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Frequency-Domain Identification of Radiation Forces for Floating Wind Turbines by Moment-Matching

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Abstract. The dynamics of a floating structure can be expressed in terms of Cummins' equation, which is an integro-differential equation of the convolution class. In particular, this convolution operator accounts for radiation forces acting on the structure. Considering that the mere existence of this operator is highly inconvenient due to its excessive computational cost, it is commonly replaced by an approximating parametric model. Recently, the Finite Order Approximation by Moment-Matching (FOAMM) toolbox has been developed within the wave energy literature, allowing for an efficient parameterisation of this radiation force convolution term, in terms of a state-space representation. Unlike other parameterisation strategies, FOAMM is based on an interpolation approach, where the user can select a set of interpolation frequencies where the steady-state response of the obtained parametric representation exactly matches the behaviour of the target system. This paper illustrates the application of FOAMM to a UMaine semi-submersible-like floating structure.

Introduction

With the rapid decrease of the easily accessible fossil fuels, the immediate shift to renewable energy systems is one of the most important challenges of the 21st century. For this reason, the installed power capacity of renewable energy plants has significantly increased in this century, more than doubling it between 2004 and 2016 [1]. Among the renewable energy sources, wind energy has one of the highest growth, producing 4.4% of the worldwide electric power usage in 2017, and 11.6% electricity in the European Union [2]. In fact, onshore wind (and solar PV) will offer, in many places, a less expensive source of new electricity than the fossil-fuel alternative without financial assistance [3].

However, onshore wind has some disadvantages with respect to offshore wind farms. On the one hand, onshore wind farms have an impact on the landscape, since they usually require to be spread over more land than other conventional power stations, and need to be built in wild and rural areas, which can lead to habitat loss. On the contrary, offshore wind is steadier and stronger than onshore, with less visual impact. However, construction and maintenance costs are higher offshore.

At the moment, most offshore wind turbines are installed in shallow water (about 30m deep), using bot tom-mounted substructures. Nevertheless, to harness the available offshore wind potential, wind farms have to be located in deeper water. To this end, and to reduce the costs related with the structure, floating support-platforms will have to be deployed to hold the turbines, for which several platform configurations can be found in the literature [4, 5].

At this stage of development, simulations that combine aerodynamic, hydrodynamic and mooring-system dynamic effect on floating wind turbines are crucial to address the possible failure conditions of such structures and, therefore, accurately modelling each of those dynamical effects is of paramount importance. In particular, the equation to describe the motion of a floating body, i.e. the equation describing the hydrodynamic interactions between the floating structure and the waves (the so-called Cummins' equation [6]), is an integro-differential equation, more precisely of the convolution class. The presence of this convolution operator represents a drawback for a number of reasons, including the fact that its numerical computation is highly inefficient, requiring considerable computational effort. To avoid such drawbacks, such a convolution operator can be approximated using a suitable parametric model (often given in terms of a state-space representation), for which several applications can be found, particularly within the wave energy literature.

In 2018, the Centre for Ocean Energy Research (COER) presented an identification strategy to compute a parametric model of the radiation convolution term of Cummins' equation [7], or the complete force-to-motion dynamics of a floating body. Such parameterisation strategy is based on recent advances in model order reduction by moment-matching, developed over several studies as, for example, [8]. The approach presented in [7] identifies a state-space model, whose frequency response exactly matches the frequency response of the target system at a set of user-selected frequencies. In fact, as a consequence of this interpolation feature, this moment-based strategy inherently preserves some of the relevant physical properties of the target floating body, such as internal stability and passivity. Motivated by the advantages behind moment-matching theory, reported in, for example, [7,9,10], a Matlab toolbox has been developed, to disseminate this moment-based identification strategy for wave energy applications [11].

The aim of the present paper is to introduce how FOAMM (Finite-Order Approximation by Moment-Matching) can be applied to compute parametric models of support platforms for offshore wind turbines, considerably reducing the computational effort related with time-domain simulation of floating structures. To illustrate the capabilities of such a toolbox, the *UMaine* semi-submersible-like floating structure [4,12] has been chosen as an application study, since its rigidly-connected multibodies represent a geometrically complex device, with frequency-response as shown in Section.

The remainder of this paper is organised as follows. Section 2 briefly introduces the equation of motion of a floating body, while the theory behind FOAMM is recalled in Section 3. Finally, an application case involving the *UMaine* structure is addressed in Section 3, whilst conclusions are encompassed in Section 4.

Equation of motion

Without any loss of generality, a single Degree of Freedom (DoF) support-platform is considered in this study. Recall that the motion of a 1-DoF support-platform can be expressed, in the time-domain, according to Newton's second law, obtaining the following linear hydrodynamic formulation [5]:

$$m\ddot{x}(t) = \mathcal{F}_r(t) + \mathcal{F}_{\hbar}(t) + \mathcal{F}_e(t) + \mathcal{F}_m(t), \tag{1}$$

where m is the mass of the structure (platform and turbine), $\ddot{x}(t)$ the acceleration of the body, $\mathcal{F}_e(t)$ the wave excitation force, $\mathcal{F}_r(t)$ the radiation force, $\mathcal{F}_{\hbar}(t)$ the hydrostatic restoring force, and $\mathcal{F}_m(t)$ the force exerted by the mooring system. The linearised hydrostatic force is given by $\mathcal{F}_{\hbar}(t) = -s_h x(t)$, where s_h denotes the hydrostatic stiffness. The mooring force is defined as $\mathcal{F}_m(t) = -b_m\dot{x}(t) - s_m x(t)$ [5], where b_m and s_m denote the damping and stiffness of the mooring system, respectively. From linear potential theory, $\mathcal{F}_r(t)$ can be modelled using Cummins' equation [6] as,

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$$\mathcal{F}_r(t) = -\mu_\infty \ddot{x}(t) - \int_{\mathbb{R}^+} k(\tau)\dot{x}(t-\tau)d\tau, \tag{2}$$

where $\mu_\infty = \lim_{\omega\to+\infty} A(\omega) > 0$ denotes the radiation added-mass at infinite frequency, and $k(t) \in L^2(\mathbb{R})$ is the radiation impulse response function. Eq. (1) can be rewritten as:

$$(m+\mu_\infty)\ddot{x}(t) + k(t) * \dot{x}(t) + s_h x(t) = \mathcal{F}_e(t) + \mathcal{F}_m(t), \tag{3}$$

where the symbol $*$ represents the convolution operator.

Since this paper is focused on the approximation of the radiation convolution term, and FOAMM identifies a parametric form using raw frequency-domain data, it is convenient to define the frequency-domain equivalent of the radiation convolution term, which can be obtained through Ogilvie's relations [15] as:

$$B(\omega) = \int_{\mathbb{R}^+} k(t)\cos(\omega t)\,dt, \quad A(\omega) = \mu_\infty - \frac{1}{\omega}\int_{\mathbb{R}^+} k(t)\sin(\omega t)\,dt \tag{4}$$

where the coefficients $B(\omega)$ and $A(\omega)$ represent the radiation damping and added-mass of the device, respectively. This set of hydrodynamic coefficients and can be efficiently obtained using any of the state-of-the-art Boundary Element Method (BEM) solvers (see [16]). The impulse response function
$k : \mathbb{R}^+ \to \mathbb{R}$ can be written as a mapping involving the radiation damping coefficient as:

$$k(t) = \frac{2}{\pi}\int_{\mathbb{R}^+} B(\omega)\cos(\omega t)\,d\omega. \tag{5}$$

with frequency-domain equivalent given by

$$K(\omega) = B(\omega) + j\omega\,[A(\omega) - \mu_\infty]. \tag{6}$$

Moment-matching-based parameterization

To keep this paper reasonably self-contained, this section provides a brief summary of the theory behind FOAMM. The interested reader is referred to [7] for an extensive discussion on the specific underlying mathematical principles.

The radiation impulse response mapping defines a linear-time invariant system completely characterised by $k(t)$, where its input is the body velocity, i.e. $\dot{x}(t)$. To be precise, the radiation subsystem Σ^k is given

$$\Sigma_k : \theta_k(t) = k(t) * \dot{x}(t), \tag{7}$$

where $\theta_K(t) \in \mathbb{R}$ is the output (radiation force) of system Σ_k.

To obtain a parametric description of (7), the velocity of the floating structure $\dot{x}(t)$ is expressed as an autonomous signal generator as,

$$\mathcal{G}_{\dot{x}} : \{\dot{\xi}_{\dot{x}}(t) = S\xi_{\dot{x}}(t), \quad \dot{x}(t) = L_{\dot{x}}\xi_{\dot{x}}(t) \tag{8}$$

where the matrix S is defined [7] as

$$S = \oplus_{p=1}^{\beta} \begin{bmatrix} 0 & \omega_{\mathrm{p}} \\ -\omega_{\mathrm{p}} & 0 \end{bmatrix} \tag{9}$$

where the symbol $\oplus$ denotes the direct sum of matrices of β matrices, and $\nu = 2\beta$, with β the number of interpolation frequencies. Note that each $\omega_p \in \mathcal{F}$, with $\mathcal{F} = \{\omega_i\}_{i=1}^{\beta} \subset \mathbb{R}^+$ represents a desired interpolation point for the moment-matching-based parameterisation process, i.e. a frequency where the transfer function of the parametric model matches the transfer function of the target system.

Following [7], the so-called moment-domain equivalent of the output of system Σ_k in (7) can be straightforwardly computed as

$$\underline{\mathcal{Y}_k} = L_{\dot{x}} \mathcal{R}^{k}, \tag{10}$$

where the matrix $\mathcal{R}^{k}$ is defined by

$$\mathcal{R}^{k} = \oplus_{p=1}^{f} \begin{bmatrix} \Re\{K(j\omega_p)\} & \Im\{K(j\omega_p)\} \\ -\Im\{K(j\omega_p)\} & \Re\{K(j\omega_p)\} \end{bmatrix} \tag{11}$$

Finally, the parametric (state-space) description

$$\widetilde{\Sigma_{k\mathcal{F}}}: \{\dot{\Theta}_k(t) = F_k \Theta_k(t) + G_k \dot{x}(t), \quad \widetilde{\theta_k}(t) = Q_k \Theta_k(t) \tag{12}$$

is a system that interpolates the target frequency response $K(j\omega)$ at the set $\mathcal{F}$, i.e. it has the *exact* same frequency response of the radiation subsystem Σ_k at the frequencies defined in the set $\mathcal{F}$, if $Q_k P_k = \underline{\mathcal{Y}_k}$, where P_k is the unique solution of the Sylvester equation

$$F_k \mathrm{P}_k + G_k L_{\dot{x}} = \mathrm{P}_k S, \tag{13}$$

and $\underline{\mathcal{Y}_k}$ is computed from equation (10). The reader is referred to [7] for the theory behind the explicit computation of the matrices F_k, G_k, Q_k in (12) fulfilling condition (13).

Application example

The selected support-platform is a *UMaine* semi-submersible-like floating structure, constrained to move in pitch. This structure is designed to support the multi-megawatt turbine *NREL offshore 5-MW baseline wind turbine* [17]. In this study, this structure is selected due to its complex geometry (illustrated in Figure 1). In other words, the radiation convolution frequency-response for this device is more geometrically complex than for other floating bodies analysed before, such as, for example, in [7,11,9]. More information about the specifications of the *Umaine* structure is provided in 1.

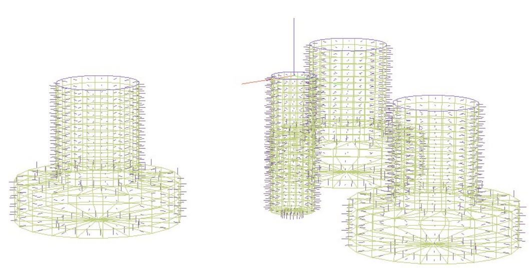

Figure 1: Low-order mesh of the UMaine semi submersible-like structure analysed in this study.

Table 1: Specifications of the structure.

Column spacing	50m
Main column diameter	6.5m
Side columns diameter	12 and 24m
Draft	20m
Mass (with ballast)	$13.5{\cdot}10^6$
Center of mass (*z*)	-13.74 from SWL

Figure 2 shows the radiation damping and added-mass in the top and bottom left-hand side figures, respectively, along with the frequency-response of the convolution operator, $K(j\omega)$, in the right-hand-side of the figure (both magnitude and phase) computed as shown in 6.

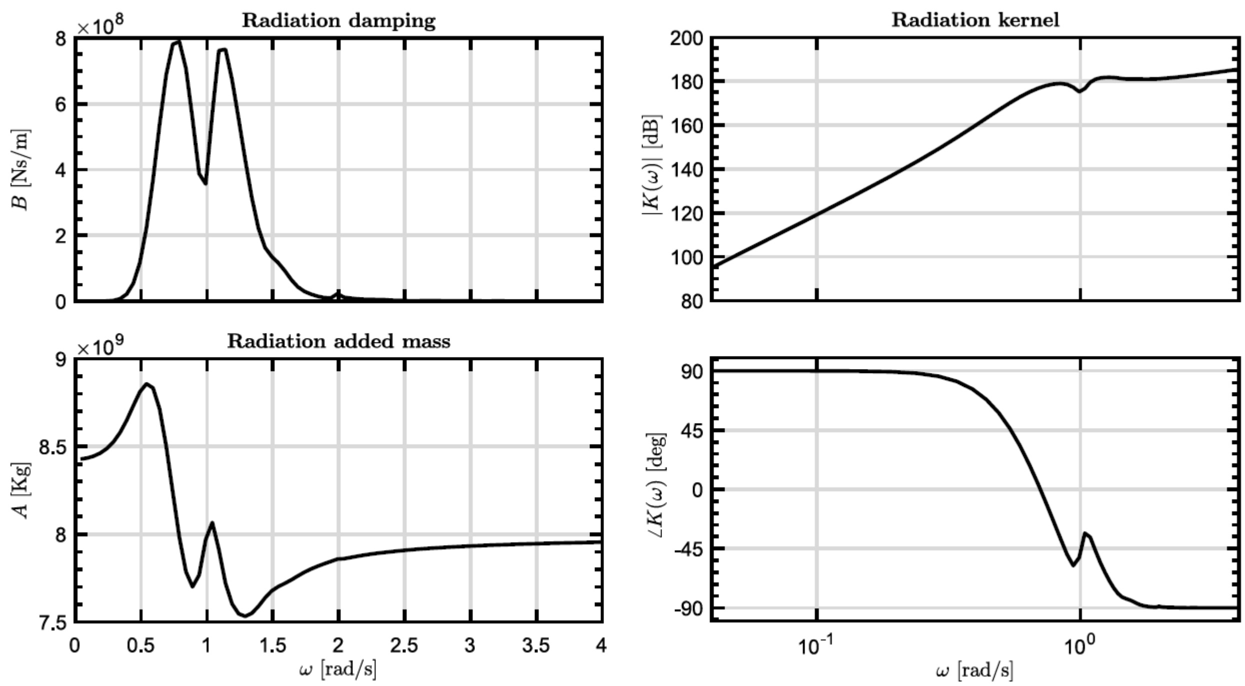

Figure 2: Hydrodynamic coefficients for the pitch motion of the UMaine semi-submersible like device analysed in this Study. The radiation damping and added-mass are represented in the left-hand-side of the figure, while the radiation force frequency-response is illustrated on the right-hand-side.

To compute the parametric model of the radiation convolution term shown in Figure 2, the software FOAMM needs to be downloaded first, which can be done for free from http://www.eeng.nuim.ie/coer/downloads/. Finally, as reported in [11], it is necessary to install the correct Matlab runtime version, which is readily provided with FOAMM. The interested reader is referred to [11] for more information on the different options and modes available on this toolbox.

The first choice for the user is the frequency range over which the parameterisation is carried out. As explained in [7], such a frequency range highly depends on the application, and it is usually conditioned by the typical sea-state characterising the location of the structure. Since this is not relevant for the current study, the frequency range is selected as $\omega_l = 0.3$ [rad/s] and $\omega_u = 3$ [rad/s].

For the selection of the interpolation frequencies, three different methods are available in FOAMM [11]. This study utilises the so-called manual identification method, where the user selects the set of interpolation frequencies. This presents several advantages since, by selecting the interpolation frequencies in a sensible manner (dynamically speaking), the accuracy of the parametric model can be considerably improved. Due to the complexity of the *UMaine* frequency-response, 7 frequencies (parametric model with order 14) are required to obtain an accept approximation, with a Mean Absolute Percentage Error (MAPE) of ≈ 0.08 %.

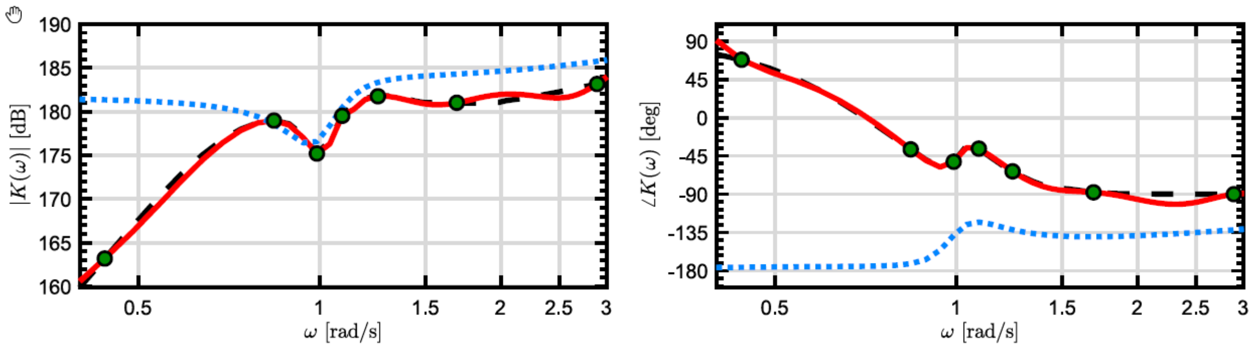

Figure 3: Frequency-response of the parametric model obtained with FOAMM for the radiation force subsystem (solid-red), along with the (target) frequency-response of the Umaine structure (dashed-black), the set of interpolation frequencies considered (green dots), and the best approximated model using subspace-methods (dotted-blue).

To provide a comparison with well-established identification strategies, a parameterisation using subspace-methods (implemented in the Matlab function *n4sid*) is considered. The most accurate model obtained with this method has a MAPE of 0.45 % of error. The frequency-response of the obtained parametric model obtained using FOAMM, along with the target frequency-domain data of the *Umaine* structure, the set of interpolation frequencies considered, and the best approximated model using subspace-methods, are shown in Figure 3.

Conclusions

This paper illustrates how to use the FOAMM toolbox to obtain a parametric model of the convolution term associated to radiation forces, for a complex-shape support- structure of a floating wind turbine. The chosen floating structure is the *UMaine* semi- submersible-like device which, due to its geometrical complexity, represents a challenge from a frequency-domain identification perspective. In fact, it is shown that, while no accurate approximation model could be obtained using well-established subspace-methods, FOAMM provides an accurate parametric description of the radiation force subsystem as a consequence of its interpolation features, which allows the user for the selection of the frequencies characterising the dynamics of the structure as interpolation points in the parameterisation process.

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