Supersymmetric AdS Solitons, Ground States, and Phase Transitions in Maximal Gauged Supergravity

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Supersymmetric AdS Solitons, Ground States, and Phase Transitions in Maximal Gauged Supergravity †

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Abstract: We review some recent soliton solutions in a class of four-dimensional supergravity theories. The latter can be obtained from black hole solutions by means of a double Wick rotation. For special values of the parameters, the new configurations can be embedded in the gauged maximal $\mathcal{N} = 8$ theory and uplifted in the higher-dimensional $D = 11$ theory. We also consider BPS soliton solutions, preserving a certain fraction of supersymmetry.

Keywords: supergravity; supersymmetry; solitons; phase transitions

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1. Introduction

Solitons play a special role in classical physics and in quantum and string theory, determining a richer structure of the full non-perturbative regime. In general, they can be regarded as solutions of non-linear differential equations, which behave, in several ways, like extended particles, representing localized stable configurations with finite energy density.

Starting from a black hole solution, a different class of exact solutions of the field equations can be obtained from a double Wick rotation of the former black hole configuration [1–11]. The new solutions generically define gravitational solitons, describing a regular spacetime configuration devoid of horizons. The existence of gravitational soliton solutions also allows the derivation of the semiclassical entropy of a rotating hairy black hole by using microscopic states counting [12–18].

Gravitational solitons were originally used as "bounce solutions" in [1] to discuss the possible instability of the pure Kaluza–Klein vacuum ground state. They feature a shrinkable circle along which fermion fields are defined with antiperiodic boundary conditions. The presence of supersymmetry stabilizes the 1-cycle in the Kaluza–Klein vacuum through periodic boundary conditions of the fermionic fields, thus preventing a transition to the bounce solution [19,20]. Generalizations of these soliton solutions have been also considered in the analysis of the semiclassical stability of non-supersymmetric AdS gravity, where solitons play a fundamental role as they can be treated as ground states for suitable field theories [2,21–23]. The negative mass of the AdS soliton has a natural interpretation as the Casimir energy of a non-supersymmetric gauge theory living on the conformal boundary. In a non-supersymmetric version of the AdS/CFT conjecture, this would indicate that the soliton is the lowest energy solution with the chosen boundary conditions, leading to a new kind of positive energy conjecture; i.e., the soliton is the unique lowest mass solution for all spacetimes in its asymptotic class [21]. In particular, since the soliton topology has circles that are not contractible at infinity but which are in the bulk, there is a failure of spinorial methods to determine a positive energy theorem, which is related to the mentioned instability in Kaluza–Klein theory [1,24].

BPS solitons (or critical solitons) in supersymmetric frameworks were originally considered in the study of quantum corrections to the classical solutions [25,26] and in the definition of a modified superalgebra including central charges [27,28], together with the possible emergence of related quantum anomalies [29–31]. BPS gravitational solitons can be constructed by the double analytic continuation of supersymmetric regular black hole solutions or naked singularities [11,32]; the obtained configurations can be interpreted as the supersymmetric extension of the AdS soliton of [2]. In general, searching for configurations that preserve some of the supercharges provides a privileged framework in studying the system evolution, since the resulting equations are typically first order as compared with the standard second-order equations of motion. It is also interesting to study under which conditions these BPS configurations are protected against quantum corrections and enjoy no-force conditions, ensuring their stability.

In the following, we review some soliton solutions in a class of four-dimensional $\mathcal{N}=2$ gauged supergravity theories with Fayet–Iliopoulos terms [33]. The soliton configurations we discuss are obtained from the hairy black hole solutions of [34] through a suitable double analytic continuation. We also analyze supersymmetric configurations, providing an explicit form for the related Killing spinors. Surprisingly, the new soliton solutions feature different BPS conditions with respect to the original black hole configurations: in particular, the BPS constraints involving the charges found in [34] do not extend to the soliton case. Moreover, the soliton Killing spinors exhibit an explicit time dependence, while the correspondent BPS black hole spinors are only radial dependent.

2. The Model

Let us restrict to a dilatonic truncation of the STU model [35–38] of the maximal SO(8) gauged $\mathcal{N}=8$ supergravity. If we take the same value for all the dilatons and make suitable identifications of the vector fields, we obtain the well-known T^3 model. The latter has an action that reads

$$\mathcal{S}=\frac{1}{\kappa}\int d^4x\,\sqrt{-g}\left(\frac{R}{2}-\frac{1}{2}(\partial\phi)^2+\frac{3}{L^2}\cosh\left(\sqrt{\frac{2}{3}}\,\phi\right)-\frac{1}{4}e^{3\sqrt{\frac{2}{3}}\phi}\left(F^1\right)^2-\frac{1}{4}e^{-\sqrt{\frac{2}{3}}\phi}\left(F^2\right)^2\right). \quad (1)$$

2.1. Original Black Hole Solution

In [34], the authors obtain a new class of $\mathcal{N}=2$ supergravity theories parametrized by an interpolating parameter ν, related to the geometry characterizing the associated special Kähler manifold [39,40]. These new theories have been shown to interpolate between all possible single-dilaton consistent truncations of the ω-deformed [41–43] gauged maximal supergravities. For special values of ν, the solutions can be suitably embedded in the gauged maximal theory. In particular, setting $\nu=-2$ determines a model that can be embedded in the dilatonic truncation (1) of the SO(8) gauged $\mathcal{N}=8$ supergravity [44,45]. This would also allow for consistently uplifting the solutions to corresponding ω-rotated models.

Let us then consider the planar electrically charged black hole configuration of [34] for $\nu=-2$:

$$e^0=\sqrt{\Upsilon(x)\,f(x)}\,dt,\quad e^1=\sqrt{\frac{\Upsilon(x)}{f(x)}}\,\eta\,dx,\quad e^2=\sqrt{\Upsilon(x)}\,d\varphi,\quad e^3=\sqrt{\Upsilon(x)}\,dz, \quad (2)$$

$$\phi=\sqrt{\frac{3}{2}}\,\ln(x)\,,\qquad A^1=-Q_1\left(x^{-2}-x_0^{-2}\right)dt\,,\qquad A^2=-Q_2\left(x^2-x_0^2\right)dt\,, \quad (3)$$

where

$$\Upsilon(x)=\frac{4\,L^2\,x}{(x^2-1)^2\,\eta^2}\,,\qquad f(x)=1+\frac{\eta^2\,(x^2-1)^3\,(3\,Q_1^2-x^2\,Q_2^2)}{6\,L^2\,x^2}\,. \quad (4)$$

2.2. Soliton Solutions

Soliton solutions in the model (1) can be obtained by applying a double Wick rotation of the form

$$t \to i\,\varphi\,, \qquad \varphi \to i\,t\,, \qquad Q_{1,2} \to i\,Q_{1,2} \tag{5}$$

to the metric of the planar black hole solution (3). The new configuration defines a gravitational soliton, i.e., a regular spacetime solution devoid of horizons, of the form [33]

$$e^0 = \sqrt{\Upsilon(x)}\,dt, \quad e^1 = \sqrt{\frac{\Upsilon(x)}{f(x)}}\,\eta\,dx, \quad e^2 = \sqrt{\Upsilon(x)\,f(x)}\,d\varphi, \quad e^3 = \sqrt{\Upsilon(x)}\,dz, \tag{6}$$

$$\phi = \sqrt{\frac{3}{2}}\,\ln(x)\,, \qquad A^1 = Q_1\left(x^{-2} - x_0^{-2}\right)d\varphi\,, \qquad A^2 = Q_2\left(x^2 - x_0^2\right)d\varphi\,, \tag{7}$$

where $\Upsilon(x)$, $f(x)$ are those in (4).

The above spacetime features a conformal boundary that is reached for $x = 1$: the configuration then consists in two inequivalent, geodesically complete regions, one for $x \in (0,1)$ and the other for $x \in (1,\infty)$. The solutions can be physically differentiated by inspecting the sign of the dilaton field ϕ.

The location of the point x_0 such that

$$f(x_0) = 0\,, \tag{8}$$

contrary to the previous black hole configuration, does not identify a horizon but the position where a φ-circle contracts in the interior of the geometry.

Expanding the metric around x_0, together with the above condition (8), we see that the request of regularity leads to the definition of a parameter Δ such that

$$\varphi \,\in\, [0,\,\Delta]\,, \tag{9}$$

where Δ is expressed as

$$\Delta^{-1} = \left|\frac{1}{4\pi\,\eta}\,\frac{df}{dx}\right|_{x=x_0} = \left|\frac{\eta\,(x_0^2-1)^2}{4\pi\,L^2\,x_0^3}\left(Q_1^2\left(1+2\,x_0^2\right) - Q_2^2\,x_0^4\right)\right|. \tag{10}$$

The magnetic flux at infinity of the soliton configuration is found to be

$$\begin{aligned}
\Phi^1_{\rm M} &= \int F^1 = \oint A^1 = Q_1\,\Delta\left(1 - x_0^{-2}\right), \\
\Phi^2_{\rm M} &= \int F^2 = \oint A^2 = Q_2\,\Delta\left(1 - x_0^{2}\right).
\end{aligned} \tag{11}$$

3. Results

In the following, we will briefly analyze BPS configurations and the phase structure of our solutions.

3.1. Supersymmetry

From an explicit calculation of the fermionic variations (Killing spinor equations), one can see that the solution preserves part of the supersymmetry if [33]

$$Q_1 = -\frac{1}{\sqrt{3}}\,Q_2\,. \tag{12}$$

When the above relation holds, the metric function reads

$$f(x) = 1 - \frac{\left(x^2-1\right)^4}{x^2} \frac{\eta^2 Q_2^2}{6L^2}. \tag{13}$$

The explicit solutions of the Killing spinor equations give us four chiral spinors; since we are working in an $\mathcal{N}=2$ theory, this in turn means that the susy configuration is $1/4$ BPS in $\mathcal{N}=2$ [46]. If instead we consider the maximal theory, the susy solution is $1/8$ BPS with respect to the $\mathcal{N}=8$ model.

3.2. Phase Configurations

It is easily shown that the phase structure of our soliton configurations can be better depicted in terms of some special physical quantities. Let us briefly analyze this characterization. A natural choice for the solution representation is to parameterize the latter by means of physical data on the boundary that are held fixed. In this regard, a suitable option is to consider fixed charges at the boundary, as we describe in the following section.

3.2.1. Fixed Charge Boundary Conditions

Let us imagine we hold fixed the ratios Q_1/η, Q_2/η and the period Δ. In this fixed charge framework, the correct energy reference to characterize the stability of a configuration is given by the free energy of the system.

In order to review the phase structure of the solution for the fixed charge framework, we introduce, for convenience, the quantities

$$q_{1,2} \equiv \frac{\Delta^2}{4\pi^2 L} \frac{Q_{1,2}}{\eta}, \tag{14}$$

corresponding to a suitable rescaling of the charges Q_1, Q_2. In terms of the latter rescaled q_1, q_2, the parameter η can be rewritten in this framework as

$$\eta = \frac{3\Delta}{2\pi} \frac{\left|\left(q_2^2 - 3q_1^2\right) - 2\left(1-x_0^2\right)\left(q_2^2 - q_1^2\right) + \left(1-x_0^2\right)^2 q_2^2\right|}{x_0\left(1-x_0^2\right)\left(3q_1^2 - q_2^2 + q_2^2\left(1-x_0^2\right)\right)}. \tag{15}$$

The free energy of the system comes from a Legendre transform of the Euclidean action of the model. The latter can be obtained by a Euclidean continuation of the metric, featuring a compact time whose range of variation corresponds to $[0,\beta]$, β being the inverse of the temperature characterizing the soliton solution [2,22]. The period β is determined by the request of absence of conical singularities in the metric and, in our case, can vary from zero to infinity (the latter corresponding to susy configurations).

The Euclidean action S_{E} has the schematic form

$$\frac{S_{\mathrm{E}}}{V} = I_{\mathrm{bulk}} + I_{\mathrm{GH}} + I_{\mathrm{BK}} + I_{\mathrm{ct}} + I_{\phi}, \tag{16}$$

with

$$V = \beta\,\Delta \int dz, \tag{17}$$

and where I_{bulk}, I_{GH}, I_{BK} are the standard bulk term, Gibbons–Hawking term and Balasubramanian–Krauss counterterm, while I_{ct} and I_{ϕ} correspond to a divergent and finite counterterm, respectively, depending on the dilaton field ϕ.

A direct computation of the above contributions gives the explicit form of the free energy, which in our case reads

$$\frac{S_{\rm E}}{V} = -\frac{\mu}{2\,L^2\,\kappa}\,, \tag{18}$$

and is then expressed in terms of the μ mass parameter. The latter has the form [33]

$$\mu = \mp\frac{4\,L^2}{3\,\eta}\left(3\,Q_1^2 - Q_2^2\right), \tag{19}$$

while, in terms of the rescaled charges (14), it reads

$$\mu = \mp\left(\frac{2\sqrt{2}\,\pi\,L}{\Delta}\right)^4 \frac{\left(3\,q_1^2 - q_2^2\right)\eta}{3}\,, \tag{20}$$

the sign depending on whether we are choosing the $x > 1$ or the $x < 1$ branch, defining the two geodesically complete spacetimes that we already mentioned in Section 2.2.

3.2.2. Susy Configurations in the Fixed Charge Framework

The supersymmetry condition (12) can be expressed in terms of the rescaled charges (14) as

$$q_1 = -\frac{1}{\sqrt{3}}\,q_2\,, \tag{21}$$

so that the metric function $f(x_0)$ in the supersymmetric case reduces to

$$f(x_0) = 1 - \frac{q_1^2}{2\,x_0^6}\left(1 + 3\,x_0^2\right)^4 = 0\,, \tag{22}$$

while the η parameter reads

$$\eta = \frac{\Delta}{2\pi}\,\frac{1 + 3\,x_0^2}{x_0\left(1 - x_0^2\right)}\,. \tag{23}$$

The latter conditions will be used in the following calculations, where we will consider the stability of the supersymmetric configurations.

In [11], a class of Einstein–Maxwell solutions was studied, with vanishing scalars $\phi = 0$ and a single vector field with charge Q. In particular, the solutions in [11] can be obtained as a limit of our hairy solitons (i.e., coupled to the ϕ scalar) for a suitable choice of the parameters, resulting in a vanishing dilaton field and a reduction of our two gauge fields to a single one:

$$\phi = 0\,, \qquad A^1 = \frac{1}{\sqrt{3}}A^2 = \frac{1}{2\sqrt{2}}A\,. \tag{24}$$

It is possible to show a matching of the boundary conditions for the Einstein–Maxwell solutions of [11] and our hairy configurations for

$$q_2^2 - 3\,q_1^2 = 0\,, \tag{25}$$

the condition being satisfied from the supersymmetric solutions of Equation (21) [33]. This means that is possible to find hairy supersymmetric and Einstein–Maxwell non-supersymmetric solutions featuring the same boundary conditions. A comparison of the free energy of the two solutions will give us interesting results about the stability of susy configurations.

The charge Q of the single vector of [11] can be put in relation to our rescaled charge by

$$q_1 = \frac{\Delta^2}{4\pi^2 L} \frac{Q}{\sqrt{8}\, L^2}\,. \tag{26}$$

In the pure Einstein–Maxwell solution, the normalized free energy reads [11]

$$\frac{F_{\rm EM}}{\Delta\,\Delta_z} = \frac{2\pi^3 L^2}{\Delta^3\,\kappa}\, X^2\,(5 - 4\,X)\,, \tag{27}$$

with $q_1^2 = 2^{-7}\, X^3\,(4 - 3\,X)$. Using the expression of free energy of the AdS soliton [2]

$$G_0 = -\frac{32}{27}\,\frac{\pi^3 L^2}{\Delta^3\,\kappa}\,\Delta\,\Delta_z \tag{28}$$

as a convenient normalization, we find instead for the supersymmetric hairy solution the explicit form [33]

$$\frac{F_\phi}{|G_0|} = \frac{27}{\sqrt{2}}\,|q_1|\,. \tag{29}$$

In order to study the stability of the solutions, we can plot the free energies of the susy and non-susy configurations. In Figure 1, we show the ratio $F/|G_0|$ as a function of q_1 for the fixed value (21), $q_1 = -\sqrt{3}\, q_2$, satisfying (25).

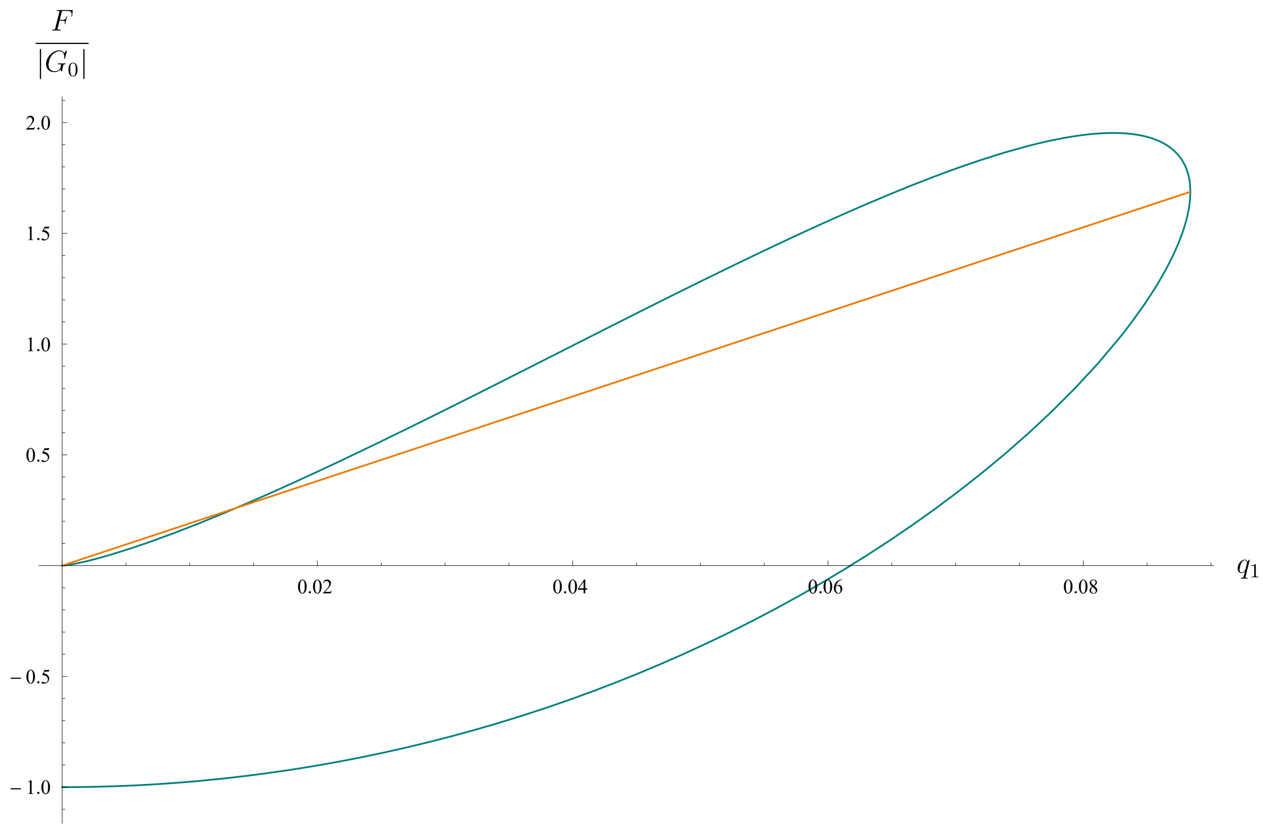

Figure 1. Rescaled free energy $\frac{F}{|G_0|}$ as a function of q_1 with the condition $q_2 = -\sqrt{3}\, q_1$. The yellow line represents the hairy supersymmetric solitons. The non-supersymmetric pure Einstein–Maxwell solutions are shown in blue. We can notice that there exist non-susy solutions featuring lower energy than susy configurations for the same boundary conditions and asymptotic charges [33].

From a direct inspection, one can see that there exists a branch of non-susy Einstein–Maxwell solutions with lower free energy than the supersymmetric hairy solution. This seems surprising, as we would expect the susy solutions to saturate a BPS bound, forbidding the existence of solutions with lower energy.

3.2.3. Stability and Positive Energy Theorem

The above results, even surprising, are not in contradiction with the consequences of the positive energy theorem [47,48]. The latter imply that the energy of a supersymmetry-preserving solution is lower than the energy of any other solution satisfying the same boundary conditions. In this regard, a necessary condition for the positive energy theorem to apply is the existence, for the non-susy solution, of an asymptotic Killing spinor that coincides, up to $O(1/r^2)$ terms at radial infinity, with the Killing spinor of the susy one.

Since, in our case, the susy hairy solution has antiperiodic boundary conditions at infinity, in order for the positive energy theorem to apply, the non-susy solutions should admit an asymptotic Killing spinor with the same properties at the boundary: this only happens for specific values of the charges at infinity, for which the energy of the non-supersymmetric solution exceeds that of the supersymmetric one, as one usually expects [33]. Then, there is no contradiction with the positive energy theorem if we include, among the boundary conditions, the constraints applying to the asymptotic Killing spinors.

4. Conclusions

We have reviewed interesting features about the stability of some special supergravity solutions. In particular, we have considered soliton configurations in a class of four-dimensional supergravity models that can be embedded in the $\mathrm{SO}(8)$ gauged maximal theory. A relevant outcome was that, for suitable choices of boundary conditions, there exist non-susy solutions featuring lower energy than susy configurations for the same boundary conditions and asymptotic charges.

The above results are non-trivial and deserve future, deeper analysis as possible, interesting frameworks to study the instability of supersymmetric configurations via quantum phase transition and related supersymmetry breaking scenarios [49].

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