exists, is denoted as $F(\omega)$, $\omega \in \mathbb{R}$. The Hermitian operator is denoted by $F(\omega)^\star$. The notation $\mathcal{RH}_\infty$ is used for the set of real rational proper and stable functions $G : s \mapsto G(s)$, $s \in \mathbb{C}$, while $\mathcal{RH}_2$ is considered for the set of strictly proper and stable functions in $\mathbb{C}$.

2. Wave energy system descriptions

The vast majority of the WEC control strategies, considered within the state-of-the-art, rely on the availability of a control-oriented model, able to capture the main dynamics underlying the wave energy conversion process. Such a model must be suitable for real-time control purposes, *i.e.* be computationally/analytically tractable.

In the light of this, we introduce, within this section, the fundamental principles leveraged within control-oriented modelling of WEC systems. Section 2.1 discusses the main tools employed to represent the wave resource. Section 2.2 offers an overview of the main relations underlying the dynamics of WEC systems, *i.e.* the Navier–Stokes equations. Section 2.3 presents linear potential flow theory, which is widely adopted within the WEC literature to produce tractable models for control design purposes. Based on linear potential flow theory, Section 2.4 introduces the celebrated Cummins' equation (or operator), virtually always adopted as the dynamical model for WEC control synthesis, while Section 2.5 offers a brief account of typical extensions to the (linear) Cummins' operator. Finally, Section 2.6 provides an overview of the use of system identification techniques for control-oriented modelling of WEC systems, discussing the main approaches followed by the state-of-the-art literature.

2.1. Modelling the wave resource

This section introduces, in brief, the underlying principles for basic modelling of the wave resource, widely employed, for example, in performance assessment and controller evaluation/tuning within the WEC literature. Consistent with the vast majority of WEC control studies, which naturally consider the dynamics of the device when the resource is suitable for energy extraction, *i.e.* in power production operating conditions (see *e.g.* Ringwood, Bacelli, & Fusco, 2014 for further detail on the different operation regions/modes for WEC systems), linear (first-order) wave theory is assumed. Note that first-order waves, though limited to waves with small height-to-spatial length ratio, constitute the most commonly applied description for wind-generated surface gravity waves, being a fundamental mathematical framework for a vast variety of marine/ocean engineering applications (Birk, 2019; Rawson & Tupper, 2001).

Let $x = [x_1, x_2, x_3]^\intercal \in \mathbb{R}^3$ be a point (coordinate) in space, and let the reference point in x_3, *i.e.* the still water level (SWL), be placed, without any loss of generality, at $x_3 = 0$. A standard assumption, within virtually all the available WEC control literature, is that of considering so-called long-crested waves (Ochi, 2005), *i.e.* waves with their crests and troughs stretching in the x_2-direction in space, rendering the fluid free-surface elevation, η, independent on the x_2-coordinate, with the x_1-axis pointing in the direction of wave propagation. If the representation of the resource considered is effectively composed of a single frequency component, the wave is commonly referred to as *regular*, and its formal description is given by

$$\eta(x_1, t) = \frac{H_w}{2} \cos(\omega_w t + \psi_w(x_1, \omega_w)), \tag{1}$$

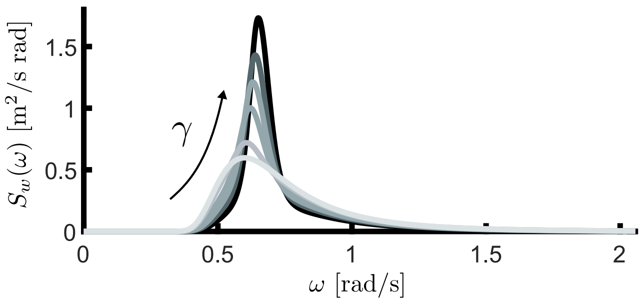

Fig. 3. Example of SDF arising from a JONSWAP representation, with increasing peak-enhancement parameter γ.

where H_w is the wave height, ω_w the wave frequency (with an associated wave period $T_w = 2\pi/\omega_w$), and where the map ψ_w defines a phase shift, which depends on the x_1 coordinate. Although clearly simplistic in nature, the monochromatic wave representation in (1) is often considered for the derivation of fundamental results, providing valuable insight into the underlying dynamics of the WEC body in water. From now on, for simplicity of exposition, we assume a given a fixed location $x_1 = x_1^*$ in space, so that the free-surface elevation can be written as a function of a single variable, *i.e.* $\eta(x_1, t) \equiv \eta(t)$.

A more comprehensive description, to that provided via the representation in (1), can be achieved by incorporating the stochastic nature of the wave phenomenon when modelling the resource. The waves resulting from this characterisation are commonly referred to as *irregular*, waves. In particular, within such a representation, the free surface-elevation η is linked with a corresponding spectral density function (SDF) $S_w(\omega)$, characterising the behaviour of ocean waves at x_1^*. Well-known modelling techniques for S_w are those provided by *e.g.* the Bretschneider spectrum (Bretschneider, 1959), for developing seas, the Pierson–Moskowitz spectrum (Pierson & Moskowitz, 1964), for fully-developed seas, and the JONSWAP spectrum (Hasselmann et al., 1973), for wind-generated seas with fetch limitations.

Within this stochastic description of the wave resource, the notions of wave height and period need to be re-defined accordingly. The concept of wave height is commonly replaced by that of *significant* wave height $\bar{H}_w$, defined as the mean wave height (though to crest) of the highest third of the observed waves characterising a sea-state. Wave period is, instead, often replaced by the concept of *peak* wave period $\bar{T}_w$, defined in terms of the period associated with the most energetic waves, for a specific location in space. Nonetheless, we clarify that other measures, different from those briefly recalled in this section, can be considered, and refer the reader to *e.g.* the well-known manuscript (Ochi, 2005) for further detail on the stochastic description of ocean waves. To provide an example of a standard wave SDF, Fig. 3 illustrates S_w for the case of a JONSWAP spectrum, where $\bar{H}_w = 2$ [m] and $\bar{T}_w = 9$ [s], and the so-called peak-enhancement parameter γ which, within the definition in Hasselmann et al. (1973), defines how narrow-banded is the resulting sea state, is such that $\gamma \in [1, 5]$.

Based on a given SDF, performance assessment for different WEC devices often necessitates of suitable (time-domain) numerical generation of η for a given sea-state, *i.e.* representative realisations of the free-surface elevation. A number of methods can be found in the literature to achieve a statistically consistent polychromatic (set of discrete components) realisation of a panchromatic (continuous spectrum) wave for evaluation purposes, including, but not limited to, techniques based on a harmonic description of η, such as those described in Mérigaud and Ringwood (2017a), or Markov-based methods (see *e.g.* Brodtkorb et al., 2000).

Finally, to conclude this brief recap, we note that, though beyond the scope of this review paper, more complex representations of ocean waves are naturally available, including those arising from higher-order (*i.e.* nonlinear) wave theory (see *e.g.* Constantin, 2017), or even multi-directional characterisations in terms of SDFs defined over $\mathbb{R}^3$ (Mérigaud, Herterich, Flanagan, Ringwood, & Dias, 2018; Ochi, 2005).

2.2. Navier–Stokes: Conservation of mass and momentum

The Navier–Stokes equations represent the (high-fidelity) 'starting point' for hydrodynamic modelling of wave energy conversion systems. In particular, this set of equations arises from the application of conservation of mass and momentum laws, and provides a representation of the motion of a fluid (or interaction between fluid and a solid structure/component) in time and space, *i.e.* for $t \in \mathbb{R}^+$ and $x \in \mathbb{R}^3$, respectively. The Navier–Stokes equations are solved in terms of both a velocity map $(x, t) \mapsto v(x, t)$ and a pressure function $(x, t) \mapsto p(x, t)$. For an incompressible fluid, these equations (Fefferman, 2006) are

$$\begin{aligned} \frac{\partial v_i}{\partial t} + \sum_{j=1}^{3} v_j \frac{\partial v_i}{\partial x_j} &= \nu \Delta v_i - \frac{\partial p}{\partial x_i} + f_i(x, t), \\ \operatorname{div}\{v\} = \sum_{i=1}^{3} \frac{\partial v_i}{\partial x_i} &= 0, \end{aligned} \tag{2}$$

with an initial condition $v_0 \equiv v(x, 0)$, where v_0 is a given C^∞ divergence-free vector field on $\mathbb{R}^3$, $\nu \in \mathbb{R}^+$ is the kinematic viscosity, $f_i(x, t)$ represents any components characterising externally applied forces (for example, gravity), and the operator $\Delta : \mathbb{R}^3 \to \mathbb{R}$ denotes the Laplacian in x.

Eq. (2) does not possess a 'generic' closed-form solution for v and p, and hence a significant effort has been made, throughout the years, to provide suitable numerical techniques, able to provide approximate solutions to the Navier–Stokes equations. This has given rise to the field of Computational Fluid Dynamics (CFD), where the set of equations in (2) is discretised in time and space and numerically solved accordingly, using a wide variety of techniques (Ferziger, Perić, & Street, 2002; Windt, Davidson, & Ringwood, 2018).

As such, techniques arising from the field of CFD represent a valuable tool for realistic (high-fidelity) hydrodynamic simulation of WEC systems, being able to effectively incorporate a full account of the main nonlinear effects characterising WEC devices, including *e.g.* viscosity and vortex shedding. Nonetheless, the computational cost associated with running CFD simulation is a major concern for control purposes, being in the order of thousands of seconds per second of simulation time (Ferziger et al., 2002), automatically prohibiting their use in control synthesis and design procedures, though providing an option for high-fidelity control system evaluation (Penalba, Davidson, Windt, & Ringwood, 2018).

Though a detailed description of CFD-based methods for WEC systems is beyond the scope of this review paper, which aims to present the main core of control-oriented studies in the WEC field, the reader is referred to Windt (2020) for further detail on the application of CFD algorithms to wave energy conversion systems.

2.3. Fundamentals of linear potential flow theory

Given the inherent complexity characterising the Navier–Stokes equations, researchers from the WEC control community tend to adopt a set of simplifying assumptions, constituting linear potential flow theory. This is often performed with the objective of providing tractable representations for control design/synthesis purposes. This section provides an overview of these assumptions, both in terms of WEC devices and the wave field, leading to the celebrated Cummins' equation. The interested reader is also referred to *e.g.* Birk (2019) and White (2011) for a thorough treatment of these topics.

2.3.1. Assumptions and boundary conditions

We consider a generic body geometry in water waves, with a wetted (submerged) surface $\mathscr{S}$. The main underlying assumptions characterising linear potential flow theory are:

(A1) The flow is frictionless (inviscid) and irrotational.
(A2) The amplitude of the body motion is significantly smaller than the dimension of the body.
(A3) Linear wave theory (as described in Section 2.1) holds.

(A1) implies that the map v in (2) can be fully characterised in terms of a potential function $(x,t) \mapsto \phi(x,t)$ such that $v = \nabla\phi$, where ∇ denotes the gradient operator. We briefly recall, in the following, the main set of boundary conditions imposed to solve for ϕ, according to linear potential flow theory, *i.e.* consistent with (A2) and (A3) above. A detailed (formal) account of these boundary conditions can be found in Papillon, Costello, and Ringwood (2020) and White (2011), while their corresponding nonlinear counterparts are explicitly defined in *e.g.* Birk (2019, Chapter 20):

- *Dynamic boundary condition:* At the undisturbed free-surface elevation, the fluid pressure has to be equal to atmospheric pressure.
- *Kinematic boundary condition:* The fluid particles on the free-surface are assumed to stay in the free-surface for all $t \in \mathbb{R}^+$, *i.e.* the component of the fluid velocity normal to the surface *must* equal the surface velocity.
- *Impermeability of the body:* The component of the fluid velocity normal to the body surface $\mathscr{S}$ has to be equal to the body velocity normal to the body surface.

The set of assumptions (A1), (A2), and (A3), together with the associated set of boundary conditions and the corresponding Laplace equation, are the basis for a family of techniques, termed boundary element methods (BEMs) (Brebbia, 1984), which numerically solve for the potential function ϕ (and, hence, the velocity map v). Unlike alternative methods, *e.g.* finite-difference (Vreugdenhil, 1989) (FDMs) and finite-element (FEMs) (Donea & Huerta, 2003), which are based on a discretisation of the entire fluid domain considered, BEM techniques (also often referred to as 'panel' methods), are based on the distribution of singularities on the boundaries of the defined domain, being significantly less computationally demanding.

When solving the linear potential flow problem for the WEC case, which naturally includes a body, two different problems need to be solved accordingly, *i.e.* the equation of motion associated with the WEC body and that characterising the fluid, both connected by the corresponding hydrodynamic effects acting on the geometry. Within BEM methods, the solution to this problem can be generally obtained via two different main approaches, namely either via explicit use of Green functions, or so-called Rankine sources, and the reader is referred to Papillon et al. (2020) for an overview of these methodologies.

Within the wider marine structures/ocean engineering literature, different BEM codes have been developed, aiming to provide an off-the-shelf solution for the computation of the potential function ϕ, both in the time- and frequency-domains. These are widely used by the WEC control community in order to compute control-oriented representations for different geometries and conversion systems, as further described within Section 2.3.2. Among the mainstream BEM codes used, WAMIT (Lee, 1995) (commercial) and NEMOH (Babarit & Delhommeau, 2015) (open-source) are worth mentioning, due to their vast use within the WEC community. A detailed comparison between these two well-adopted BEM codes, for the case of wave energy systems, can be found in Penalba, Kelly, and Ringwood (2017).

2.3.2. Main forces under linear potential flow theory

Let the local frame of reference for the WEC body (with the origin located at the corresponding centre of mass) be defined by a coordinate vector z in $\mathbb{R}^N$, where $N \in \mathbb{N}$ denotes the number of modes of motion, *i.e.* degrees-of-freedom (DoF), considered to describe the dynamics of the WEC system. Though a single WEC system is considered in this section, we note that the modelling procedure involved for multi-body devices, or arrays of devices, can be done analogously, in a relatively straightforward manner, as detailed in *e.g.* Folley and Forehand (2016).

Under linear potential flow theory, as described in Section 2.3.1, the equation of motion for a generic WEC system can be derived via Newton's second law, as[1]

$$m\ddot{z} = f_e + f_r + f_{re} + f_u, \tag{3}$$

where $m \in \mathbb{R}^{N\times N}$ is the so-called mass-inertia matrix of the WEC system, $z(t) \in \mathbb{R}^N$ denotes the displacement vector of the body as a function of time, $f_e(t) \in \mathbb{R}^N$ is the wave excitation force, $f_r(t) \in \mathbb{R}^N$ is the radiation force, $f_{re}(t) \in \mathbb{R}^N$ is the hydrostatic restoring force and, finally, $f_u(t) \in \mathbb{R}^N$ represents the control input, exerted via an associated power take-off (PTO) system. Each contribution to Eq. (3) is described in the following:

- *Wave excitation* f_e: Defined as the force acting on the body when it is held fixed in the presence of waves. As such, the excitation $f_e = 0$, $\forall t$, in the absence of incident waves. This effect can be written, within linear potential flow theory assumptions, as

$$f_e = \eta * k_e, \tag{4}$$

where $k_e(t) \in \mathbb{R}^N$ is the *excitation impulse response function*, and η is the free-surface elevation at the location of the centre of mass of the device. In particular, k_e arises from direct superposition of diffraction effects, and a linearised account of so-called dynamic Froude–Krylov forces (see *e.g.* Penalba, Giorgi, & Ringwood, 2017).

- *Radiation effects* f_r: Defined as the hydrodynamic force applied from the fluid to the body in the absence of incident waves. In particular, the motion of the WEC body generates a time-dependent fluid pressure which, integrated over $\mathscr{S}$, creates the force

$$f_r = -m_\infty \ddot{z} - \dot{z} * k_r, \tag{5}$$

where the matrix $k_r(t) \in \mathbb{R}^{N\times N}$ is the (causal) *radiation impulse response function*, containing the memory effects associated with the fluid response, and $m_\infty \in \mathbb{R}^{N\times N}$ the so-called added-mass (infinite frequency) asymptote (see *e.g.* Falnes, 2002). Note that the impulse response function k_r effectively constitutes a hydrodynamic coupling between different modes of motion. In other words, the waves radiated by each DoF affect the overall dynamics of the device.

- *Restoring force* f_{re}: Defined as the force arising from the difference between gravitational and buoyancy forces. For this linear potential theory case, it follows that:

$$f_{re} = -s_h z, \tag{6}$$

where $s_h \in \mathbb{R}^{N\times N}$ is commonly referred to as the restoring coefficient matrix, with $z = 0$ considered to be the equilibrium condition.

- *Control input* f_u: Defined as the force exerted by the PTO system on the device. As per discussed in Section 1, the design of f_u is fundamental to achieve efficient energy extraction, being able to significantly affect the performance of any given WEC system across a wide variety of sea state conditions. Note that, consistent with the vast majority of the WEC control literature, we generally consider mechanical power as the key measure for the design of f_u. As a result, computation of the optimal energy-maximising law f_u, according to the energy-based control objective in Section 3.1,

[1] From now on, the dependence on t is dropped when it is clear from the context.

does not explicitly involve the definition of any particular PTO system, retaining some level of generality. Nonetheless, we refer the reader to *e.g.* Ahamed, McKee, and Howard (2020), Penalba and Ringwood (2019) and Têtu (2017) for further detail on available PTO systems for WEC devices, including the main modelling assumptions involved.

Eq. (3), commonly known as Cummins' equation, forms the basis for much control-oriented modelling of wave systems, as discussed in Section 2.4. The impulse response maps k_e and k_r can be effectively derived in terms of the solution provided by the family of BEM methods described in Section 2.3.1, since both impulse responses can be readily written in terms of the potential function ϕ (see *e.g.* Papillon et al., 2020). Note that, clearly, both k_e and k_r are geometry dependent, *i.e.* both excitation and radiation effects naturally depend on the shape of the WEC itself. It is important to clarify that, as per BEM operation, the corresponding characterisation of k_e and k_r is performed in a non-parametric fashion, *i.e.* a set of data points (either in the time- or frequency-domain) is available, rather than a closed-form expression for such kernels. This issue is addressed in Section 2.4, where Eq. (3) is considered to derive (parametric) models compatible with control design procedures. We can note that some attempts have been made to provide parametric descriptions from first principles, for relatively simple WEC shapes, including studies by Abdulkadir and Abdelkhalik (2023), Havelock (1954), and Yeung (1981).

2.4. Control-oriented models: Cummins' equation

Before effectively presenting the dynamical WEC system arising from Cummins' Eq. (3), we note that the excitation force term f_e in (4) (*i.e.* as described within linear potential flow theory) does not explicitly depend on any WEC motion variable, but only on the free-surface elevation (which is an external variable for the device). As such, from now on, we directly consider f_e as the external (uncontrollable) variable affecting the WEC system, which can be numerically generated by simply 'filtering' the wave elevation η according to the associated excitation kernel k_e.

From Eq. (3), and following the arguments posed immediately above, the equation of motion of the WEC device can be expressed in terms of the following system $\mathscr{G}$:

$$\mathscr{G} : \begin{cases} \ddot{z} = \bar{M}\left(-\dot{z} \ast k_r - s_h z + f_e + f_u\right), \\ y = \dot{z} = v, \end{cases} \tag{7}$$

where $\bar{M} = M^{-1}$, with $M = m + m_\infty$, and, without any loss of generality, the output y is set to be the velocity vector associated with the motion of the device $\dot{z}$, in line with the energy-maximising optimal control problem for WECs, defined in Section 3.

Though effectively linear in nature, system $\mathscr{G}$ in (7) is not particularly convenient for control/estimation purposes, nor for computationally efficient motion simulation and performance assessment, due to the presence of the (non-parametric) convolution term in k_r, arising from BEM codes. As such, a common practice among WEC control-oriented modelling is to compute an approximation for the convolution operator in terms of a finite-dimensional linear system. In other words, and given that the impulse response function k_r fully characterises a corresponding LTI system, the underpinning idea is that of finding a continuous-time radiation sub-system $\mathscr{R}$ written, in state-space form, as

$$\mathscr{R} : \begin{cases} \dot{\Theta}_r = A_r \Theta + B_r \dot{z}, \\ k_r \ast \dot{z} \approx C_r \Theta + D_r \dot{z}, \end{cases} \tag{8}$$

with $\Theta(t) \in \mathbb{R}^{n_\mathscr{R}}$ and the set of matrices $\{A_r, B_r, C_r, D_r\}$ of appropriate dimensions. Standard algorithms from the field of system identification can be employed to compute the approximating system (8), starting from the set of datapoints obtained via BEM analysis either in the time- or frequency-domains. Well-adopted strategies in the field, to compute such an approximation, are those presented in *e.g.* Faedo, Peña-Sanchez, and Ringwood (2018a, 2020a), Rogne, Moan, and Ersdal (2014) and Taghipour, Perez, and Moan (2008), while a comparison study for different techniques can be found in Pena-Sanchez, Faedo, and Ringwood (2019).

To guarantee a physically consistent representation of the WEC conversion process via (7), the approximating system (8) must respect a number of fundamental dynamical properties, pertaining to the radiation characteristic (see *e.g.* Taghipour et al., 2008). In particular, preserving the underlying passive behaviour of the map $\dot{z} \mapsto k_r \ast \dot{z}$ is fundamental to guarantee consistency of the overall input–output WEC system, including *e.g.* internal stability. Preserving the physical properties of the radiation map is not only fundamental for representative simulation, but also to guarantee the existence and uniqueness of a wide number of energy-maximising optimal control algorithms available in the literature, having a strong impact on the convexity of the associated energy-based objective function (see *e.g.* Bacelli & Ringwood, 2014b; Faedo, Giorgi, Ringwood, & Mattiazzo, 2022b; Scruggs, Lattanzio, Taflanidis, & Cassidy, 2013). Available studies within the marine engineering community, explicitly addressing the radiation passivity property when providing an approximation as in (8), include (Faedo, Peña-Sanchez, Carapellese, Mattiazzo, & Ringwood, 2021; Faedo, Peña-Sanchez, & Ringwood, 2018b; Hatecke, 2015; Scruggs et al., 2013).

Finally, we close this section by noting that, with the approximation provided in (8), and by defining a state vector $x := [z^\top \quad \dot{z}^\top \quad \Theta^\top]^\top$, a state-space description for the overall WEC system $\mathscr{G}$ in (7) follows immediately, *i.e.*

$$\begin{aligned} \dot{x} &= Ax + B(f_e + f_u) \\ y &= C_v x, \end{aligned} \tag{9}$$

with

$$A := \begin{bmatrix} 0 & \mathbb{I}_N & 0 \\ -\bar{M}s_h & -\bar{M}D_r & -\bar{M}C_r \\ 0 & B_r & A_r \end{bmatrix}, \quad B := \begin{bmatrix} 0 \\ \bar{M} \\ 0 \end{bmatrix},$$
$$C_v := \begin{bmatrix} 0 & \mathbb{I}_N & 0 \end{bmatrix}.$$

Here the dimension of the space characterising (9) is $n_\mathscr{G} = 2N + n_\mathscr{R}$.

2.5. Extensions to Cummins-based models

Though the operator in (7), expressed in state-space form in (9) via suitable approximation of the radiation effects in (8), is convenient from both representational and computational perspectives, it is derived under linear potential flow theory assumptions (as described in Section 2.3.1), which can be limiting (*i.e.* non-representative) for WEC systems under controlled conditions. This argument is, in fact, discussed in length within this paper, in Section 5 (Paradox 1). In an effort to broaden the scope of application of Cummins' formulation, a common practice is to 'append' a number of relevant additional forces affecting the WEC system, via relatively simple (though often nonlinear) analytical representations. We provide, in the following, a brief description of the main effects that researchers within the WEC community tend to superimpose on Cummins' formulation in an attempt to relax the somewhat restrictive assumptions characterising linear potential flow theory:

- *Nonlinear viscous drag:* Aiming to provide a simplified representation of the viscous force, generated by shear stress, an additional viscous term can be added to (7), by means of the so-called Morison equation (Morison et al., 1950), *i.e.*

$$f_v = \frac{1}{2}\rho C_d A_c \dot{z}|\dot{z}|, \tag{10}$$

where ρ denotes the fluid density, C_d is the so-called *viscous drag coefficient*, and A_c is the characteristic area of the body. This representation for viscous effects has been considered, within the WEC control literature, in *e.g.* Bacelli, Genest, and Ringwood (2015), Faedo, Giorgi, Ringwood, and Mattiazzo (2022a), Karthikeyan, Previsic, Scruggs, and Chertok (2019) and O'Sullivan, Sheng, and Lightbody (2018). However, the determination of a suitable value for C_d for a WEC is not always straightforward (Giorgi & Ringwood, 2017b), since the Morison equation was derived for piles within a stream.

- *Nonlinear restoring forces*: Due to the fact that the motion of the WEC system can be exaggerated when maximising energy absorption, the assumption of having a 'constant' cross-sectional wetted surface (a condition in which the restoring force is effectively linear) can cease to hold. To amend this issue, a nonlinear representation of the restoring can be incorporated into Cummins' equation to provide a more realistic restoring force representation. In particular, and given the nature of the restoring phenomenon (which is linked to the static Froude–Krylov (FK) force — see *e.g.* Giorgi & Ringwood, 2017a), polynomial parameterisations are commonly employed, *i.e.*

$$f_{re} = -s_h z + \sum_{i=2}^{P} \gamma_i z^i, \tag{11}$$

with $\{\gamma_i\}_{i=2}^{P} \subset \mathbb{R}^{N\times N}$. Examples of WEC control papers, incorporating such effects, are Faedo et al. (2022b), Mérigaud and Ringwood (2018) and Todalshaug et al. (2016). However, some care must be taken to maintain consistency between static and dynamic FK forces, especially if nonlinear FK forces are also considered (Giorgi & Ringwood, 2018).

- *Mooring forces*: The vast majority of proposed wave energy systems can be classified as offshore floating devices (Czech & Bauer, 2012), and hence need to be 'confined' to remain within specific locations. As such, a vital component, which guarantees the proper functioning of such WEC devices, is the mooring system, being responsible for solving the station-keeping problem. The effect of moorings on the overall system response can be significant, potentially exhibiting a strongly nonlinear behaviour under certain conditions (Davidson & Ringwood, 2017; Paduano et al., 2020), and a rigorous inclusion of the significant dynamics within tractable WEC models is, naturally, not straightforward. Numerical high-fidelity solvers, such as Orcaflex (ORCINA, 2023), are effectively available, although, analogously to CFD, their computational burden precludes direct application within control-oriented modelling. Finally, note that, though analytical models for relatively generic mooring systems do exist (see Davidson & Ringwood, 2017), their complexity is often high, which (at least partially) explains why the incorporation of mooring dynamics in typical WEC control studies is generally absent (with some notable exceptions, such as *e.g.* Paduano, Carapellese, Pasta, Faedo, & Mattiazzo, 2022; Richter, Magaña, Sawodny, & Brekken, 2013). By way of example, Narayanan, Yim, and Polo (1998) expresses the behaviour of mooring forces in terms of the map

$$f_m = k_{m_1} z + k_{m_3} z^3, \tag{12}$$

where z represents surge displacement in (12), and the set of coefficients $\{k_{m_1}, k_{m_3}\} \subset \mathbb{R}$ is computed based on a least squares approximation procedure, using the model in Gottlieb and Yim (1992) as 'target' mooring force expression. An additional pathway towards generating control-oriented models of mooring effects is pursued in *e.g.* Cerveira, Fonseca, and Pascoal (2013) and Paduano et al. (2022), where system identification techniques are employed to compute parametric expressions of f_m, based on data generated with the aid of high-fidelity mooring solvers (see also the discussion provided in Section 2.6).

In summary, the (augmented) input-to-state nonlinear Cummins-based model can be described as

$$\dot{x} = Ax + B(f_e + f_u + f_{nl}), \tag{13}$$

where f_{nl} summarises the nonlinear hydrodynamic effects added to Cummins' equation, *e.g.*, (10)–(11). Finally, we note that a number of other nonlinear potential flow formulations are available (Davidson & Costello, 2020; Papillon et al., 2020), with some computationally efficient solutions, especially for axisymmetric devices (Giorgi & Ringwood, 2018).

2.6. Data-based modelling

Another pathway towards control-oriented models, popular within the WEC field, is that of *data-based* modelling, where mathematical models are directly computed offline, based on data-sets constructed with data arising either from high-fidelity simulators (*e.g.* numerical wave tanks Davidson & Ringwood, 2017), or experimental activities with devices at different scales. This practice has its roots in the field of system identification, and its popularity within the WEC field is fairly recent (Penalba, Giorgi, & Ringwood, 2017). Although a detailed review of data-based WEC modelling is beyond the scope of this study, which is primarily focused on the development of control technology, we provide a brief account of the main studies that can be found within the state-of-the-art, in the paragraphs below.

The studies performed in Davidson, Giorgi, and Ringwood (2016) and Giorgi, Davidson, and Ringwood (2016) provide a comprehensive theoretical and practical analysis of the principles of system identification for the WEC application, including insight into the definition of the experiments required to generate sufficiently representative data for data-based modelling. The data is generated within a numerical wave tank for diverse persistently exciting inputs, where a black-box modelling approach is pursued, in both linear and nonlinear identification frameworks, and compared accordingly. Davidson, Giorgi, and Ringwood (2015) pursues a similar data generation approach, but the resulting modelling process is based on a grey-box approach, where a Cummins-like structure (as in Eq. (7)) is employed as starting structure. Another application of grey-box procedures can be found in Bakar, Green, Metcalfe, and Ariff (2014), where an Unscented Kalman Filter (UKF) is designed to estimate the parameters of a pre-defined WEC model structure.

The studies (Bacelli, Coe, Patterson, & Wilson, 2017; Bacelli, Nevarez, Coe, & Wilson, 2019; Coe, Bacelli, & Forbush, 2021) consider experimental tests on small-scale WEC prototypes, where the input/output data-set is used to produce linear control-oriented models, following a black-box approach. The study in Faedo, Pasta, et al. (2022) follows an analogous procedure, but with an aim to (experimentally) characterise a subset of the dynamics of an inertial wave energy converter, which is shown to be sufficient for optimal control design. Other black-box approaches can be found in Farajvand, García-Violini, and Ringwood (2023) and Giorgi, Davidson, Jakobsen, Kramer, and Ringwood (2019) (based on synthetic, *i.e.* numerical, data) and Faedo et al. (2023) and García-Violini, Peña-Sanchez, Faedo, Ferri, and Ringwood (2023) (based on experimental results). Identification of OWC systems has been considered in Gkikas and Athanassoulis (2014), Rosati, Kelly, and Ringwood (2021) and Stappenbelt, Fiorentini, Cooper, Zhu, and Nader (2011), with the latter two studies pursuing inherently nonlinear modelling strategies. Control-oriented mooring identification has been considered in *e.g.* Cerveira et al. (2013) and Paduano et al. (2022), where the data-sets are generated with aid of high-fidelity mooring solvers.

Finally, we note that data has also been used to compute nonlinear reduced models for WEC systems in Faedo, Piuma, et al. (2021) and Papini, Piuma, Faedo, Ringwood, and Mattiazzo (2022), where data-driven model reduction by moment-matching is explicitly adopted for the wave energy conversion case. Exploiting similar techniques, Faedo et al. (2022b) accommodate nonlinear Froude–Krylov effects for real-time optimal control purposes, based on data generated from a high-fidelity hydrodynamic solver.

3. Control of wave energy systems

As highlighted in Section 1 (Introduction), the general objective of control, in the wave energy context, is to maximise converted energy. However, the ultimate arbiter as to whether a renewable energy technology, or a particular wave energy converter, will achieve commercial success is the cost of energy, specifically the Levelised Cost of Energy (LCoE). LCoE, dealt with in more detail in Section 3.1, also considers costs, to which control actions may also contribute. The remainder of this section brings the reader through the fundamentals of WEC control (Section 3.2) through to more advanced WEC control techniques in Sections 3.3 to 3.6. Though impossible to deal with the wide variety of WEC systems, some generic models will be considered, with WEC arrays specifically dealt with in Section 3.5.

3.1. Control objectives

Since the raw energy resource is entirely free, but the cost of conversion to a useful form is not, the primary metric governing the performance of WEC system is the cost of energy. There may also be considerations as to when this energy is available (considering the demand profile), or the consistency of the supply of energy from the waves (often considering additional storage), which influences dispatchability, but a number of these issues are beyond the scope of the current control-focussed analysis. Let us start by assuming that we wish to minimise the LCoE over the WEC/project lifetime:

$$\mathrm{LCoE} = \frac{\mathrm{PV(CapEx)} + \mathrm{PV(OpEx)}}{\mathrm{PV}(E_c)}, \tag{14}$$

where the present value (PV) of a quantity is calculated as:

$$\mathrm{PV(CF)} = \sum_{y_r=y_r^0}^{Y_r} \frac{\mathrm{CF}(y_r)}{(1+R_d/100)^{y_r}}, \tag{15}$$

with CapEx denoting capital costs, OpEx denoting operational costs, R_d is the discount rate, E_c is captured energy, and Y_r is the project lifetime (in years). In most control-focused studies, (14) is usually distilled to maximisation of captured energy:

$$E_c = \int_0^T v(t) f_u(t) \mathrm{d}t, \tag{16}$$

subject to:

$$\begin{aligned} |z(t)| &\le z_{max} \\ |f_u(t)| &\le f_{max} \\ |v(t)| &\le v_{max} \end{aligned} \tag{17}$$

for all $t \in [0, T]$, since the quantities involved in (16) and (17) are measurable in real time. While the assumption that control *actions* have little effect on the system capital cost (though there is a marginal fixed cost for control equipment) for a given system configuration may be reasonable, there is significant interplay between control performance and PTO limits (which impact CapEx), as articulated in Section 6. However, given that the typical action of WEC control is to exaggerate device motion, there may also be an impact on OpEx. While some attempts have been made to articulate the effect of control actions on system health (fatigue Nielsen, Pedersen, Andersen, & Ambühl, 2017, damage Arredondo-Galena, Ermakov, Shi, Ringwood, & Brennan, 2023), the relatively long timescales in fatigue/damage assessment do not easily lend themselves to metrics which can be employed in real-time.

3.2. WEC control fundamentals

To establish the fundamentals of WEC control, we consider, throughout this subsection, that the WEC motion can be described in terms of the linear Cummins' operator in (7) (equivalently in (9)). Based on the energy-maximising control objective posed in Section 3.1, *i.e.* with f_u designed according to the following map

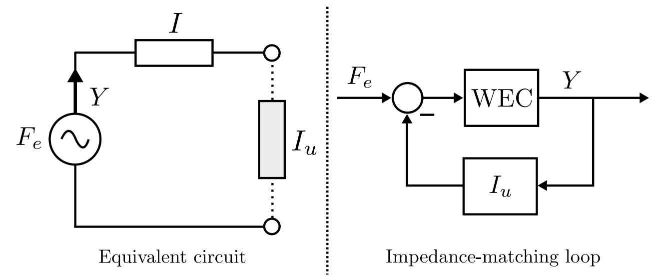

Fig. 4. Circuit representation for the WEC system within the scope of impedance-matching.

$$f_u \leftarrow \max_{f_u} E_c(f_u), \tag{18}$$

without considering, for the moment, state and input constraints, a frequency-domain approach can be adopted, to gain useful insight into the nature of the control solution arising from (18). In particular, such a perspective permits the application of a fundamental principle, well-known within the electrical and electronic engineering community, to achieve maximum power transfer: the *impedance-matching* principle (see *e.g.* Thomas, 1976).

Let us assume, for simplicity of exposition, that $N = 1$ throughout this section, *i.e.* a single-DoF WEC system is considered. An extension of the presented theory to a general class of multi-DoF systems can be found in Faedo, Carapellese, Pasta, and Mattiazzo (2022). A direct application of the Fourier transform to Cummins' Eq. (7) yields,

$$\jmath M Y(\omega) + K_r(\omega) Y(\omega) + \frac{s_h}{\jmath\omega} Y(\omega) = F_e(\omega) + F_u(\omega). \tag{19}$$

It follows immediately that[2]

$$Y = \frac{1}{I}\left(F_e + F_u\right), \tag{20}$$

where, adopting an analogous notation to that commonly used within electrical/electronic engineering applications, $I(\omega)$ is defined as the WEC *intrinsic impedance*, as

$$I = K_r + \jmath\left(\omega M + \frac{s_h}{\omega}\right). \tag{21}$$

The relation posed in (20), together with the impedance definition in (21), resembles a standard circuit representation, represented according to the schematic illustration provided in Fig. 4. In particular, the wave excitation force, F_e, can be seen as an external source affecting the WEC system, the latter being characterised with the impedance I. Furthermore, letting the control force be expressed in terms of an associated control impedance I_u, *i.e* $F_\mathrm{u} = -I_u Y$, the PTO action can be seen as a control load to the overall WEC device which, according to the control objective in (18), needs to be designed to achieve maximum power transfer from the wave resource F_e. From this perspective, the design of the WEC control force can be addressed by leveraging fundamental impedance-matching theory: Maximum power transfer from the source is achieved for (20) when the control load I_u is designed such that

$$I_u = I^\star, \tag{22}$$

i.e. as the complex-conjugate of the WEC intrinsic impedance I, for all ω (that is, in a broadband sense). The choice of control load in

[2] From now on, the dependence on ω is dropped when clear from the context.

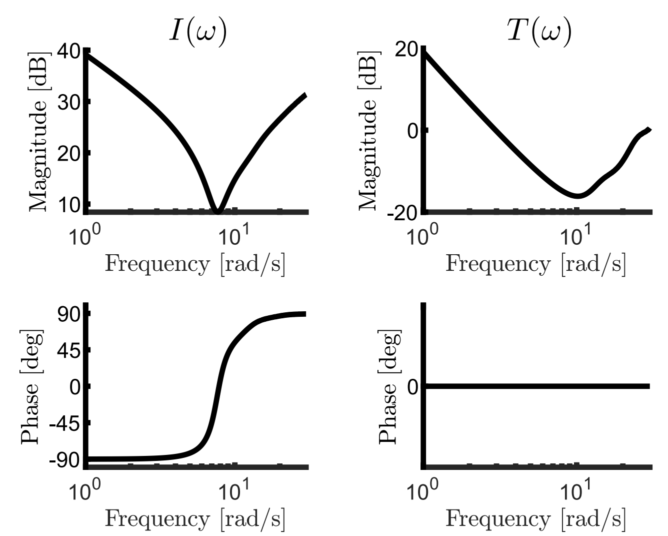

Fig. 5. Intrinsic impedance (left) and resulting optimal response (right) for the small-scale prototype of the so-called Wavestar WEC,.

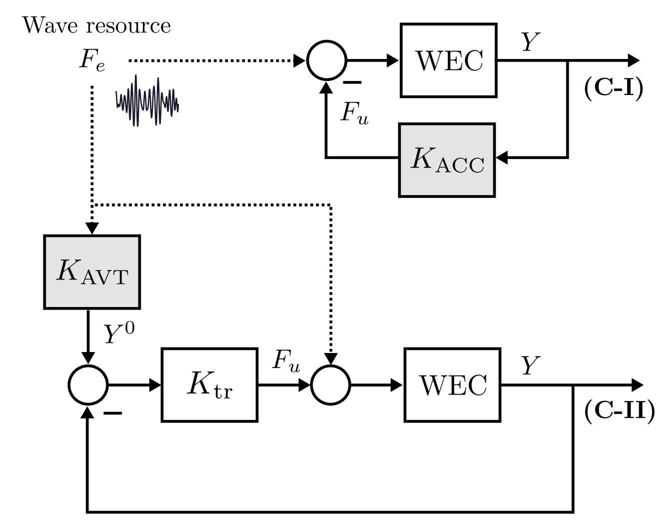

Fig. 6. Main control architectures for impedance-matching-based control solutions available in the WEC literature.

(22) generates an associated frequency-domain closed-loop response $F_e \mapsto Y$, which can be written as

$$T = \frac{1}{I + I^{\star}} = \frac{1}{2\mathfrak{R}(K_r)}, \tag{23}$$

where $T(\omega) \in \mathbb{R}^+$, $\forall\omega$, due to the positive real nature of K_r (which characterises the passive radiation system with impulse response k_r - see Section 2.4). By way of example, Fig. 5 illustrates the frequency response function characterising the intrinsic impedance for the small-scale prototype of the prototype Wavestar WEC, as considered for the wave energy control competition in Ringwood et al. (2019), along with the corresponding optimal closed-loop response (23).

Note that T in (23) essentially represents an ideal zero-phase filter (see *e.g.* Paarmann, 2006), meaning that, in optimal energy-maximising (steady-state) conditions, the velocity (output y) of the WEC system is a (real) scaled version of the wave excitation force input f_e. This scaling effect is, however, not constant, but frequency-dependant, according to the map in (23). Following this line of reasoning, two well-known optimal conditions have been widely adopted (see *e.g.* Faedo, García-Violini, Peña-Sanchez, & Ringwood, 2020), constituting the basis for a large variety of control strategies in the WEC field:

- *Optimal phase condition*: The instantaneous phase of y, under the control condition in (22), is synchronised with that of f_e.
- *Optimal amplitude condition*: The instantaneous amplitude of y, under the control condition in (22), is that of f_e scaled according to the map T in (23).

The conditions presented in this section, until this point, are derived entirely in the frequency-domain, without paying any specific attention to the effective implementation of the associated control force. As a matter of fact, a number of issues arise when attempting to implement the controller defined by the impedance-matching criterion in (22). In particular, one might, as a first attempt, be readily tempted to consider the analytic continuation of I_u in (22) to all of $\mathbb{C}$, though this would inevitably result in an inherently non-causal control structure, due to the nature of the complex-conjugate operator (and its corresponding analytic continuation (see *e.g.* Fuhrmann, 1989). A detailed discussion on the non-causality of I_u can be found in a number of studies, including *e.g.* Scruggs (2010).

Although inherently non-causal, the derivation provided by the impedance-matching principle has inspired a large number of techniques based on the idea of providing implementable control structures which approximate, in some sense, the frequency-domain condition expressed in (22). Within this impedance-matching approximation framework, two main architectures are commonly employed[3] (see *e.g.* Ringwood, 2020; Ringwood, Mérigaud, Faedo, & Fusco, 2020), following the designation suggested by Hals, Falnes, and Moan (2011a), as illustrated schematically in Fig. 6.

The controller structure **C-I** is typically referred to as *approximate complex-conjugate* (ACC) control, where the feedback structure $K_{\text{ACC}} \in \mathcal{RH}_\infty$ is virtually always designed such that $K_{\text{ACC}} \approx I^{\star}$ for $\omega \in \mathscr{W}$, where $\mathscr{W}$ is a subset of frequency range of interest. This set is commonly linked to the operating conditions of the associated WEC, *i.e.* excitation input wave spectra S_w as described in Section 2.1, and can be either composed of a (carefully selected) finite set of points, or a dense interval in $\mathbb{R}$. Examples of different ACC controllers, arising from approximation of the impedance-matching condition presented in this section, are Faedo, Pasta, et al. (2022), Previsic, Karthikeyan, and Scruggs (2021) and Song et al. (2016).

In contrast, the controller structure **C-II** in Fig. 6 is commonly termed *approximate velocity tracking* (AVT) control, since an optimal velocity profile y^0 is first generated, via suitable approximation of the closed-loop condition in (23) by means of $K_{\text{AVT}} \in \mathcal{RH}_\infty$. This is performed in a similar fashion to the ACC control case, *i.e.* for a carefully selected set $\mathscr{W}$. Note that an extra design degree-of-freedom is considered in the AVT case, where an 'inner' tracking controller K_{tr} is required to track the provided optimal profile, ultimately providing the injected control force f_u. Note that the AVT control structure requires knowledge of the wave excitation force f_e, which is not available in practice. As such, an estimate of f_e needs to be computed, as explicitly discussed within Section 4.1 of this paper. Cases of impedance-matching-based WEC controllers, adopting such an architecture, are Fusco and Ringwood (2012) and Garcia-Rosa, Kulia, Ringwood, and Molinas (2017).

[3] Other architectures are also available, including *e.g.* feedforward force controllers, such as those in Carapellese, Pasta, Paduano, Faedo, and Mattiazzo (2022), García-Violini, Pena-Sanchez, Faedo, Windt, and Ringwood (2020) and García-Violini et al. (2023).

3.3. Causal sub-optimal WEC controllers

The optimal energy maximisation control for WECs differs from conventional optimal control problems. Firstly, in conventional optimal controllers developed for setpoint tracking, disturbances generally drive the system state away from the reference setpoints or trajectories and, therefore, need to be attenuated (Zhan, Chen, Steffen, & Ringwood, 2022). In WEC control, the presence of persistent wave excitation, treated as the exogenous disturbance in the control formulation, essentially brings kinetic energy to the float, which is beneficial and must be utilised to achieve desirable energy conversion performance. Secondly, as discussed in Section 3.2, the WEC optimal control problem is inherently a non-causal problem. When used in WEC optimal control formulation, information about the short-term incoming wave can help to significantly increase the energy conversion rate (Zhan & Li, 2018), more than doubling for some WEC designs in particular sea states (Li, Weiss, Mueller, Townley, & Belmont, 2012). These unique features challenge the direct application of conventional control theory.

Causal sub-optimal control strategies have been developed for wave energy converters, which avoid using a wave predictor and decide the WEC control action purely based on the current and past information of the WEC devices. Some WEC causal sub-optimal controllers use only easy-to-measure feedback signals from motion sensors, such as displacement and/or velocity. Several studies (Babarit & Clément, 2006; Babarit, Duclos, & Clément, 2004; Falnes, 2001; Hoskin & Nichols, 1987) applied optimal control theory to determine the optimal parameters for a WEC latching controller.

When the WEC can be described by a linear model (9), several studies (Schoen, Hals, & Moan, 2011; Scruggs, 2010; Scruggs et al., 2013) formulated the causal sub-optimal control problem into a non-standard linear quadratic optimal control problem, with a closed-form analytic solution. Stochastic sea states can be incorporated in the control formulation to estimate produced energy, but cannot be used to improve energy conversion, as in the non-causal controllers described in Section 3.4. In contrast to optimal latching strategies, linear causal sub-optimal control uses full-state feedback, including those non-directly measurable states associated with the radiation dynamics. This implies the need for a state observer, which is discussed in Section 4. Zhan, Huijberts, Na, and Li (2016) incorporate the current wave information into the causal sub-optimal control formulation, resulting in a closed-form controller, consisting of a state-feedback part and an additional feedforward part of the current wave. Nie et al. (2016) generalise the application of linear causal sub-optimal control to a WEC described by nonlinear modes using a time-varying linearised model and a state-dependent control parameter. Zou and Abdelkhalik (2020b) developed a similar time-varying causal linear quadratic control for a 3-DOF WEC. Although an extended Kalman filter was designed to provide future wave information, this control framework does not use predictions to improve performance and, therefore, belongs to the causal sub-optimal control category. Considering monochromatic waves, Nielsen, Zhou, Kramer, Basu, and Zhang (2013) propose a causal design strategy, which can approximate the non-causal optimal solution with minor performance degradation.

Safety constraints (17) can also be considered in the causal sub-optimal formulation. Zhan et al. (2016) applied a set analysis method to assess the potential risk of violating safety constraints. Considering only stroke limits, Scruggs (2017) proposed a two-stage control design. The linear causal sub-optimal controller is augmented with a nonlinear passivity-based outer-loop design to prevent stroke saturation with guaranteed control stability.

By solving constrained online-optimisation problems recursively at each time step, model predictive control (MPC) provides a natural control mechanism to optimally handle constraints (17) for WECs. By minimising the deviation between an unconstrained linear causal sub-optimal control, and a signal considering constraints, Zhan, He, and Li (2017) developed a control structure, which uses feedback MPC to cope with constraints. Important control safety features can be guaranteed, such as robust constraint satisfaction, recursive feasibility, and stability.

3.4. Optimal predictive (non-causal) controllers

As discussed in Sections 3.2 and 3.3, the control formulation must incorporate non-causal information of f_e to find the *true* optimal control signal for WECs. With the state-of-the-art wave prediction technique (see Section 4), many optimal predictive (non-causal) control strategies have been developed for WECs, which show significant performance improvement over causal controllers. The extra cost required to obtain the non-causal wave information, e.g. sensors and microprocessors, is relatively minor compared with the benefit of using predictive control. Recent progress in wave prediction, detailed in Section 4, makes implementing a non-causal control structure more economically viable (Ringwood et al., 2014).

3.4.1. Time domain

MPC is a well-developed advanced control technique that naturally deals with non-causality, constraints and nonlinearity, the efficacy of which has been demonstrated over a wide range of control applications (Mayne, 2014; Mayne, Rawlings, Rao, & Scokaert, 2000; Qin & Badgwell, 2003). MPC has been applied to WECs for over a decade. Gieske (2007) applied a linear MPC to the Archimedes wave swing WEC. Other pioneer researches focused on point-absorber WEC (also assuming linear dynamics), e.g. Brekken (2011), Cretel, Lewis, Lightbody, and Thomas (2010), Cretel, Lightbody, Thomas, and Lewis (2011) and Hals, Falnes, and Moan (2011b).

Compared with conventional MPC problems for setpoint tracking, WEC MPC is a non-conventional problem, where the control objective function, usually defined as the converted energy to maximise performance (hence reducing LCOE, as per (14)), is not always positive definite, which leads to implications on *constraint satisfaction* and *convexity*.

Generally speaking, the explicit handling of constraints within a optimal constrained formulation (as opposed to a constrained optimal formulation) guarantees the safety of WEC operation. When the MPC is 'recursively feasible', i.e., a sequence of optimal signals can be found repeatedly for the associated constrained optimisation problem, constraints (17) are handled in a rolling-based manner. However, there are possible scenarios that the incoming wave excitation force f_e is so significant that no control solution f_u can be founds which simultaneously satisfies constraints on stroke and force limits (see Bacelli & Ringwood, 2013b; Zhan, Li, & Bailey, 2019 for details).

For WEC MPC to be real-time implementable, *convexity* of the resulting MPC problem is essential so that efficient solvers can be used to solve the online optimisation problem within one sampling interval. Convexity also guarantees a unique global solution which, with the inherent robustness of MPC (Fang & Chen, 2022; Monasterios & Trodden, 2018), prevents discontinuous closed-loop control signals. However, with a non-definite cost in the MPC formulation, the resultant MPC may not always be convex, even for a simple linear case (see Li & Belmont, 2014 for details).

Considering a WEC with linear dynamics, Li and Belmont (2014) proposed a convexification for the cost function by modifying the cost function by adding penalties on PTO force f_u, heave displacement z, and the slew rate of f_u, but optimising this biased objective (with added penalties) may degrade performance, as identified in Zhan, Li, and Bailey (2019). Another study (Kody, Tom, & Scruggs, 2019) provided a condition on convexity for linear WEC MPC, i.e. when the passivity requirement for WEC modelling is satisfied, the resultant linear MPC for WEC is naturally convex, assuming a sufficiently long horizon, which addresses the convexity issue without affecting energy production performance. The recursive feasibility property can be guaranteed using similar approaches that have been used in conventional robust MPC, such as constraint tightening approaches (Zhan, Li, & Bailey, 2019), which essentially provides a safe operational sea state range, characterised by an upper bound on f_e.

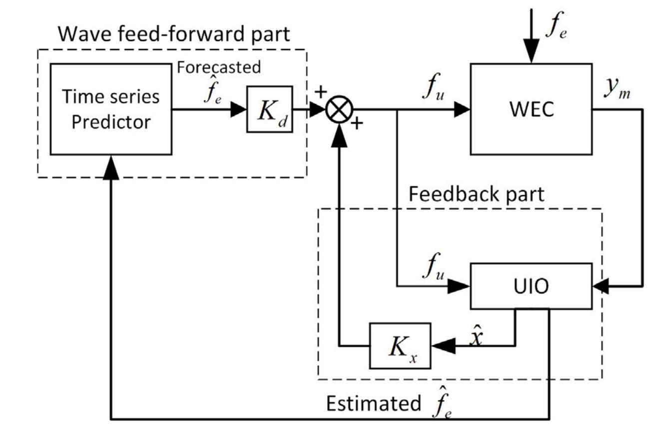

Fig. 7. Illustrative diagram for the linear predictive (non-causal) optimal control framework with an unknown input observer (UIO in the diagram) to estimate $\hat{x}$ and $\hat{f}_e$, and a time-series predictor for future $\hat{f}_e$ prediction.

Instead of directly considering the energy maximisation objective in cost functions, an alternative two-step approach was developed in Hals et al. (2011a) where initial unconstrained optimal velocity trajectories were generated, followed by a MPC to track the optimal velocity trajectory and to handle constraints. In this manner, the design of WEC MPC follows the standard design procedure of conventional tracking MPC.

At the expense of not being able to explicitly handle constraints, the non-causal WEC MPC problem, which requires repeated solution of online optimisation, will reduce to an unconstrained linear non-casual optimal control (LNOC) problem, which can be solved with

$$f_u = K_x x + K_d \boldsymbol{f}_e \tag{24}$$

consisting of a linear state-feedback part, and a feedforward part utilising a short-term wave forecast, can be implemented straightforwardly with offline-calculated coefficients. In (24), K_x and K_d are the feedback and feedforward control coefficients, respectively; $\boldsymbol{f}_e$ is a short-term forecasting sequence of f_e with a horizon n. An illustrative diagram for LNOC is shown in Fig. 7, with LNOC successfully applied to a number of WECs, e.g. M4 with multiple DOFs and PTOs (Liao, Stansby, Li, & Moreno, 2021).

Targeting WECs described by nonlinear dynamics, Ling, Bosma, and Brekken (2019) locally linearised the nonlinear model and then applied linear MPC, while Richter, Magana, Sawodny, and Brekken (2012) and Sergiienko, Cocho, Cazzolato, and Pichard (2021) developed a nonlinear MPC (NMPC) that generates optimal control action by directly solving the associated nonlinear programming problem. NMPC was also designed and investigated using optimal control theory, such as that based on Pontryagin's minimum principle (PMP) (Nielsen et al., 2017), in a continuous-time formulation and, based on Bellman's equation via dynamic programming (DP), in a discrete-time formulation Li et al. (2012). Methods based on optimal control theory usually have a common drawback of high computational cost. Specifically, DP-based methods suffer the well-known 'curse of dimensionality', while PMP-based methods must solve a nonlinear Hamilton–Jacobi–Bellman (HJB) equation online, which is challenging in a real-time receding horizon context. Adaptive dynamic programming (ADP) can reduce the computational load of DP by using a function approximation structure to approximate the solution of the HJB equation. Based on this principle, Zhan, Na, and Li (2019) developed a model-based noncasual ADP for WECs, which uses online policy iterations to find the optimal control policy. However, there is no universal way to find an appropriate function approximation, which limits the application of ADP to WECs with relatively simple dynamics.

In a time domain formulation, guaranteeing *constraints* and *convexity* for WECs described by nonlinear dynamics is more complex than in linear cases. The invariant sets for nonlinear systems are generally difficult to obtain Köhler, Müller, and Allgöwer (2018), which makes the 'constraint-tightening' approach challenging to implement for WECs with nonlinear dynamics. Regarding convexity, NMPC, in general, is non-convex, which means (i) there is no efficient solver guaranteed to find a global minimum solution at each time instant; and (ii) the NMPC may quickly switch between multiple local minima, leading to a discontinuous input signal. Constraints and penalties on the changing rate of input signal have been added, in an attempt to resolve the discontinuous input signal issue (Li et al., 2012). However, how to formulate NMPC for WECs in the time domain, guaranteeing smooth input trajectories with optimised energy output and constraint handling, remains an open question.

3.4.2. Alternative controller parameterisations

In an effort to overcome some of the main issues affecting standard linear and nonlinear MPC formulations (see Section 3.4.1), researchers from the WEC control field have leveraged techniques from the more general field of direct optimal control theory. As in the case of standard MPC, direct methods, also often referred to as *'first discretise then optimise'* techniques, discretise the variables involved in the WEC optimal control problem, and attempt the maximisation of the resulting nonlinear program (NP) directly, by exploiting numerical optimisation techniques.

Initially, two families of direct optimal control techniques gained popularity within the WEC field, namely *spectral* and *pseudospectral* methods. These two strategies, which belong to the family of so-called mean weighted residual methods (Finlayson & Scriven, 1966), have been traditionally used for the numerical approximation of the solution of partial differential equations by discretising and minimising an associated residual function. Their effective use in control procedures can be traced back to the Aerospace community (see *e.g.* Ross & Karpenko, 2012). The underpinning idea of these methods can be informally outlined as follows (the interested reader is referred to Boyd (2001) and Shizgal and Shizgal (2015) for a detailed discussion on these topics). Assume that the dynamics associated with the WEC system can be described in terms of the differential equation $\dot{x} = f(x, u)$. Then, the variables x and u are assumed to belong to a given function space $\mathscr{X}$, typically either Hilbert or Sobolev. A series expansion of x_Q and u_Q is sought, in terms of a finite-dimensional subset $\mathscr{X}_Q \subset \mathscr{X}$, with coefficients $\mathcal{X}_Q = \{\alpha\}_{i=1}^{Q}$ and $\mathcal{U}_Q = \{\beta\}_{i=1}^{Q}$ of appropriate dimensions, respectively. Subsequently, a residual function $\mathscr{R} := \dot{x}_Q - f(x_Q, u_Q)$ is defined accordingly, and projected onto a finite-dimensional subset $\tilde{\mathscr{X}}_Q \subset \tilde{\mathscr{X}}$, where $\tilde{\mathscr{X}}$ is potentially different to the space $\mathscr{X}$, considered to describe the state and input variables. Such a projection produces a set of algebraic equations $\tilde{\mathscr{R}}$ in the coefficients $\mathcal{X}_Q$ and $\mathcal{U}_Q$. The set of equations $\tilde{\mathscr{R}}$, together with x_Q and u_Q, are explicitly considered to parameterise the WEC optimal control problem, which is now transcribed to a finite-dimensional NP.

If the set of functions $\mathscr{X}_Q$ coincides with that used for the corresponding projection of the residual map, *i.e.* $\tilde{\mathscr{X}}_Q$, the method is known as *spectral*. If, on the contrary, the set $\tilde{\mathscr{X}}_Q$ is composed of translated Dirac-delta functions, the method is commonly referred to as *pseudospectral* (often also referred to as *collocation* method). Naturally, the function spaces involved in the transcription process will depend on the overall nature of the control problem to be solved. In the WEC case, different spaces have been considered within the state-of-the-art, as discussed in the following paragraphs. Furthermore, unlike MPC, spectral and pseudospectral methods, as applied in the WEC field, are often based on functions defined over the entire control horizon, *i.e.* they have global, rather than local, support.

Fourier-type functions are those predominant within the WEC field, with Bacelli, Ringwood, and Gilloteaux (2011) being the pioneering paper implementing a spectral formulation for a self-reacting point absorber device. The choice of this set of functions is clearly motivated by the nature of the WEC process itself, and the surrounding wave field, which can be reasonably described in terms of trigonometric polynomials of a given fundamental frequency (see *e.g.* Mérigaud & Ringwood, 2017a, 2017c). In fact, Fourier-type expansions have shown to be very efficient in computational terms, since reduced approximation spaces (in terms of dimensions) are often required to have an accurate representation of state and input variables, *i.e.* a handful trigonometric polynomials are sufficient to compute an accurate numerical solution of the associated control problem, given the harmonic nature of the problem. Studies, which consider this type of approach in the *spectral* case are *e.g.* Abdelkhalik, Robinett, et al. (2016) and Bacelli and Ringwood (2014b) for single linear devices, while Bacelli, Balitsky, and Ringwood (2013), Bacelli and Ringwood (2013a), Garcia-Rosa, Bacelli, and Ringwood (2015) and Westphalen, Bacelli, Balitsky, and Ringwood (2011) consider the case of arrays of linear WEC systems (see also Section 3.5). This inherent numerical efficiency has been exploited to provide solutions for nonlinear WEC systems in Auger, Merigaud, and Ringwood (2018), Mérigaud, Ngo, Nguyen, Sabiron, and Tona (2020), Mérigaud and Ringwood (2018) and Mérigaud and Tona (2020), which, as discussed in Section 3.4.1, can be potentially prohibiting for NMPC. For an in-depth discussion on computational aspects for this family of WEC Fourier spectral methods, see Mérigaud and Ringwood (2017b). Fourier-type expansions have been also exploited within a pseudospectral approach, including the studies presented in *e.g.* Abdelkhalik et al. (2018), Bacelli et al. (2015), Bacelli and Ringwood (2014a), Paparella and Ringwood (2016) and Tom, Yu, Wright, and Lawson (2017), and even combined with standard adaptive control procedures in Davidson, Genest, and Ringwood (2018).

Though inherently efficient due to their connection with the WEC absorption process, Fourier-type expansions have a particular drawback: Trigonometric polynomials are, by nature, periodic functions, presenting issues for real-time implementation, which inherently requires receding-horizon computation. In other words, the approximation of state and input variables by means of periodic functions in a receding-horizon fashion, inherently presents the well-known Gibbs phenomenon, artificially induced by considering a short time window. This has been, nonetheless, solved in Auger et al. (2018) and Mérigaud and Ringwood (2018), by exploiting classical windowing techniques (see *e.g.* Prabhu, 2014), hence forcing state and input variables to be compactly supported within the receding window. The connection between Fourier-type functions, and the WEC process itself, does not only represent an advantage from a computational perspective, but also facilitates handling of the 'original' energy objective function, *i.e.* spectral/pseudospectral methods based on Fourier descriptions do not require (in general) the adoption of regularisation terms to guarantee convexity of the associated NP (at least in the linear case (Bacelli & Ringwood, 2014b) - see also the discussion in Faedo, Olaya, & Ringwood, 2017).

Other variations, apart from Fourier functions, do exist within the WEC application field. In particular, and aiming to solve the periodicity issue underlying trigonometric expansions in a receding-horizon window, Genest and Ringwood (2016b) exploit the family of half-range Chebyshev polynomials. These functions, originally proposed in Huybrechs (2010) are, in essence, an extension of the well-known Fourier series for non-periodic functions, and have been shown to be computationally efficient for the WEC application. While providing a swift implementation in a receding-horizon setting (see Genest & Ringwood, 2016a) for a comparison study against MPC for a point absorber WEC system), the technique in Genest and Ringwood (2016b) does not enjoy the same existence and uniqueness guarantees offered by Fourier-type expansions. Finally, Legendre-based pseudospectral methods are applied in Herber and Allison (2013) and Li (2017), where the latter exploits the differential flatness of the adopted (nonlinear) model to enhance the computational properties of the strategy.

Another family of direct optimal control techniques, effectively originated within the WEC field by Faedo, Scarciotti, Astolfi, and Ringwood (2018), is *moment-based* control. These techniques are based on the concept of *moments*, originally formulated within the field of model reduction by moment-matching (see *e.g.* Astolfi, 2010; Astolfi, Scarciotti, Simard, Faedo, & Ringwood, 2020; Faedo, Scarciotti, Astolfi, & Ringwood, 2021c). Moments are, in essence, mathematical objects which describe, under the right circumstances, the steady-state response map of a general class of (linear and nonlinear) systems. In particular, moment-based control is constructed on the basis of two fundamental stepping stones, in the spirit of (geometric) nonlinear regulation theory (see *e.g.* Isidori, 1995): An implicit form description of the input(s) affecting the WEC system, and a corresponding invariant manifold describing the steady-state response of the device for the defined class of inputs. Informally, and referring to a WEC system described in terms of the differential equation $\dot{x} = f(x, u)$ (as in the case of spectral/pseudospectral methods within this section), the input map u is described in implicit form, via an exogenous system (often referred to as *signal generator*) *i.e.* $\dot{\xi} = s(\xi)$ and $u = l(\xi)$. Under some technical assumptions (Astolfi, 2010; Faedo, Scarciotti, Astolfi, & Ringwood, 2021b), there exists a mapping $\pi : \xi \mapsto \pi(\xi)$ which, for a given trajectory of the signal generator, coincides with the steady-state response of the WEC system, that is $x_{ss}(t) = \pi(\xi(t))$.

Both the implicit representation of u, together with the associated mapping π, can be used to parameterise the WEC optimal control problem in terms of the state-vector of the signal generator ξ, effectively obtaining a finite-dimensional problem. Apart from the underlying computational advantages (Faedo et al., 2022a, 2021b), the particular representation offered by moment-based theory has facilitated a number of theoretical results, often not offered/discussed within spectral and pseudospectral methods. For instance, Faedo et al. (2022b, 2021b) provides a framework for moment-based control for a general class of nonlinear WEC systems, explicitly elucidating conditions for the existence and uniqueness of control solutions, by showing that the objective function is consistently mapped to a general class of convex functions, by virtue of a suitable choice of the associated signal generator. Furthermore, a receding-horizon formulation is developed in Faedo, Peña-Sanchez, and Ringwood (2020b) for real-time control implementation, validated experimentally in Faedo et al. (2023), including corresponding wave excitation estimation and forecasting (see Section 4). Array extensions of moment-based control for WECs have been also formulated, in Faedo, Scarciotti, Astolfi, and Ringwood (2019, 2021a) (see also Section 3.5). Further exploiting the relationship between moments and steady-state, Faedo, García-Violini, Scarciotti, Astolfi, and Ringwood (2019) and Faedo, Mattiazzo, and Ringwood (2022) provide a robust formulation of moment-based control considering both input (wave excitation force) and system uncertainties, respectively, as further discussed in Section 3.6.1. Finally, a combination between moment-based control and higher-order sliding modes has been proposed in Mosquera, Faedo, Evangelista, Puleston, and Ringwood (2022), and validated experimentally in Faedo, Mosquera, Evangelista, Ringwood, and Puleston (2022), using a hardware-in-the-loop system for WEC emulation.

3.5. Control of WEC arrays

As discussed within Section 1, the deployment of WEC systems is likely to happen in array configurations (also often referred to as 'parks' or 'farms'), in an effort to proportionally reduce the associated costs of installation, operation, and maintenance (Göteman, Giassi, Engström, & Isberg, 2020; Robertson, Hiles, Luczko, & Buckham, 2016), which helps in reducing the overall CapEx and OpEx, contributing to the reduction of LCoE. This procedure involves the deployment of a large number of WEC systems in a common area, arranged according to a given layout configuration (see *e.g.* Yang et al. (2022) for further detail on layout choices and associated optimisation).

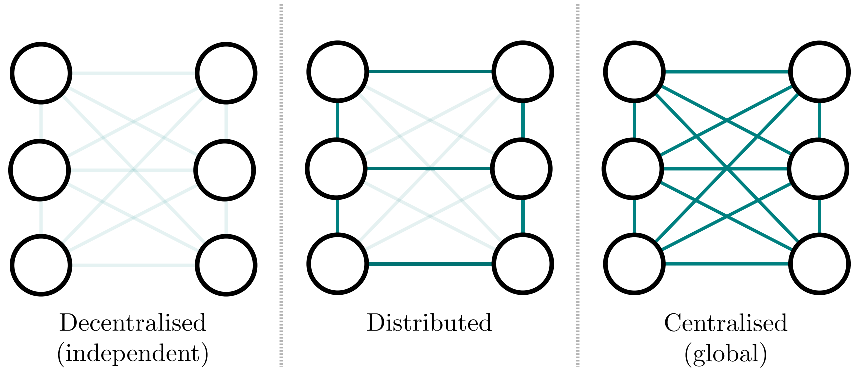

Fig. 8. Schematic illustration of decentralised, distributed, and centralised control of WEC arrays.

Devices in close proximity naturally interact with each other: Each WEC in the array has the capability to modify the surrounding wave field, affecting the (hydro)dynamics of nearby devices accordingly. These hydrodynamic interactions can affect the overall performance of the array, having the potential for both positive or negative effects on the power absorption capabilities of neighbouring devices (see *e.g.* Babarit, 2013). As such, consideration of these interactions can become particularly relevant within the control design procedure, and hence different approaches can be found in the state-of-the-art to handle the WEC array control scenario.

Early studies in WEC array control can be found in Evans (1979) and Thomas and Evans (1981), based upon frequency-domain optimal conditions (*i.e.* the impedance-matching principle presented in Section 3.2). Thomas and Evans (1981) effectively incorporates motion constraints into the control design procedure being, to the best of our knowledge, the first result with practical impact proposed within the state-of-the-art of WEC array control.

Contemporary studies often include more sophisticated control techniques, in either a decentralised (often also called 'independent'), distributed, or centralised (often referred to as 'global') control formulations, as depicted in Fig. 8. Within the WEC array case, the term decentralised is used for controllers which 'ignore' the hydrodynamic interactions between devices in the array, and hence the associated control computation depends on the dynamics of a single device only. While less computationally demanding, this can often lead to suboptimal performance in terms of energy absorption, especially if the devices are located reasonably close to each other for economic purposes (*e.g.* sharing of mooring systems). Centralised controllers, in contrast, aim to incorporate the complete hydrodynamic interactions affecting the WEC array, providing superior performance with respect to their decentralised counterparts. There is, clearly, a price to pay in terms of computational burden, which can often preclude real-time implementation, particularly in the case of controllers involving an online optimisation procedure, such as *e.g.* MPC-based solutions. Distributed controllers are somehow 'half-way' in between decentralised and centralised formulations, making use of local models which incorporate the most significant interactions between devices in the control computation procedure, within a limited 'neighborhood' of a given WEC. This formulation can approach the performance of a centralised controller, depending on the array configuration/spacing, but with a significant reduction in computational requirements. Such an approach is not unrelated to the decentralised control of network systems (Zhang, Li, & Li, 2022). We further note that WEC arrays have been also modelled (and controlled) as multi-agent systems (see *e.g.* Xie & Liu, 2017), as presented in Pereira, de Oliveira Valério, and Beirão (2021) for a farm of inertial pitching conversion systems.

Frequency-domain centralised controllers, in the spirit of Section 3.2, can be found in Folley and Whittaker (2009) and Wu et al. (2016), where the impedance-matching principle is used to compute the optimal power production of a given WEC array. Parametric control laws, optimised in a centralised fashion, can be found in Lyu, Abdelkhalik, and Gauchia (2019) (passive damper), Zou and Abdelkhalik (2020a) (proportional–integral controller optimised by leveraging a surrogate model), and Wang, Engström, Leijon, and Isberg (2016) (time-varying damping in constrained conditions). The study in Pasta, Veale, et al. (2021) also exploits a parametric control law (proportional–integral), which is optimised by adopting a tailored genetic algorithm (GA), where each device in the array is considered as an individual within the GA evolution procedure. Based on the optimal phase condition presented in Section 3.2, Thomas et al. (2018) propose a latching control strategy, where the corresponding latching times for each WEC in the array are optimised via model-free collaborative learning.

As per the single device case (see Sections 3.2 to 3.4), a number of MPC-based solutions can be found in the literature for WEC arrays, often built upon extensions of the techniques presented for an isolated device. Centralised MPC formulations can be found in Oetinger, Magaña, and Sawodny (2015), Zhang, Zhang, et al. (2022) and Zhong and Yeung (2019, 2022). The formulation presented in Zhong and Yeung (2019, 2022) is based on MPC for a single device presented in Zhong and Yeung (2018), which incorporates a set of constraints on the rate of change of the associated PTO control inputs. The study in Zhang, Zhang, et al. (2022) is somewhat unique in the sense that modelling is approached by exploiting tools from the theory of complex networks (see *e.g.* Latora, Nicosia, & Russo, 2017), *i.e.* the WEC array is modelled in terms of a graph, and characterised accordingly. A decentralised version of the MPC in Oetinger et al. (2015) can be found in Oetinger, Magaña, and Sawodny (2014), offering a corresponding comparison between both formulations in terms of energy-absorption performance. Finally, within the MPC solution space, the study (Li & Belmont, 2014) proposes all three scenarios, *i.e.* decentralised, distributed, and centralised MPC, including a critical comparison in terms of key parameters, such as relative absorbed energy and computational demand associated with each approach.

In line with the alternative controller parameterisations reported in Section 3.4.2, (Bacelli et al., 2013; Bacelli & Ringwood, 2013a; Garcia-Rosa et al., 2015; Westphalen et al., 2011) propose centralised and decentralised control formulations based on spectral methods, using trigonometric basis functions. Within these studies, the benefit of centralised over decentralised control is studied in detail, including motion and control constraints, by virtue of the computationally efficient spectral formulation. Within moment-based control, Faedo, Scarciotti, et al. (2019) and Faedo et al. (2021a) present an array formulation of the framework introduced in Faedo, Scarciotti, et al. (2018), illustrating the benefits of this approach on a farm of devices by including a comparison with a benchmark WEC array controller.

Finally, indirect optimal control methods have also been exploited for the WEC array scenario, particularly in Abdulkadir and Abdelkhalik (2022a, 2023) in a centralised formulation, where a bang-singular-bang solution is proposed, based on the strategy developed in Abdulkadir and Abdelkhalik (2022b). Note that this controller introduces a constraint on the sign of the mechanical instantaneous power, often desired to avoid large reactive power flow in the conversion chain.

3.6. Dealing with model uncertainty

It is clear, from various studies (Penalba, Giorgi, & Ringwood, 2017), that there is a significant challenge in (a) providing a model of suitable fidelity, and (b) ensuring that the model is of a suitable analytical/computational complexity, for model-based WEC control design. Inevitably, there are compromises in the accuracy of model-based WEC control designs, while the sensitivity of particular WEC control structures (see Fig. 6) to specific hydrodynamic modelling errors has also been highlighted (Ringwood et al., 2020). Some solutions to these difficulties are now presented.

3.6.1. Robust control

Robust control design provides a potential mechanism to deal with modelling inaccuracy (unmodelled dynamics) as well as model uncertainty. This permits the possibility of employing a simpler (e.g. linear) system model/controller while synthesising a controller which will guarantee robust stability, or give the best achievable performance, over the full range of operating conditions. Inevitably, the greater the range of model inaccuracy/uncertainty covered, the more conservative will be the control action, limiting the attainable performance.

In traditional (setpoint following, or regulation) control applications, elaborate robust control synthesis procedures have been developed, dealing with both structured (Stein & Doyle, 1991) and unstructured (Francis & Zames, 1984) uncertainty. However, given the predefined (complex conjugate) relationship between the WEC 'transfer function' and the feedback controller K_{ACC} in Fig. 6, there is little flexibility in altering the system sensitivity properties through loop shaping. Equally, the design of K_{AVT} (Fig. 6) in the feedforward AVT controller needs to effectively implement a complex conjugate relationship with the WEC system description. However, given that the lower velocity tracking AVT loop is a more traditional setpoint following loop, the application of traditional robust control techniques can be effected.

One of the earliest applications of robust control to a wave energy system is described in the study by Schoen et al. (2011), who employ a fuzzy controller to deal with system uncertainty.

In Garcia-Violini and Ringwood (2021), a linear pseudospectral control method is employed which maximises the worst case performance over an uncertainty set, with a method described in Farajvand, Grazioso, García-Violini, and Ringwood (2023) to determine the associated nominal model and uncertainty region for a given application. In a similar vein, the robust approach in Faedo, García-Violini, et al. (2019) casts the robust control problem in the moment domain, again utilising best worst case performance as a metric, while the associated study in Faedo, Mattiazzo, and Ringwood (2022) is somewhat unique in dealing with uncertainty in the estimate/forecast of f_e provided to the control algorithm. Lao and Scruggs (2020) employ a multi-criterion approach to simultaneously optimise energy capture performance while ensuring robust stability. The resulting optimisation problem is non-convex, but a specific optimisation technique is suggested for its solution. Jama, Wahyudie, and Noura (2018) employ an 'ultra-local' model to account for mismatch between the main model-based MPC controller and the system. However, it could be argued that this controller is better classified under 'data-driven', rather than robust control, since measured data is used to tune the local model. A tube-based robust MPC approach is reported by Zhang and Li (2022), utilising an unstructured uncertainty term. A robust causal MPC WEC controller is detailed in Zhan et al. (2017)

Abdelrahman and Patton (2017) propose a robust WEC controller, though the emphasis is on optimal position following for the linear generator, with the optimal (AVT) command provided by the method described in Guo, Patton, Abdelrahman, and Lan (2016). Fusco and Ringwood (2014) also focus on robust control of the velocity tracking loop, with the velocity setpoint provided by a controller based on the core system characteristics, which has some insensitivity to modelling error.

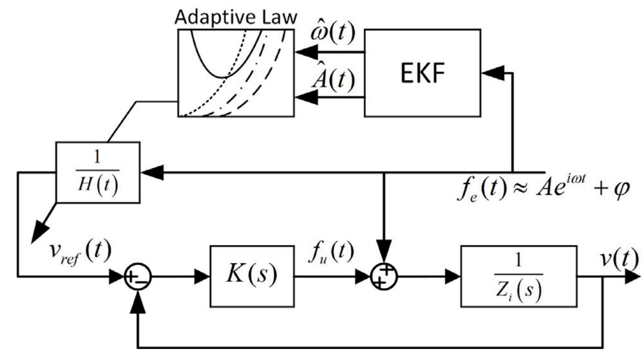

Fig. 9. The control structure of a 'simple and effective controller' (Fusco & Ringwood, 2012). An adaptive mechanism is designed based on the information provided by the EKF, which helps ensure the optimal velocity profile by adapting the feedforward gain $1/H(s)$, which depends on the instantaneous wave frequency ans amplitude.

Though not explicitly termed 'robust control' techniques, the excellent robustness of sliding mode techniques is also noteworthy. However, in general, like a number of other studies, they are generally relegated to the task of velocity following within the wave energy domain e.g. Mosquera, Evangelista, Puleston, and Ringwood (2020) and Zou, Song, and Abdelkhalik (2023).

3.6.2. Adaptive control

Adaptive control involves the use of data to update the parameters of a model-based controller either directly (implicit) or indirectly (explicit). In the wave energy case, as in many other application areas, adaptive control can be used to determine unknown parameters (often termed self-tuning), or track variations in the system dynamics, due either to environmental effects, or to maintain the relevance of the controller parameters at the current operating condition (adaptation). The latter scenario is typical when, for instance, the system is truly nonlinear, but an adaptive linear model is used to track the system dynamics, in the spirit of gain scheduling (Leith & Leithead, 2000).

In the simplest case, an adaptive mechanism can be used to adapt the parameters of an impedance-matching controller to handle the change of incoming waves such that the resonance frequency for the WEC with an adaptive controller matches the dominant frequencies of the incoming wave to maximise energy. Assume the excitation force $f_e(t)$ is a narrow-banded harmonic process

$$f_e(t) = \mathcal{A}(t)\cos(\omega(t)t + \psi(t)), \tag{25}$$

where the amplitude $\mathcal{A}$, frequency ω and phase ψ are time-variant variables, which can be updated using an extended Kalman filter based on real-time measurements (Budal & Falnes, 1982; Fusco & Ringwood, 2010). With the real-time updated wave model (25), an adaptive control philosophy can be designed with the structure shown in Fig. 9 (see Fusco & Ringwood, 2012; Ringwood et al., 2014 for the details). Alternative techniques, including the Hilbert-Huang transform and a frequency-locked loop, have also been explored for tracking the instantaneous wave frequency, with their impact on control performance assessed by Garcia-Rosa, Ringwood, Fosso, and Molinas (2019).

Like robust control, adaptive mechanisms can also be designed to handle uncertainties and model mismatches for model-based controllers (Åström & Wittenmark, 2013). The main differences between robust control and adaptive control are:

- Robust control requires prior information about the error bound, within which the control law will be fixed, and

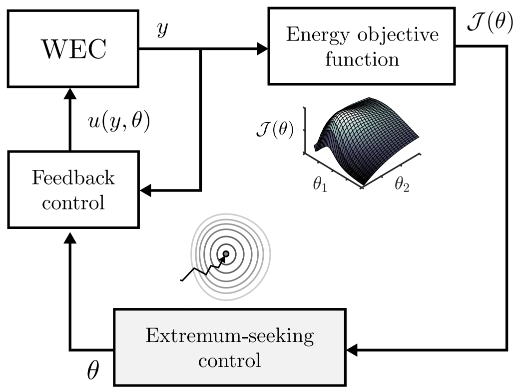

Fig. 10. Schematic of ESC as applied in wave energy. θ represents a set of parameters, optimised via the ESC algorithm to maximise energy absorption.
Source: Adapted from Moens de Haste, Pasta, Faedo, and Ringwood (2021).

- adaptive control does not need error-bound information, but the control law is constantly adapted.

Korde, Robinett, and Wilson (2015) implemented an adaptive trajectory-tracing control, using mechanical and hydrodynamic parameters updated via an online estimation mechanism, to cope with the slow variation of the (true nonlinear) WEC dynamics. Davidson et al. (2018) developed an adaptive control scheme, consisting of online parameter estimation using a recursive least square (RLS) algorithm, which constructs a 'most representative' linear model, and a receding horizon pseudospectral control to maximise the performance, verified in a CFD-based numerical wave tank. Zhan, Wang, Na, and Li (2018) developed a adaptive hierarchical MPC scheme, where an online cascaded identification mechanism is developed to adapt the excitation force dynamics on the top layer and, on the bottom layer, an MPC is developed to solve the constrained energy maximisation control problem.

3.6.3. Model-free control

Given the potentially large level of uncertainty associated with linear WEC modelling, and the computational complexity associated with high-fidelity models, an ideal scenario, within the wave energy conversion field, would be to avoid the use of a model at all, *i.e.* to exploit model-free control strategies. Although a number of data-driven techniques have been adopted within the field, the 'truly' model-free strategies considered, which exploit online data *only* for control decision, belong to the family of extremum-seeking control (ESC) (see *e.g.* Krstić & Wang, 2000). Briefly summarising, ESC comprises a family of techniques designed with the objective of optimising a parametric control law in terms of a given criterion solely based on the availability of online (measured) data, as schematically depicted in Fig. 10. Naturally, having the capabilities of incorporating 'non-traditional', *i.e.* economic, objectives in a relatively straightforward fashion, ESC found its way to energy systems, being often considered as a 'silver bullet' in *e.g.* solar energy.

To the best of our knowledge, the pioneering application of ESC within the WEC field is that proposed in Hals et al. (2011a), where a continuous-time ESC is designed, following the seminal work (Krstić & Wang, 2000), and evaluated for a point absorber WEC system. A similar approach is adopted in Garcia-Rosa, Lizarralde, and Estefen (2012) but inherently designed in discrete-time. Sun et al. (2018) and Zhao et al. (2019) propose a discrete-time solution based on non-standard numerical optimisation techniques, namely the so-called 'flower pollination' approach (Yang, Karamanoglu, & He, 2014). The comparison study in Parrinello et al. (2020) offers a critical performance evaluation of a series of ESC algorithms, including perturbation-based (Krstić & Wang, 2000), sliding mode (Pan, Özgüner, & Acarman, 2003), self-driving (Straus, Krishnamoorthy, & Skogestad, 2019), relay (Olalla, Arteaga, Leyva, & El Aroudi, 2007), and least-squares (Hunnekens, Haring, van de Wouw, & Nijmeijer, 2014) ESC strategies. Finally, Pasta, Faedo, et al. (2021) presents an extension of the least-squares technique proposed in Parrinello et al. (2020), which is able to accommodate soft constraints within the control objective.

As discussed in detail within Moens de Haste et al. (2021), though ESC has the major advantage of being completely model-free, being particularly appealing for the WEC application, it comes with its own pitfalls, some of which have been addressed within the state-of-the-art. Nonetheless, the currently adopted controller parameterisations, which are limited by the very nature of the (time-varying) WEC objective function, are rather simplistic to operate in real (and potentially broadband) sea states, hence presenting sub-optimal results with respect to most model-based techniques. Furthermore, (hard) constraint handling is not straightforward without the availability of a dynamical model to predict the behaviour of the WEC system to effectively enforce limitations, which compromises the optimality of ESC in constrained scenarios. This can, at least partially, explain why ESC has not yet been broadly adopted within the field.

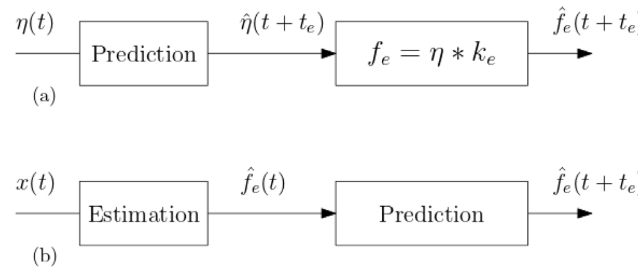

Fig. 11. Possibilities for prediction of the wave excitation force f_e: (a) using measurement/estimation of the free surface, and (b) using an unknown input estimator.

4. Estimation and prediction

As discussed in Section 3, to maximise the energy converted by a WEC, many WEC energy maximisation control strategies require current and/or future knowledge of the wave excitation force f_e, which is a non-measurable quantity. Moreover, many optimisation-based controllers, e.g. predictive controllers (see Section 3.4), require forecasting of wave excitation force, up to several wave periods. The specific prediction horizon required is a strong function of the system dynamic, specifically the radiation impulse response (Fusco & Ringwood, 2011b). This section reviews existing approaches for wave elevation/wave excitation force estimation and prediction.

Two methods may be identified to determine future values of f_e, as shown in Fig. 11:

(a) Determine a wave forecast (using one of the methods detailed in Section 4.2), from which f_e is calculated, using (4), or
(b) The current value of f_e is estimated (using one of the techniques mentioned in Section 4.1) which, in combination with past estimates, can be extrapolated into the future (using one of the methods detailed in Section 4.2).

4.1. Excitation force estimation

4.1.1. Using measurement or analytic calculation

It is possible to directly measure f_e or analytically calculate f_e, but only for some exceptional cases. For example, considering regular (monochromatic) wave conditions, by exploiting the Haskind relations, Newman (1962) derived an analytical expression of f_e for a fixed submerged body, which is the integral of the pressure over the wetted surface of the WEC hull measured by affixed pressure sensors. Later, in Newman (1965), the method is generalised to a moving body with a constant forward speed. Those analytic methods can provide a precise estimate of f_e, but cannot be directly used to estimate f_e for a WEC in operational mode, mainly because the submerged surface is also affected by other hydrodynamic and hydrostatic effects, e.g., the wave radiation and restoring forces (f_r, f_h), detailed in Section 2. Using the WEC model, Guo, Patton, Jin, and Lan (2018) calculated f_e by subtracting all other hydrodynamic effects, estimated from motion information, from the total wave force, calculated by integrating the pressure over the wetted surface, for benchmark purposes. However, this approach assumes accurate knowledge of the system model.

For WECs that can be described by a linear hydrodynamic model (9), a critical benchmark study comparing most of the state-of-the-art $f_e(t)$ estimation methods can be found in Peña-Sanchez, Windt, Davidson, and Ringwood (2019). The benchmark study is based on a WEC with essential linear dynamics, which discusses performance, required information, possible delays, and computation time, in detail, using a BEM-based numerical wave tank for benchmark calculations.

4.1.2. Via observer design

A more practical approach is to design an observer to estimate f_e, using a WEC dynamic model and measured motion information of the device. Recall the dynamic WEC equation

$$\begin{aligned} \dot{x} &= Ax + B(f_e + f_u + f_{nl}), \\ y_m &= C_v x, \end{aligned} \tag{26}$$

where y_m is the measured WEC motion information. In (26), both the state and excitation force f_e are to be estimated by the observer. The estimation problem of both x and f_e falls into the category of a *disturbance observer problem*; a comprehensive review on disturbance observer design can be found in Chen, Yang, Guo, and Li (2015). In the wave energy application, different disturbance observer techniques have been applied to WECs to estimate f_e, but most of the studies focus on WECs with linear dynamics, including those based on a Kalman filter (KF) with a random walk model (Liao et al., 2021; Nguyen & Tona, 2017), a KF with a harmonic oscillator model (Cavaglieri, Bewley, & Previsic, 2015; Garcia-Abril, Paparella, & Ringwood, 2017; Kracht, Perez-Becker, Richard, & Fischer, 2015), unknown input observer (Abdelrahman & Patton, 2019), receding horizon estimation (Nguyen & Tona, 2017), fast adaptive unknown input estimation (Abdelkhalik, Zou, et al., 2016), sliding model observer (Zhang, Zeng, & Li, 2019), unified linear input and state estimator (Coe & Bacelli, 2017) and moment based approach (Cunningham, Faedo, & Ringwood, 2019).

Disturbance observers can also use information from pressure sensors. For example, Abdelkhalik, Zou, et al. (2016) and Abdelkhalik, Zou, Robinett, Bacelli, and Wilson (2017) developed an extended-KF to estimate f_e using the recursively updated measurement of device position, as well as the pressure on the WEC hull.

4.1.3. Using the mapping $\eta \rightarrow f_e$

A third method to estimate f_e relies on available wave elevation information, exploiting the relationship between wave excitation force f_e and the measured free surface elevation η, established from linear potential theory as detailed in (4). This type of approach, developed in Yu and Falnes (1995), is termed the 'Wave-to-Excitation-Force (W2EF)' approach in the benchmark study (Guo et al., 2018).

Similar to the LTI approximation of the radiation force mapping (5), a control-oriented state-space model $\mathscr{E}$, which maps η to f_e, can be found as

$$\mathscr{E} : \begin{cases} \dot{\Theta}_e = A_e \Theta_e + B_e \eta, \\ f_e = k_e * \eta \approx C_e \Theta_e + D_e \eta, \end{cases} \tag{27}$$

with $\Theta(t) \in \mathbb{R}^{n_\mathscr{E}}$ and a set of matrices $\{A_e, B_e, C_e, D_e\}$ of appropriate dimension. The coefficients of model $\mathscr{E}$ can be found in a similar way to those for the radiation dynamics, detailed in Section 2.4. With (27), the relationship between the measurement/estimate/prediction of the free surface elevation η and the estimate/prediction of the wave excitation force f_e can be established.

4.2. Prediction of excitation force

With the convolution expression (4), or the W2EF model (27), we can obtain an f_e prediction from prediction of the free surface elevation, following the process illustrated in Fig. 12. Therefore, although this subsection focuses on excitation force prediction, results on wave-by-wave free surface elevation forecasting will be presented here. Since f_e is simply a low-pass filtered version of η, similar time series techniques (as described in Section 4.2.2) can be used to forecast both η and f_e. Consequently, in the following, we will use 'wave prediction' to alternatively describe the prediction of wave elevation or wave excitation force.

However, forecasting sea water level, significant wave height, or more generally the sea state, for longer time scales, e.g. ranging from hours to days, will not be discussed here. The readers may refer to Reikard, Pinson, and Bidlot (2011) for long time-scale forecasting of such quantities.

4.2.1. Deterministic sea wave prediction

Originally developed for quiescent period prediction (Belmont, Horwood, Thurley, & Baker, 2006), but subsequently applied to the wave energy application (Belmont, 2010) deterministic sea wave prediction (DSWP) uses sea elevation measurements taken at one or multiple locations at a certain distance from the WEC, as shown in Fig. 13(a). Using information on wave propagation and direction, wave prediction can be obtained at the WEC location several wavelengths into the future. The number of measurement points required will depend mainly on the directional propagation of the wave. This approach is often termed DSWP. For fast real-time calculation, Belmont et al. (2014) developed DSWP using a linear oceanographic wave model

$$\begin{aligned} \eta(p_x, p_y, t) = \sum_{n=1}^{N_p} \sum_{r=1}^{R_s} \mathcal{A}(\omega_n, \theta_r) \cos[k_n p_x \cos(\theta_r) \\ + k_n p_y \sin(\theta_r) - \omega_n t + \Theta(\omega_n, \theta_r)] \end{aligned} \tag{28}$$

where p_x, p_y are the spatial coordinates; N_f are R_s are the numbers of frequency used and significant storm directions, respectively; k_n is the wave number; $\mathcal{A}(\omega_n, \theta_r)$ and $\Theta(\omega_n, \theta_r)$ are the directional magnitude and phase-lift spectrum, respectively. The key concept of DSWP is to identify the coefficients in (28) using the measured data and then use (28) to predict future wave profiles. DSWP can typically achieve accurate prediction ($\geq 80\%$) for a prediction horizon of 30 s (Abusedra & Belmont, 2011; Connell et al., 2015).

4.2.2. Time series methods

The time series approach is solely based on extrapolating past measurements into the future, as illustrated in Fig. 13(b), based on the principle that ocean waves, within the operational sea state region of a WEC, can be modelled as a stationary linear Gaussian process, for short periods (up to 30 min). With sea state changes, the time series model parameters can be adapted to reflect the changing environment, as appropriate.

Fusco and Ringwood (2010) compared a range of time series models for wave forecasting, including harmonic models, an extended Kalman filter (EKF), and both linear and artificial neural network (ANN) based autoregressive (AR) models. The broad conclusion was that a simple AR wave predictor shows good accuracy in low-frequency swell waves, which are the sea states with the most energy generation potential, which led to a number of successful applications (Fusco & Ringwood, 2011a; Liao, Stansby, & Li, 2020). Pena-Sanchez and Ringwood (2017) compared AR with an autoregressive moving average (ARMA) model,

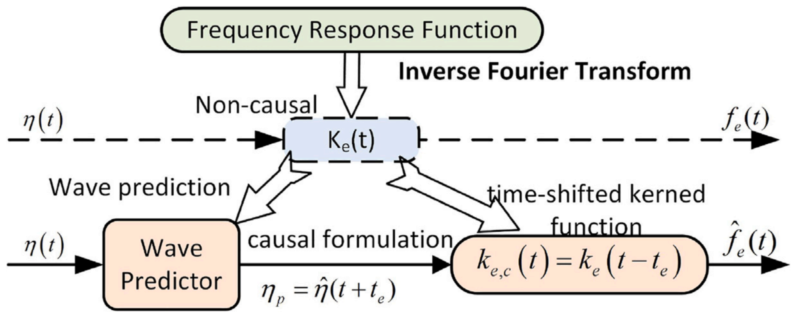

Fig. 12. Schematic diagram of the W2EF approach (Guo et al., 2018).

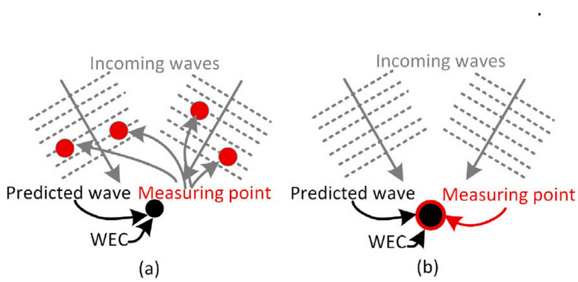

Fig. 13. Two main methods for wave forecasting: (a) Using distant measurements based on an oceanographic wave model. (b) Based on the measurements of a single point based on a time-series model (Peña-Sanchez, Mérigaud, & Ringwood, 2020).

and determined no significant performance difference between AR and ARMA models. Shi, Patton, and Liu (2018) predicted wave elevation based on a Gaussian process (GP) model, which can use wave spectral and past wave information to predict future waves. Mérigaud et al. (2018) show that the GP-based predictor is theoretically the best estimator, assuming the linear stationary Gaussian wave assumption holds. This GP-based predictor has the equivalent structure to the direct multistep (DMS) predictor. Compared with a standard AR model, which uses a unique set of parameters for one-step-ahead prediction, and carries out the multi-step-ahead prediction in a recursive manner, DMS uses different sets of parameters for each horizon and, therefore, can directly predict multiple steps ahead without the need for iteration. However, the optimisation problem associated with such prediction error models (PEMS) can be more challenging (Ljung, 1999).

Peña-Sanchez et al. (2020) compared a range of time-series-based methods using natural wave data. In real-sea conditions, the achievable prediction accuracy of a time-series-based predictor was shown to be modest, with all predictors' goodness-of-fit (GoF) dropping to 20% for one (pseudo) wave period ahead, with or without an online low-pass filter to filter out the high-frequency components of the wave data. This result contrasted with previous results (Fusco & Ringwood, 2010) using offline data (over 90% accuracy for more than wave period ahead), largely due to the negative phase effect of a realist online filter. However, in practice, the achievable accuracy depends on the nature of the sea spectrum, with high accuracy achievable in narrow-banded seas. Other time-series wave prediction methods are based on nonlinear AR models (Desouky & Abdelkhalik, 2019), including neural networks (Mahmoodi, Nepomuceno, & Razminia, 2022). However, it is unclear whether either free surface, or excitation force time series demand a nonlinear prediction model, especially when a WEC device is operating in the power production region (eliminating extreme sea states, which induce nonlinear waves).

4.3. Estimation and forecasting for arrays

As discussed in Section 3.5, WECs deployed in arrays can reduce CapEx and OpEx due to shared infrastructure. Compared with an isolated WEC, the f_e estimation and forecasting problem for an array of WECs becomes significantly more complex since, in addition to the incident waves, with devices both diffracting and radiating (due to their own movement) waves. Furthermore, the diffracted waves will depend on the wave orientation (variable) relative to the array orientation (fixed).

An initial approach considers the estimation and forecasting problem for f_e from an individual device perspective. This approach ignores the interactive diffraction and radiation effects when designing the f_e estimator and predictor, and the design process follows a similar route to the isolated WEC case, discussed in Sections 4.1–4.2.

The second approach is to view this problem from a global/array perspective, assuming information on the full array is available to each device. In this manner, Peña-Sanchez, Garcia-Abril, Paparella, and Ringwood (2018) established a state-space model for the array of WECs from a multi-DoF Cummin's equation, which has a similar expression to a model for a WEC with multiple DoFs, i.e., (9) or (13). The interactive effects are modelled in the off-diagonal of each relative coefficient matrix. Then, estimation and forecasting of f_e for the WEC array are similar to those for a single WEC, using the multi-DoF representation. Peña-Sanchez et al. (2018) developed a KF estimator and AR predictor, which shows a performance improvement of up to 45%, compared with the independent approach. Interestingly, the achievable performance of the array estimate/forecast is shown to be equivalent to that for an isolated device, with the extra information of the array state compensating for the more complex wave field.

Inspired by cooperative control in the multi-agent system theory, Zhang et al. (2023) formulated the centralised estimation problem into a multi-agent estimation problem, where local f_e estimators are designed for each device, using only part of the array motion information. Various communication graphs and array layouts are compared. Zhang et al. (2023) showed that, compared with the centralised estimator, local estimators can improve the computational efficiency, but shows a drop in estimation performance.

5. Paradoxes in wave energy control

'Control' is a broad church and encompasses many potential objectives, all with the intention of altering the behaviour of a system through external action, usually involving the implementation of a computer control algorithm. The main body of control science tends to deal with regulation and setpoint tracking problems, while renewable energy applications typically use energy maximisation as a control objective. While control systems for some renewable energy application areas are relatively mature (e.g. solar PV and wind), there are some additional important differences in the wave energy application, mainly due to the reciprocating energy flux in oscillating WEC systems, and the time-varying multi-harmonic nature of the wave excitation (for example, see Ringwood and Simani (2015). Here, we examine the contrasts between setpoint tracking/regulation problem and the wave energy maximising problem for oscillating bodies (note that some WEC systems perform rectification of the energy flux at the hydrodynamic stage e.g. overtopping devices), presenting the essential departures as a set of 'paradoxes'.

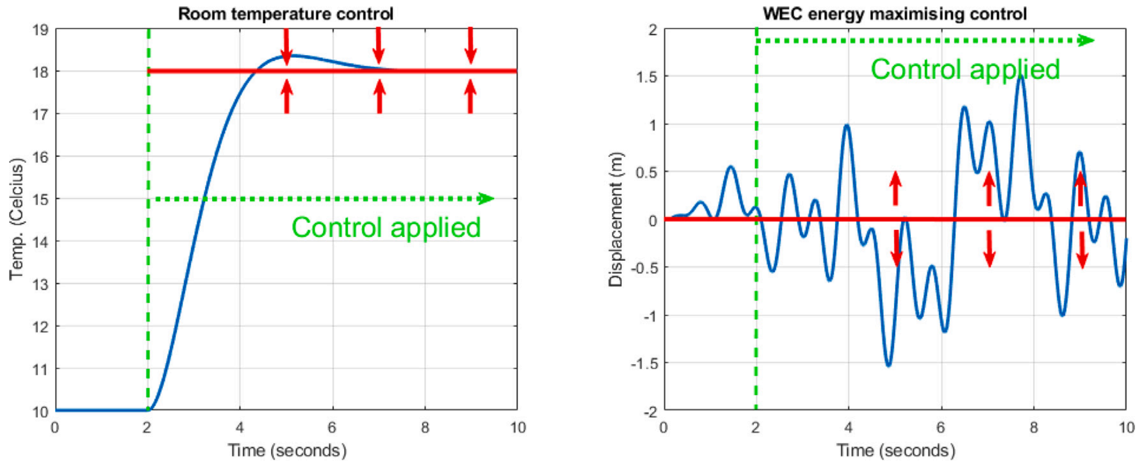

Fig. 14. Contrast in linearisability of traditional regulation and energy maximising control applications.

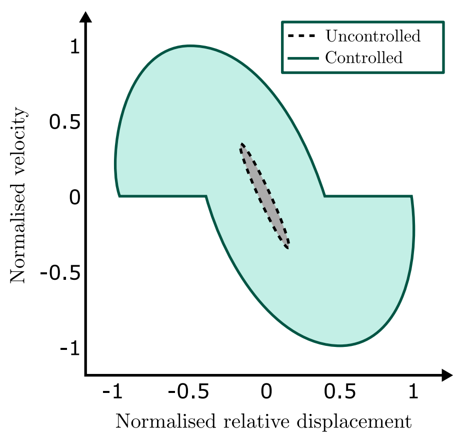

Fig. 15. Phase space occupied by an uncontrolled, and latching controlled, WEC for the same wave excitation.

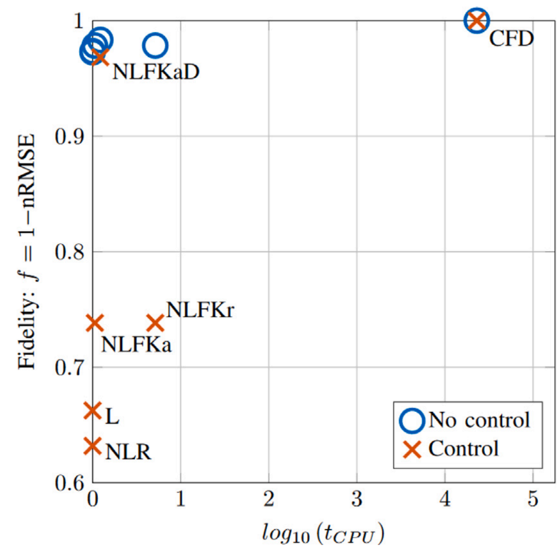

Fig. 16. Fidelity Vs computational complexity of various WEC models under uncontrolled, and controlled, conditions (Giorgi, Penalba, & Ringwood, 2016). Key: CFD = CFD model, adopted as the 'gold standard'; L = linear model; NLR = model with nonlinear static Froude–Krylov (buoyancy) force; NLFKa/NLFKr = model with static and dynamic nonlinear FK forces, with algebraic/remeshed calculation; NLFKaD = model with static and dynamic nonlinear FK forces + viscous drag.

Paradox 1: Violation of linearising assumptions

In traditional regulatory control, the objective is to minimise the variance of the tracking error, generally leading to tight operation around an equilibrium point, at which linearisation can be reasonably well justified. Even if the setpoint changes, a linearised model can be adapted, or a parameter scheduling approach adopted. In contrast, in the oscillatory WEC control case, the objective is to *exaggerate* the device motion to maximise the objective in (16). This contrast in 'linearisability' is illustrated in Fig. 14.

To illustrate the difference in operational space, Fig. 15 shows the phase space occupied by an uncontrolled (heaving buoy) WEC, and one controlled using latching, clearly illustrating the exaggerated motion of the controlled device. It should also be noted that latching is a relatively passive form of control i.e. no power is injected into the system, as is the case for reactive control. In terms of how this motion exaggeration affects the validity of various linear/nonlinear modelling approaches, Fig. 16 shows how the validity of the linear, and weakly nonlinear, models falls away as control is applied, with additional illustration of the relative computational complexity of each model.

However, the degree to which a linear model can provide an adequate (e.g. for model-based control design) WEC representation will ultimately depend on the device itself, the range of sea conditions encountered (over which the controller is supposed to operate), and the aggressiveness of the controller. For example, resistive control (see Section 3.2 is significantly less aggressive than reactive (complex conjugate, MPC, etc.), and constrained control may also serve to limit device excursions, and velocity. However, it is probably true to say that many WEC control studies adopt a linear model (which may validate well under uncontrolled tank tests) which is subsequently used for model-based control design, without assessing the impact of the control action on the model validity.

Paradox 2: Non-causal control solution

Since the fundamental optimal WEC controller involves the complex conjugate of the system intrinsic impedance (see Section 3.2), and the intrinsic impedance is stable and proper, the optimal panchromatic (complex conjugate) controller is alternatively either non-causal or unstable (Scruggs, 2010). While the use of a single-frequency controller circumvents this problem, in the ACC structure of Section 3.2 such a controller has significant limitations in a broadband sea. As a result, truly optimal WEC controllers always require advance knowledge of the wave excitation force, f_e.

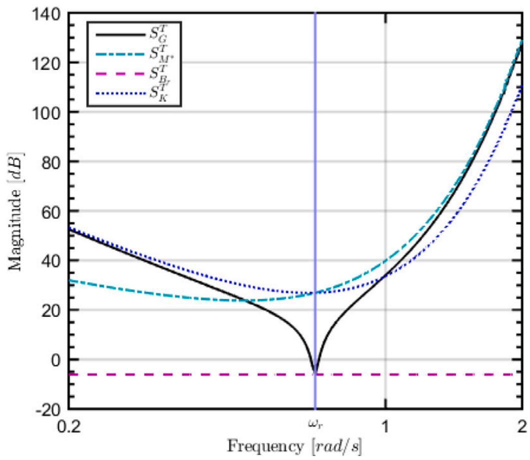

Fig. 17. Closed-loop sensitivity to variation in the overall WEC model (S_G^T), variation in inertia (S_M^T), variation in damping (S_B^T), and variations in restoring constant (S_K^T). Note the sensitivity values considerably in excess of 0 dB away from the frequency where the controller is inactive i.e. $\omega_w = \omega_r$.

Paradox 3: Sensitivity degeneration

It is well established that, for traditional regulation/servo control problems, the action of a controller serves to reduce the sensitivity of the closed-loop system to errors/uncertainty in the system model (Kreindler, 1968). Indeed, some control synthesis procedures specifically exploit this property, in a frequency selective way (Zames & Francis, 1983). However, due to the fundamental nature of the relationship between the WEC and controller, and not unrelated to the nature of the control objective, the closed-loop sensitivity properties of WEC control systems can be rather surprising. For example, Fig. 17 shows a set of closed-loop sensitivity functions for the ACC controlled system in Fig. 6.

It can be noted from Fig. 17 that, when the controller is active (i.e. the wave frequency is distant from the device resonant frequency, ω_r), all sensitivity functions (overall and with respect to individual model parameters) approach alarmingly high values, well beyond open-loop sensitivity values ($S_G^G = 1$). Unfortunately, it is not possible to manipulate the controller gain at selective frequencies, as can be carried out for traditional regulator/servo systems, due to the fixed relationship between G and I_u implied by (22).

The closed-loop sensitivity function S_G^T plays a very important role in traditional regulatory control loops, and has a degree of importance in WEC control, given that the closed-loop transfer function T specifies the relationship between the excitation force F_e and the (optimal) device velocity. However, perhaps more important is the effect of system/controller mismatch errors on the energy conversion performance of the system. To this end, the reader is referred to Ringwood et al. (2020), which documents the effect of real (damping) and imaginary (inertia, restoring) modelling errors on energy capture, for both ACC and AVT control configurations. In summary, the ACC controller is considerably more sensitive to inertial and restoring force errors than AVT, with the AVT more sensitive to damping errors. It is also somewhat counter-intuitive that the feedforward structure of the AVT controller has better sensitivity properties that the (feedback) ACC controller, in some respects at least. Additionally, however, the AVT has sensitivity to excitation force estimation/prediction errors, which are not an issue for the ACC controller, since direct output feedback of velocity is used.

To some extent, the sensitivity issues articulated in this subsection can be mitigated by appropriate robust controller synthesis procedures, such as those proposed in Faedo, García-Violini, et al. (2019), Faedo, Mattiazzo, and Ringwood (2022) and Garcia-Violini and Ringwood (2021).

Paradox 4: Persistent excitation

While many of the 'paradoxes' associated with WEC control are unfavourable in comparison with traditional regulatory control, one feature that is potentially advantageous is the characteristic that wave energy devices are normally subject to persistent wave excitation. The exception to this is during calm sea conditions but, in such a circumstance, WECs would not be operational, including their energy maximising controllers.

Persistence of excitation is a requirement for various data-based and data-driven control strategies and a requirement for the stability of, for example, adaptive control schemes (Narendra & Annaswamy, 2012) and extremum-seeking controllers (Krstić & Wang, 2000). In general, such schemes normally require the addition of a 'dither' signal on the manipulated variable to provide sufficient variance on the system output to satisfy convergence requirements, which has the undesirable effect of increasing the output variance. In the wave energy application, a persistent external excitation is present as a disturbance which, in the wave energy case, is not rejected by the controller, but rather enhanced (Zhan et al., 2022). However, care must be taken that the nature of a particular wave disturbance satisfies the criteria for stability and convergence of any data-based or data-driven control scheme.

6. Control co-design

Control co-design (Deshmukh, Herber, & Allison, 2015; Garcia-Sanz, 2019) refers to the simultaneous design of a controller for a system and optimisation of some, or all, of the physical system parameters. For a wave energy system, such system parameters may relate to the physical WEC geometry (including inertia), WEC array layout, or PTO parameters and/or constraints. Traditionally, in the vast majority of application areas, application specialists are tasked with the primary system design, after which a controller is designed for the system. However, since (particularly) model-based control design involves an assessment of the total system dynamics, including both those arising from the physical system, and those imposed by the control system, informed decisions can be made as to whether to alter the physical system dynamics, or use parameterised control force, to (economically/efficiently) achieve particular overall system behaviour. The principle of control co-design can be easily demonstrated with a classical mass–spring–damper system (not totally unrepresentative of a wave energy converter!):

$$M\ddot{x}(t) + B\dot{x}(t) + kx(t) = F_u(t), \tag{29}$$

and let:

$$F_u(t) = M_c\ddot{x}(t) + B_c\dot{x}(t) + K_c x(t), \tag{30}$$

so that:

$$(M - M_c)\ddot{x}(t) + (B - B_c)\dot{x}(t) + (K - K_c)x(t) = 0, \tag{31}$$

with characteristic equation:

$$p^2 + 2\zeta\omega_n p + \omega_n^2 = 0. \tag{32}$$

Alteration in the system characteristic parameters, ζ and ω_n, can be equivalently achieved by manipulation of either the system (M, B, k) or the controller (M_c, B_c, k_c), but with the following differences:

(a) Sufficient control force must be available in F_u to implement the desired (M_c, B_c, k_c) values,
(b) The cost of changing (M, B, k) Vs (M_c, B_c, k_c) may be different (bearing in mind (a) above), and
(c) Adjusting the system characteristics via (M_c, B_c, k_c) gives greater flexibility, since these parameters can be adjusted in real-time, in response to variations in the system parameters, or the ambient conditions e.g. the wave spectrum, in the WEC case (to keep the device in resonance with the predominant wave frequency).

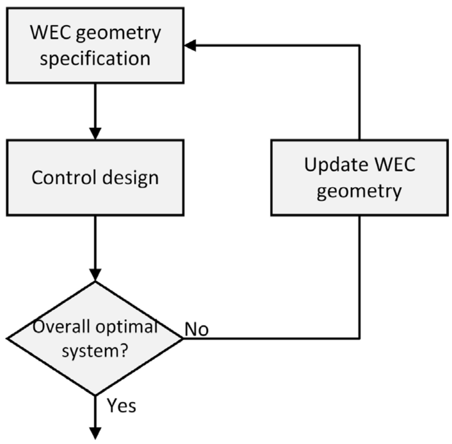

Fig. 18. Co-design philosophy which includes WEC geometry and controller.

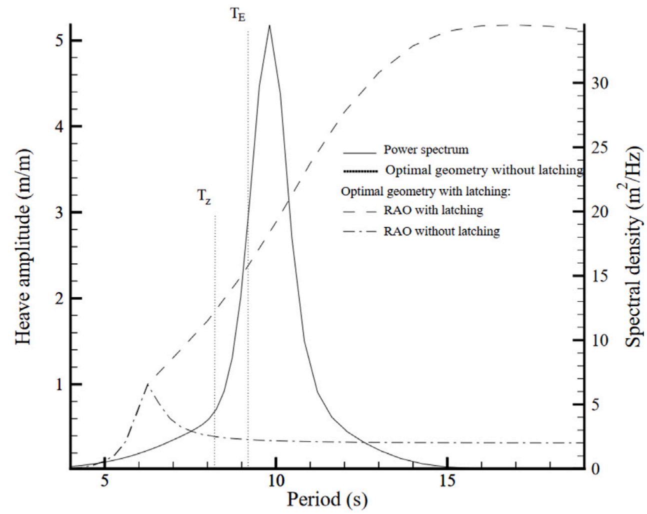

Fig. 19. Optimisation of height and radius of a cylindrical WEC. Note the position of the uncontrolled WEC resonance period (6.2 s) far away from the energy period T_E (9.2 s) of the wave spectrum.

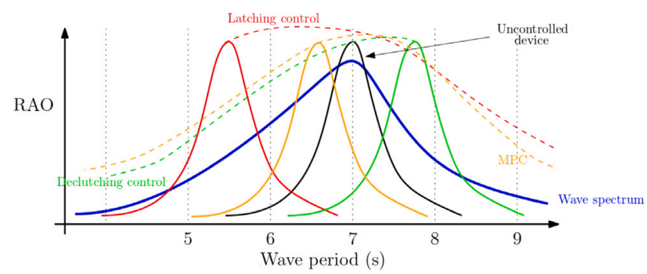

Fig. 20. Indicative plot of total RAO (system + control) coverage, for different controller types. In general, if no control applied (solid lines), the device resonance is aligned with the peak wave frequency; latching slows down the device, so WEC resonance is placed at $T_r < T_w$; declutching speeds up the WEC response, so $T_r > T_w$; MPC has the ability to both speed up, and slow down the response, so $T_r \sim T_w$. The dashed lines indicate how the RAO can be extended by control action.

Relatively recently, the 'co-design' moniker has been applied to WEC systems (e.g. Coe, Bacelli, Olson, Neary, & Topper, 2020), though the principle has been employed for the past decade or so, including examination of the interaction between optimal WEC control and (a) WEC geometry optimisation (Section 6.1 and (b) WEC array layout optimisation (Section 6.2). One of the challenges in WEC control co-design is evaluation of the comparative merits of controller adjustment Vs physical system adaptation. This is approached, to some extent, in Section 6.3, where a version of LCoE is used as a common figure of merit.

In an ideal world, all of WEC array layout, individual geometry optimisation, and PTO constraints would be optimised together with the control system. However, this is a challenging computational problem, with appropriate (fidelity/complexity) representation of the hydrodynamic model key to the achievability, and utility, of the results. Such are the aspirations of the HetWEC project (HetWEC, 2022). One study, which includes both array layout and device geometry optimisation, is that by Penalba, Touzón, Lopez-Mendia, and Nava (2017), where the controller (consisting of a simple damper) is adapted for each geometry/array configuration.

6.1. WEC geometry co-design

WEC geometry optimisation has been examined in relative detail over the past decade (Garcia-Teruel & Forehand, 2021; Guo & Ringwood, 2021a). However, the interaction between the geometry optimisation problem and the optimal control solution, as illustrated in the geometry/controller co-design procedure of Fig. 18, has been relatively less well examined. One of the earliest such studies is that of Gilloteaux and Ringwood (2010), where a simple cylindrical WEC, operating in heave only, was optimised in the presence of a controller. One of the key results was that the device, in the knowledge of latching control, is made smaller, knowing that latching can cover slower wave periods, as shown in Fig. 19. Fig. 20 gives an indication of how different controller types might optimally place the device (uncontrolled) RAO, in a co-design process. The clear indication is that the optimal device geometry is quite sensitive to the nature of the control philosophy adopted.

The idea of geometry co-design is extended in Wang and Ringwood (2021), where ballast distribution is also concurrently optimised. Garcia-Rosa and Ringwood (2016) examined the optimal (cylindrical) WEC geometries which result from different control systems (uncontrolled, latching, declutching, pseudospectral), demonstrating up to a 500% variation in optimal WEC radius, and 100% variation in optimal WEC draft, purely as a result of particular control algorithm selection. The same authors, in Garcia-Rosa et al. (2015), also show that the optimal device geometry is sensitive to the PTO constraints, implemented by MPC and MPC-like (e.g. pseudospectral) controllers. Similar conclusions, regarding the influence of displacement constraints on optimal geometry, were also reached by McCabe (2013).

However, typically, many geometry co-design studies employ a simple control representation (e.g. a frequency-independent damper, as in Shadman, Estefen, Rodriguez, & Nogueira, 2018), which probably reflect the computational challenges of WEC control co-design.

6.2. Array layout co-design

Again, WEC array layout optimisation has been studied to an extent, though usually agnostic to the form of control, if any, employed. Some array optimisation studies employ a classical spring–damper PTO model (Child & Venugopal, 2010; Mercadé Ruiz et al., 2017), but many array optimisation studies focus mainly on the optimisation method(s) employed (Neshat, Alexander, Sergiienko, & Wagner, 2020), rather than on interactions between key system variables. Interestingly, a study by de Andrés, Guanche, Meneses, Vidal, and Losada (2014), that purports to examine the factors influencing optimal array layout, omits the control aspect entirely. One study, which specifically examines the interplay between the controller and the optimal array layout, from a power production perspective, is that of Garcia-Rosa et al. (2015), where significant sensitivity of the optimal array to the controller type is demonstrated, as shown in Fig. 21. A very recent study (Abdulkadir & Abdelkhalik, 2023) optimises both array layout and individual

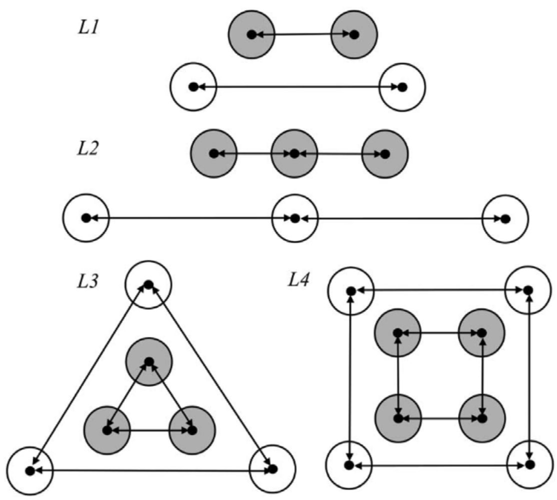

Fig. 21. Optimisation of a number of different array configurations (scale 1:100), with passive control (shaded) and global (pseudospectral) array control (unshaded). The globally controlled arrays are consistently more widely spaced, optimising the benefit of inter-device radiated waves.

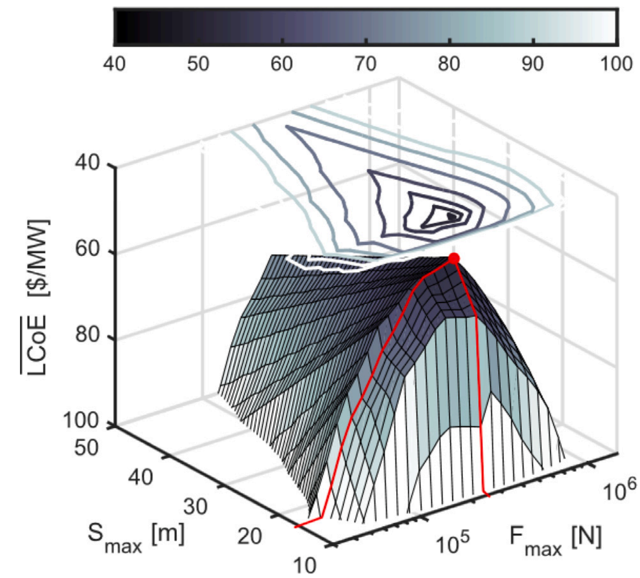

Fig. 22. Optimisation of force (F_{max}) and stroke (S_{max}) constraints with a version of LCoE as a performance objective.

device geometry, with the nested optimisation loops requiring a very computationally efficient model and control solution. Finally, we note, within WEC array layout co-design, the use of centralised/decentralised energy-maximising control solutions (as described within Section 3.5) effectively has an impact in the final outcome of the co-design process, since the control solution itself is different for each of these design approaches, having distinctive performance capabilities (see also Bacelli et al., 2013).

6.3. PTO co-design

Various parameters of the PTO system should/could be considered as a control co-design problem. These include the generator power capacity, torque capacity and, where a linear generator is employed, stroke limits. In relation to the latter, Peña-Sanchez, García-Violini, and Ringwood (2022) optimise both stroke and force limits for a linear generator, where a direct relationship between these limits and capital cost (CapEx) is established, allowing a trade-off between power production receipts and cost of constraints, as shown in Fig. 22. A further PTO control co-design is presented by O'Sullivan and Lightbody (2017), who introduce field-weakening into the PTO generator, considering the desired action of the MPC controller and the losses incurred in the PTO.

7. Conclusions and perspectives

Wave energy control, with the associated activities/ technologies of modelling, estimation and forecasting, has a significant role to play in improving the commercial viability of wave energy systems. Significant progress has been made over the past five decades since the pioneering work of Budal and Falnes (1975), Evans (1976) and Salter (1974). Ultimately, the objective of reducing LCoE, while achieving smooth and dispatchable power output, is the prime factor in the commercialisability and acceptability of wave energy. To that end, wave energy has been shown to have desirable persistence (Fusco et al., 2010) and complementary (Bhattacharya et al., 2021) characteristics compared to wind, while combined wind/wave platforms (Wang et al., 2022) present an opportunity to share capital costs, with wind/wave platforms also presenting interesting combined control problems and opportunities (Fenu et al., 2020).

Regarding LCoE, there is an inherent difficulty in using this as a direct control objective, given the disparity between the relatively long time interval upon which OpEx can be accurately assessed, and the immediacy of required control actions. However, some studies (Arredondo-Galena et al., 2023; Nielsen et al., 2017) are emerging which attempt to quantify the potentially deleterious effects of WEC control actions which could contribute to OpEx. Continued efforts are required to move towards LCoE as the ultimate control objective. One further aspect, deserving of increased attention and directly affecting LCoE, is the potential for fault-tolerant control in wave energy systems, given the relative remote deployment locations and the difficulty of maintenance, and the relatively harsh environmental conditions at favourable wave energy sites. Some studies are now beginning to appear in this area (Benbouzid, Amirat, & Elbouchikhi, 2020; González-Esculpi, Verde, & Maya-Ortiz, 2021; Xu, Chen, Yang, & Zhu, 2022; Zhang, Zeng, & Gao, 2022).

Control co-design has become somewhat formalised over the past decade and, given the relative immaturity and lack of convergence of wave energy technology, presents an excellent opportunity to advance the performance of wave energy systems, including arrays, considering the central role played by control in enhancing the performance of such systems.

Finally, considering the difficulty of accurate hydrodynamic modelling for WECs and WEC arrays, and the sensitivity of controllers to modelling errors, there is increasing interest in data-driven WEC/array control. However, the difficulty of prompt evaluation of the control objective in stochastic seas, combined with the need to take immediate control action and observe strict system physical constraints, conspire to keep this a challenging prospect. Nevertheless, well-tuned (by data) simpler controllers may see greater industrial acceptability, and achieve comparable (if not better) performance, than (potentially) high-performance model-based designs based on hydrodynamic models of dubious fidelity.

In summary, the control of wave energy systems is an important, multi-faceted, and challenging area, with considerable opportunity for further development.

CRediT authorship contribution statement

John V. Ringwood: Conceptualization, Methodology, Investigation, Formal analysis, Software, Writing – original draft, Writing – review & editing, Visualization. **Siyuan Zhan:** Conceptualization, Methodology, Investigation, Formal analysis, Software, Writing – original draft, Writing – review & editing, Visualization. **Nicolás Faedo:** Conceptualization, Methodology, Investigation, Formal analysis, Software, Writing – original draft, Writing – review & editing, Visualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article

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