Schauder Regularity Theory for Degenerate and Singular PDEs

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This thesis is devoted to the study of regularity results for degenerate and singular partial differential equations (PDEs) that fail to satisfy the standard uniform ellipticity condition. The degenerate or singular behavior arises from coefficients that vanish or blow up as powers of the distance to some characteristic manifolds.

In the first part of the thesis, we investigate parabolic PDEs that are degenerate or singular on hypersurfaces. We establish a complete Schauder theory, proving that the smoothness of the problem's data is inherited by the solution. These equations exhibit strong connections to the study of nonlocal operators via the Caffarelli-Silvestre extension method, to the analysis of free boundary problems, and to the development of Boundary Harnack-type principles.

The second part of the thesis addresses elliptic operators where degeneracy occurs on lower-dimensional manifolds. In this context, we establish optimal regularity results, ensuring the continuity of the gradient of the solutions. These equations represent a relatively new direction in the literature; we highlight their compelling connections to critical points of semilinear PDEs, harmonic measures on lower-dimensional manifolds, and harmonic maps with prescribed singularities in general relativity.