# Pretending Ignorance Is Bliss: Competing Insurers with Heterogeneous Informational Advantages

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The availability of big data and analytics expertise provides insurers with informational advantages over policyholders in estimating risk. We study competition between heterogeneously informed insurers, showing that their information may or may not be revealed in equilibrium. We find that all equilibria are profitable and that noninformative equilibria entail risk pooling and possibly efficiency. In informative equilibria, the signaling problem interacts with the screening problem that arises endogenously from insurers' revelation of information, implying underinsurance. Our main insights are robust to changes in insurers' information precision and market concentration and to the presence of two-sided asymmetric information and withdrawable contracts. (*JEL* D43, D82, G22)

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In the last few years, the insurance industry has repeatedly hit the headlines because of the technological changes that are reshaping the traditional business model. The ever-increasing availability of large datasets and advances in big data analytics and artificial intelligence are fueling the informational advantages of insurance companies, which are better equipped than policyholders to assess risk through their expertise and access to data. These technological advances have stimulated a new generation of insurance models in which risk-relevant information is held not only by policyholders but also by insurers (e.g., Brunnermeier, Lamba, and Segura-Rodriguez 2020; Villeneuve 2005). The implications for competition of the interplay between more informed insurers have been investigated by Villeneuve (2005) in a context in which insurers have identical (perfect) information about the policyholder's risk. In practice, however, the difficulties underlying the process of risk estimation impose that insurers' informational advantages are often imperfect, leading to different assessments of risk (e.g., because of the reliance on different algorithms or data sources).[1]

In this paper, we add dispersed information to Villeneuve (2005), focusing on a duopoly insurance market and modeling competition between insurers who receive a private, imperfect signal about a representative policyholder's risk. After observing the signal, each insurer offers a menu of contracts, which may or may not convey his private information, resulting in informative and noninformative equilibria, respectively.[2] In the case of informative equilibria, the signaling content of insurers' offers allows the policyholder to infer insurers' information. Hence, when choosing a contract, the policyholder holds market information and is more informed than insurers are when they issue their offers. In this respect, the signaling problem endogenously generates a screening problem, and their interplay determines novel and nonstandard results in terms of equilibrium characterization. Because of the presence of a screening problem, we find that informative equilibria are inefficient, analogous to Rothschild and Stiglitz (1976). At the same time, because of the presence of

[1] For instance, privacy laws requiring that medical records are not released to outsiders without the consent of the patient increase the probability of mistakes in the estimation of risk by insurers. Moreover, companies may learn about the risk of their policyholders by observing claims records and contract choices but will not freely share this private information with rival firms (see, e.g., Fombaron 1997). As a consequence, rival firms do not have access to accident histories.

[2] While we frame our analysis focusing on insurance markets, our results may be relevant in other domains as well where principals competing under exclusive contracts are privately but imperfectly informed about hidden parameters that influence both their payoffs and agents' preferences (common values). This is the case, for instance, for credit markets in which privately informed banks compete for borrowers, for investors holding heterogeneous assessments on the potential cash flows originated by a security (or investment project), or for consultants holding heterogeneous assessments of the probability of success.

signaling, strictly positive profits emerge in equilibrium for some (and possibly all) insurers. These results emerge even for negligible amounts of signal imperfection and differ from those obtained when insurers are identically informed (Villeneuve 2005), in which case the need to screen the policyholder based on market information is absent and profits can be zero.

We also find that there exist strictly profitable noninformative equilibria, in which insurers make identical offers. In contrast to informative equilibria, these equilibria entail risk pooling and may be *ex ante* fully efficient. This is because the policyholder may reject undercutting deviations, thus hindering competition, when she holds optimistic out-of-equilibrium beliefs about insurers' estimates of risk. The signaling content of deviations sustains noninformative equilibria, exactly as is the case for pooling equilibria in signaling games. In this respect, the existence of noninformative equilibria in our setup extends the findings of Villeneuve (2005) for the case of identically informed insurers.

An important implication of our results concerns the different effects of heterogeneously informed principals for noninformative and informative equilibria. In the former, heterogeneous insurers' information is consistent with full efficiency; in the latter, several incentive compatibility constraints must be satisfied, causing inefficiency. Therefore, the presence of heterogeneity could result in information revelation being less attractive for insurers.

Interestingly, although our model requires strong assumptions to guarantee equilibrium existence (as is often the case in screening and signaling insurance models of competition), some of our informative and noninformative equilibria are robust to equilibrium refinements, such as the Intuitive and D1 criteria, and survive even when we add asymmetric information on the policyholder's side and let the model converge to Rothschild and Stiglitz (1976).

Furthermore, we show that an increase in signal precision, by affecting the estimation of risk, induces upward pressure on insurance premiums (for informative equilibria) and higher profits for insurers with a favorable estimation of risk (for noninformative equilibria). These effects stem from the fact that more precise insurers' information increases the risk assessment associated with an unfavorable signal, at the same time reducing the estimation of risk of an optimistic policyholder and thus her willingness to pay for undercutting deviations.

We also find that enabling insurers to withdraw loss-making offers after having observed those of their competitors may improve the efficiency of informative equilibrium outcomes, different from what occurs in pure adverse selection models, such as Wilson (1977) and Hellwig (1987). Since market information emerges endogenously from insurers' offers, the latter can withdraw their contracts after they have learned about the policyholder's risk from observing all the offers. Interestingly, the possibility of withdrawable contracts does not affect the multiplicity and profitability of noninformative equilibria because of the role of out-of-equilibrium beliefs in hindering competition.

Finally, we show that there exists an upper bound to the number of firms consistent with the existence of equilibria and that, in the case of noninformative equilibria, a larger industry dispersion may entail larger equilibrium premiums and profits, consistent with the observed unstable relationship between market concentration and profitability emphasized by the empirical literature (Dafny, Duggan, and Ramanarayanan 2012; Dauda 2018; Hyman and Kovacic 2004). While building on the tradition of competitive screening and signaling models, our results suggest that modeling information differently could be key in understanding the empirical shortcomings of standard insurance models à la Rothschild and Stiglitz (1976).

From a policy point of view, our results may contribute to the ongoing debate about the impact of new technologies in the insurance industry, and how they can threaten the insurance business. To the extent that the new technologies allow for a more granular and precise assessment of risk, paving the way for personalized policies, one major concern is that they may unhinge the pooling of risk that is at the core of the existence and profitability of insurance markets. In 2017, The Economist wrote that the "coming revolution in insurance"could wreak havoc on a so-far relatively "complacent industry", boosting competition and eroding profits (Economist, 2017). Our paper suggests that these conclusions might be unwarranted by showing that risk pooling and persistent profitability are fully consistent with competition among insurers holding an imperfect informational advantage over policyholders.

## 1. Related Literature

The theoretical literature on insurance markets typically assumes that policyholders have an informational advantage over insurers. Only a few studies have focused on more informed insurers. Among them, Villeneuve (2005) models competition between homogeneous insurers who can perfectly observe the policyholder's riskiness, showing that full efficiency and actuarially fair outcomes can always be achieved, and finding that profitability and risk pooling are possible in equilibrium. While the contractual design problem in Villeneuve (2005) is one of signaling because of the assumption of homogeneous information, by assuming dispersed insurers' information, we have both a signaling and a screening problem.[3] This difference has far-reaching implications, as actuarially fair outcomes can no longer be achieved under dispersed information. Related literature has explored the role of similar signaling mechanisms outside the scope of insurance markets. Hartman-Glaser and Hebert (2020) focus on credit markets and show that there can exist a pooling equilibrium in which information is not disclosed and lenders offer unindexed securities, provided that lenders are better informed about the quality of an index measuring credit market conditions. Lenders in Hartman-Glaser and Hebert (2020), similar to insurers in Villeneuve (2005) but different from our insurers, are symmetrically informed. This has important implications for the profitability of equilibria: while (noninformative) equilibrium profits may be nil in Hartman-Glaser and Hebert (2020), the presence of heterogeneously informed insurers always implies strictly positive equilibrium profits in our setup.

The competition between informed principals under dispersed information has been the subject of a large body of literature in which principals' informational advantage concerns either technological (cost function) parameters or consumer preferences (willingness to pay). For the first class of problems, Vives (2011) focuses on supply function competition under private information in a linear-quadratic setup, showing that positive profits emerge in equilibrium due to the information role of prices. Our contribution differs from that of Vives (2011) as he considers a passive competitive demand, while the policyholder selects among alternative contracts in our setup. More importantly, in his paper the uncertainty is on suppliers' cost functions, while in our setup it is on the agent's risk type, thus affecting both the latter's willingness to pay and the cost of insurance. For the second class of problems, several contributions on expert advice and credence goods study competition between informed sellers when uncertainty concerns consumers' willingness to pay (e.g., Emons 1997; Hertzendorf and Overgaard 2001; Wolinsky 1995). In contrast to our setup, these contributions exclusively focus on private values rather than on common values.

The assumption of dispersed information is also standard in the auction literature (e.g., Abraham et al. 2020; Milgrom and Weber 1982). The bidders in an auction reveal their private evaluations through separating equilibrium strategies. Therefore, the auctioneer (i.e., the *ex ante* less informed party) can extract all bidders' information, as the policyholder does in informative equilibria in our setup. However, there are at least two substantial differences between the auction literature

[3] Note that our setup is fully consistent with Harsanyi's approach to incomplete information games, which entails that each player has more information about her type.

and our approach. First, in auctions, the contract is designed by the less informed party (the auctioneer), whereas in our setup the opposite occurs. Pooling emerges in our framework because of the signaling content of insurers' offers, while it typically does not in the auction literature. Second, while the competition among bidders favors the appropriation of the transaction surplus by the auctioneer (on the supply side of the market), the competition between insurers in our model favors the emergence of a policyholder's surplus (on the demand side of the market), still preserving insurers' profits.[4] The latter follows from the specific nature of our problem that, when insurers' information is revealed, merges a screening problem with a signaling problem.

Our contribution is also related to the literature on mechanism design with one informed principal (Maskin and Tirole 1990, 1992; Myerson 1983) investigating the implementation of second-best mechanisms that are efficient and optimal for each type of principal (e.g., Dosis 2022; Severinov 2008).[5] That literature focuses on nondelegation (centralized) mechanisms in which the principal commits to an allocation scheme as a function of his communication in a later stage, thus allowing the principal to effectively determine the contract to be implemented (Maskin and Tirole 1992). Instead, we study a delegation game in which the contract is chosen by the agent within a menu offered by the principals. As noted by Galperti (2015) in a common agency framework, the set of equilibrium allocations in delegated games with multiple informed principals departs from those of nondelegation games for two reasons. First, in multiprincipal settings, the applicability of the revelation principle comes into question (Epstein and Peters 1999; Martimort and Stole 2002). Therefore, offering incentive compatible contract menus rather than direct mechanisms may lead to different equilibrium outcomes. Second, with many informed principals, signaling through offers becomes an essential strategic component of the game (Galperti 2015). The signaling of insurers' information in informative equilibria allows the agent to collect market information from observing contractual offers, requiring incentive compatibility to hold across agent types. This reduces the efficiency of the equilibrium outcomes compared to the typical outcomes of the informed principal approach.

Brunnermeier, Lamba, and Segura-Rodriguez (2020) investigate a monopolistic insurance setup in the spirit of the informed principal approach, in which both the insurer and the policyholder have private

[4] On competition between sellers in security auctions, see Gorbenko and Malenko (2011).

[5] The same approach has been used in different frameworks, such as those entailing transferable utility (Balkenborg and Makris 2015) or risk neutrality (DeMarzo and Frankel 2022).

information.[6] They find that the insurer may have an incentive to pool risk in some states to retain part of his informational advantage and that equilibrium profits may be large. Although focusing on different information and market structures, both our contribution and that of Brunnermeier, Lamba, and Segura-Rodriguez (2020) build a strong case in favor of recognizing the fundamental role of insurers' information and consumers' beliefs in shaping the insurance industry.

Finally, our results on profitability and risk pooling are consistent with a large body of empirical literature. The profitability of insurance companies and their underlying driving forces have been the subject of heated debate (e.g., Cabral, Geruso, and Mahoney 2018; Robinson 2004; Sommer 2017). Furthermore, insurance contracts often entail premiums that purposely ignore risk-relevant information (Finkelstein and Poterba 2014).[7] For instance, Cawley and Philipson (1999), who found no evidence of a significant relationship between insurance prices and risk in life insurance, conjectured that insurers may have an informational advantage over policyholders. Different streams of theoretical literature, in addition to those on more informed insurers discussed above, have attempted to reconcile the theory of insurance markets with empirical evidence.[8] In particular, several authors have noted how the assumption of nonexclusive contracts allows for the existence of profitable pooling equilibria by relying on latent contracts used strategically to prevent deviations by competitors (see, e.g., Attar and Chassagnon 2009; Attar, Mariotti, and Salaniè 2011; Bisin and Guaitoli 2004). We depart from these papers by building instead on exclusive contracts, which allows us to account for the many instances in which profitable exclusive offers entailing risk pooling are observed in practice, especially when insurers' informational advantage is more likely to be significant.[9]

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[6] The informed principal approach with screening under common values has been applied to a variety of contexts, including procurement (Martimort and Sand-Zantman 2006), sales (Koessler and Skreta 2016, 2019), security design (DeMarzo, Frankel, and Jin 2021), bank bailouts (Philippon and Skreta 2012), investment funding (Morellec and Schurhoff 2011), dividend stickiness (Guttman, Kadan, and Kandel 2010), monopoly pricing (Ottaviani and Prat 2001), job market matching (Inderst 2005), and innovation funding (Bouvard 2014).

[7] Finkelstein and Poterba (2014, p. 710) note how insurance companies in the U.K. annuity market "voluntarily choose not to price based on risk-related buyer information that they collect," such as the annuitant's place of residence, although this may help to predict future mortality. This is starkly at odds with traditional competitive insurance models, showing that risk types must be separated in equilibrium.

[8] The departure from the standard competitive mechanism may also be the result of search and switching costs (as highlighted by Hortaçsu and Syverson (2004) for the U.S. mutual fund industry) or financial and product market frictions (as shown by Koijen and Yogo 2015, 2016 for life insurance, where insurance prices are used to relax regulated leverage constraints).

[9] Automobile insurance contracts, for example, are almost always exclusive, as noted by Chiappori and Salanié (2000).

## 2. The Model

We focus on a duopoly in which two insurers, $i \in \{1,2\}$, are better able than a representative policyholder to assess individual risk and offer exclusive, nonwithdrawable insurance contracts. The policyholder has no private information about her characteristics, is endowed with initial wealth $W$ and faces two possible states of the world, $\ell$ if a loss $L \in [0,W]$ is observed, and $n\ell$ if no loss occurs. The probability that $\ell$ occurs is $p \in [0,1]$.

The policyholder can insure herself against the loss $L$. Let $\Omega$ be the contract space, which includes full-insurance and underinsurance contracts. Each insurance contract $c \equiv (w_\ell, w_{n\ell}) \in \Omega$ specifies the policyholder's wealth $w_\delta \in [W-L,W]$ in the two possible states $\delta \in \{\ell, n\ell\}$. The no-insurance (i.e., autarky) contract $\underline{c} = (W-L,W) \in \Omega$ constitutes the policyholder's outside option. The policyholder's preferences are represented by the utility function $u: \mathbb{R}_+ \to \mathbb{R}_+$ defined over wealth in each state $\delta \in \{\ell, n\ell\}$ and continuously differentiable. The policyholder is risk averse, and the von Neumann-Morgenstern expected utility is $Eu_p(c) \equiv pu(w_\ell) + (1-p)u(w_{n\ell})$, for any implemented contract $c = (w_\ell, w_{n\ell})$ and $p \in [0,1]$. Most of the results in the paper hold regardless of the specific instantaneous utility function considered. However, to ease the analysis, when dealing with the existence of equilibria, we focus on a utility function entailing constant absolute risk aversion, adopting the standard CARA specification

$$u(w) = \frac{1-e^{-\beta w}}{\beta}, \tag{1}$$

where $\beta \in \mathbb{R}_+$ denotes the degree of (absolute) risk aversion.

Insurers are risk neutral. Insurer $i$'s profit, for $i \in \{1,2\}$, is a linear function $\pi^i: \mathbb{R}_+ \to \mathbb{R}_+$ defined over wealth in each state $\delta \in \{\ell, n\ell\}$. The expected profit of insurer $i \in \{1,2\}$ when contract $c = (w_\ell, w_{n\ell})$ is implemented, given a loss probability $p$, is $E\pi_p^i(c) = p(W-L-w_\ell) + (1-p)(W-w_{n\ell})$. The loss probability $p$ depends on the environment in which the insurers and the policyholder operate, which can be either dangerous ($d$) or safe ($s$) with probabilities $q$ and $1-q$, respectively. Let $p_\theta$ denote the specific loss probability in environment $\theta \in \{d,s\}$, and assume with no loss of generality that $p_d > p_s$.

The realization of individual states $\delta \in \{\ell, n\ell\}$ is commonly observed, whereas the value of $\theta \in \{d,s\}$ is unobservable, although each insurer $i$ receives independently and privately an imperfect signal $\hat{\theta}_i \in \left\{\hat{s}, \hat{d}\right\}$ about $\theta$. The signal $\hat{\theta}_i$ identifies an insurer's type and it is not observable by the other insurer or by the policyholder. We refer to the vector of signals $(\hat{\theta}_1, \hat{\theta}_2)$ as a signal profile. Each signal $\hat{\theta}_i$ is correct with probability $\alpha \in (1/2,1)$, so that $\alpha$ indicates the precision of the signal received by an insurer: $\alpha \equiv \Pr(\hat{\theta}_i = \hat{s}|\theta = s) = \Pr(\hat{\theta}_i = \hat{d}|\theta = d)$ for $i \in \{1,2\}$. Since signals are independent and of equal precision, the number of insurers who receive signal $\hat{\theta}_i = \hat{s}$, denoted by $\sigma \in \Sigma = \{0,1,2\}$, is a sufficient statistics for the loss probability. Namely, $\sigma = 0$ if both firms receive signal $\hat{d}$ ; $\sigma = 1$ if one firm receives signal $\hat{s}$ and the other receives signal $\hat{d}$; and $\sigma = 2$ if both firms receive signal $\hat{s}$. Throughout the paper, we summarize, without loss of generality, the signal profile $(\hat{\theta}_1,\hat{\theta}_2)$ by $\sigma$. $p_\sigma \in \{p_0,p_1,p_2\}$ denotes the loss probability conditional on $\sigma$, that is

$$p_\sigma = \sum_{\theta \in \{d,s\}} p_\theta \Pr(\theta|\sigma), \tag{2}$$

where $\Pr(\theta|\sigma) \in [0,1]$ denotes the probability that the environment is $\theta$ given that $\sigma$ insurers receive signal $\hat{s}$. Each insurer $i$ offers a menu $\mathcal{C}_i$ of exclusive and nonwithdrawable contracts, conditional on what is revealed to the policyholder about the signal profile.[10] $\mathcal{C} = (\mathcal{C}_1,\mathcal{C}_2)$ denotes the vector of menus offered by the two insurers.

The timing of the game is as follows: 0) Nature chooses the environment $\theta$, and draws – independently and from a common distribution – each insurer's signal $\hat{\theta}_i$; 1) each insurer $i$, $i \in \{1,2\}$, privately observes $\hat{\theta}_i$ and updates his prior belief on $\theta$ conditional on $\hat{\theta}_i$; 2) each insurer $i$ offers a menu of publicly observable contracts $\mathcal{C}_i$; 3) the policyholder observes all offers at no cost, updates her beliefs, and selects one contract $c \in \mathcal{C} = (\mathcal{C}_1,\mathcal{C}_2)$ or the no-insurance contract $\underline{c}$; 4) the accepted contract is implemented and payoffs are received.

This timing entails that the policyholder and the insurers rely on different estimates of the loss probability. The *ex ante* loss probability estimated in stage 0 is $\bar{p} = p_d q + p_s(1-q)$. In stage 1, insurers assess the *ad interim* loss probability $p_{\hat{\theta}_i} = p_s Pr(s|\hat{\theta}_i) + p_d Pr(d|\hat{\theta}_i)$, based on their private signal. With a slight abuse of notation, we write $p_{\hat{\theta}}$ instead of $p_{\hat{\theta}_i}$ whenever it is not confusing. In stage 3, the policyholder updates her beliefs based on the observed offers, which may reveal (or not reveal) market information about the signals received by insurers. Let $\mu_i(\mathcal{C}) = \Pr(\hat{\theta}_i = \hat{s}|\mathcal{C})$ be the *ex post* probability (obtained using Bayes' rule) that the policyholder assigns to $\hat{\theta}_i = \hat{s}$ given the vector $\mathcal{C}$ of contract menus she observes. The vector $\mu(\mathcal{C}) \equiv (\mu_1(\mathcal{C}),\mu_2(\mathcal{C}))$ represents the policyholder's belief about the signal profile $(\hat{\theta}_1,\hat{\theta}_2)$. Note that the *ex post* loss probability depends on the characteristics of the emerging equilibrium. If all insurers' types offer the same contract menu (a pooling noninformative equilibrium), the policyholder learns nothing regarding insurers' signal profiles and relies on the *ex ante* loss probability. Conversely, in separating (informative) equilibria, the policyholder infers

[10] Sections 3 and 4 clarify the structure of these menus, which depends on the noninformative versus informative nature of the equilibrium.

insurers' type by looking at contract menus and exploits this information to update the *ex ante* loss probability.

### 2.1 Equilibrium and beliefs

We focus on symmetric equilibria with pure strategies in which all insurers of the same type offer the same menu. Thus, a perfect Bayesian equilibrium (PBE) can be defined as follows.

**Definition 1.** A symmetric perfect Bayesian equilibrium is (a) a vector of contract menus $\mathcal{C}^e = (\mathcal{C}_1^e, \mathcal{C}_2^e)$, where $\mathcal{C}_i^e \in \left\{ \mathcal{C}_{\hat{\theta}_i}^e \right\}_{\hat{\theta}_i \in \{\hat{s}, \hat{d}\}}$, $i \in \{1,2\}$, depending on the type of insurer $i$, and (b) a belief mapping $\mu^e(\mathcal{C})$ such that:
1. insurers' strategies are sequentially rational, so that for any insurer $i$ of type $\hat{\theta}_i$, the menu $\mathcal{C}_{\hat{\theta}_i}^e$ maximizes expected profits given the strategy profile of the insurer $j \in \{1,2\}$, $\mathcal{C}_j^e$, $j \neq i$, and the policyholder's strategy;
2. for any given information set, the policyholder's equilibrium strategy selects the contract belonging to $\mathcal{C}^e$ that maximizes her expected utility given the belief mapping $\mu^e(\mathcal{C})$;
3. beliefs are consistent with the Bayes' rule when relevant.

The notion of perfect Bayesian equilibrium requires the definition of a belief system based on Bayesian updating on the equilibrium path, and arbitrarily defined off the equilibrium path. We assume that the policyholder's equilibrium beliefs out of the equilibrium path are independent of insurer $i$'s specific deviation from the equilibrium; that is, $\tilde{\mu}_i = \mu_i^e(\mathcal{C}_i, \mathcal{C}_j)$ for all $i$ and $\mathcal{C}_i \neq \mathcal{C}_i^e$. Although this implies homogeneous beliefs across the contract space, we show in Internet Appendix A.1 that the key properties of our equilibria are also preserved when adopting different equilibrium refinements (i.e., the intuitive criterion and the D1 criterion). As is standard in the pertinent literature, we also assume that a deviation by insurer $i$ reveals nothing about the type of the other insurer. Moreover, given symmetry between firms, the policyholder forms the same beliefs about the deviating firm, regardless of the identity of the deviator, namely, $\tilde{\mu}_1 = \tilde{\mu}_2 = \tilde{\mu}$, where $\tilde{\mu} \in [0,1]$ denotes the policyholder's "degree of optimism", namely, the probability that she assigns to the event that a deviating insurer is of type $\hat{s}$ when a deviation is observed. The degree of optimism includes fully optimistic ($\tilde{\mu} = 1$) and fully pessimistic ($\tilde{\mu} = 0$) beliefs, which are the almost exclusive focus of the literature on informed insurers (see, e.g., Seog 2009; Villeneuve 2005).[11]

[11] Our definition can also encompass passive beliefs, under which a policyholder retains prior beliefs about the type of the deviating insurer. Such beliefs are immediately obtained by setting $\tilde{\mu} = \alpha(1-q) + (1-\alpha)q$.

The assumption that insurers have private information about the riskiness of the environment in which they operate gives rise to two opposite scenarios, depending on whether or not insurers' information is revealed in equilibrium. The resultant informative or noninformative equilibria are analyzed in the following sections.

## 3. Noninformative Equilibria

We focus first on noninformative equilibria, in which the insurers' offers do not carry information about the signals they received. Since any difference in the contract menus $\mathcal{C}^e_{\hat{s}}$ and $\mathcal{C}^e_{\hat{d}}$ offered in equilibrium by the $\hat{s}$ and $\hat{d}$ insurers, respectively, would convey information regarding the insurers' types, noninformative equilibria necessarily require that $\hat{s}$ and $\hat{d}$ insurers offer identical menus (i.e., $\mathcal{C}^e_{\hat{s}}=\mathcal{C}^e_{\hat{d}}$), which implies pooling on the insurers' types in the first stage.

Since the policyholder does not observe insurers' types in equilibrium, she cannot make inferences about the signal profile. Then, in equilibrium there is only one type of policyholder, who maintains her *ex ante* beliefs about risk even after observing the insurers' offers (i.e., $\mu^e(\mathcal{C}^e)=1-q$ on the equilibrium path). This implies that $p_{\mu^e(\mathcal{C}^e)}=\bar{p}$ for all noninformative equilibria $\mathcal{C}^e$. Given that observationally there is only one type of policyholder, the outcome of noninformative equilibria entails a degenerate menu $\mathcal{C}^e$ represented by a single contract $c^e\in\Omega$, for all $i\in\{1,2\}$ and $\hat{\theta}_i\in\{\hat{s},\hat{d}\}$. $\mathcal{E}^*\subset\Omega$ denotes the set of noninformative equilibrium contracts. Note that, as no information is conveyed in equilibrium, insurers update their beliefs on the environment based on their own signal $\hat{\theta}_i$ only. The loss probability estimated by insurer $i$ when signal $\hat{\theta}_i$ is received is the *ad interim* loss probability $p_{\hat{\theta}_i}$.

The set of possible noninformative equilibria is potentially infinite, as it depends on several factors, particularly on the set of beliefs off the equilibrium path. To ease the exposition, throughout the paper we restrict the set of admissible contracts to those implying nonnegative profits if accepted.[12]

### 3.1 Characterization

An equilibrium contract $c^e\in\mathcal{E}^*$ must satisfy three conditions. First, the contract must be profitable, or fair, for all types of insurers, so that the insurers' participation constraint

$$E\pi_{p_{\hat{\theta}}}(c^e)\geq 0 \tag{3}$$

[12] We generalize the analysis in Internet Appendix A.2, showing that any individually rational allocation can be sustained as a noninformative equilibrium when allowing for (latent) contracts entailing nonpositive profits if accepted.

holds for all $\hat{\theta}_i \in \{\hat{s},\hat{d}\}$. Second, it must be acceptable for a policyholder who relies on the prior estimation of the loss probability, so that the policyholder's participation constraint

$$Eu_{\bar{p}}(c^e) \geq Eu_{\bar{p}}(\underline{c}) \tag{4}$$

holds. Third, any deviation that is acceptable by the policyholder must be less profitable than the equilibrium contract for both $\hat{s}$ and $\hat{d}$ insurers, given the policyholder's beliefs. For this third condition, note that the set of admissible deviations from a noninformative equilibrium include single contracts only and not menus, given that there is just one type of policyholder, regardless of her out-of-equilibrium belief about the loss probability. Nonetheless, insurers can offer different contracts depending on their own type, $\hat{s}$ or $\hat{d}$. For all practical purposes, it is sufficient to focus only on the most profitable deviation for a $\hat{\theta}_i$ insurer, which we write as $c^{dev}_{\hat{\theta}_i}$. Formally, the contract $c^{dev}_{\hat{\theta}_i}$ can be defined as

$$c^{dev}_{\hat{\theta}_i} \equiv \arg\max_{c_i \in \Omega} E\pi_{p_{\hat{\theta}}}(c_i) \quad s.t \quad Eu_p(c_i) \geq Eu_p(\underline{c}), \tag{5}$$

with $p = p_{\mu^e(c)}$, for any $c = (c_i, c_{-i}) \in \Omega$ and $c_{-i} \in \mathcal{E}^*$.[13] Any deviation from $c^e$ guarantees to a $\hat{\theta}_i$ insurer expected profits that are at most equal to those induced by $c^{dev}_{\hat{\theta}_i}$, that is, $E\pi_{p_{\hat{\theta}}}(c^{dev}_{\hat{\theta}_i})$.[14] If the equilibrium expected profits are greater than the upper bound of the deviation profits, that is,

$$\frac{1}{2} E\pi_{p_{\hat{\theta}}}(c^e) \geq E\pi_{p_{\hat{\theta}}}(c^{dev}_{\hat{\theta}_i}) \text{ for all } \hat{\theta}_i \in \left\{\hat{s},\hat{d}\right\}, \tag{6}$$

then any acceptable and profitable deviation can be ruled out. For (6) to hold, appropriate restrictions on beliefs are needed, as established by the following lemma.

**Lemma 1.** In any noninformative equilibrium, out-of-equilibrium beliefs are sufficiently optimistic, that is, $\tilde{\mu} > 1 - q$.

When the policyholder is sufficiently optimistic about her risk, she expects a lower insurance premium than insurers are willing to grant. Thus, the offer by a deviating insurer of marginal undercuts from noninformative equilibria is rejected by the policyholder, who optimistically demands a greater discount. The greater the

[13] Slightly abusing notation, we consider beliefs as a function of a single contract rather than a menu.

[14] Note that $c^{dev}_{\hat{\theta}_i}$ is preferred to autarky, but not necessarily to $c^e$. However, the following Lemma 1 will show that it is not the case in equilibrium.

policyholder's degree of optimism is, the higher the requested premium reduction and the lower the profitability of the deviation. Optimistic beliefs then hinder the effectiveness of the competitive mechanism, thus preventing the emergence of an actuarially fair outcome. We show in Internet Appendix A.1 that, under appropriate conditions, optimistic beliefs are consistent with the Intuitive and D1 criteria. The reason is that the policyholder overestimates risk in deviation when a deviating $\hat{s}$ insurer is believed to be of type $\hat{d}$, thus increasing her willingness to pay. Conversely, when a deviating $\hat{d}$ insurer is believed to be of type $\hat{s}$, the policyholder underestimates the real risk, which in turn reduces her willingness to pay. Thus, since a $\hat{s}$ insurer is more likely to obtain larger profits in deviation, both refinements require that the probability of a deviating insurer being assessed to be of type $\hat{s}$ is larger than that of being of type $\hat{d}$, consistently with our definition of optimistic beliefs.

### 3.2 Existence

A noninformative equilibrium exists if and only if Conditions (3), (4), and (6) hold. Understanding when this is the case for a generic utility function is cumbersome. Nonetheless, for the CARA specification in Equation (1), the existence of noninformative equilibria entailing a positive level of insurance can be established under relatively mild conditions on the degree of the policyholder's risk aversion, as shown in Proposition 1.[15]

Let $\underline{\beta}$ be implicitly defined by $W - p_{\hat{d}}L + \frac{1}{\beta}\ln\left((1-\bar{p})e^{-\beta W} + \bar{p}e^{-\beta(W-L)}\right) = 0$, which ensures that the participation constraints of both the insurers and the policyholder are met. Additionally, we define $\bar{\beta}$ as $\bar{\beta} = \min\{\bar{\beta}_{\hat{s}}, \bar{\beta}_{\hat{d}}\}$, where $\bar{\beta}_{\hat{s}}$ and $\bar{\beta}_{\hat{d}}$ are the unique solutions of the no-deviation constraints (6) taken with equality, for $\hat{s}$ and $\hat{d}$ insurers, respectively.

**Proposition 1.** Under the CARA specification (1), an (*ex ante*) efficient contract $c$ is a noninformative equilibrium if and only if there exist $\underline{\beta}$ and $\bar{\beta}$ such that $\underline{\beta} \leq \bar{\beta}$, and $\beta \in [\underline{\beta}, \bar{\beta}]$. The set of equilibrium contracts $\mathcal{E}^*$ is nonempty if and only if the insurers' information precision $\alpha$ is large and the loss probability in the safe environment, $p_s$, is small.

[15] The existence of pooling equilibria is in contrast with Rothschild and Stiglitz (1976), who show that risk pooling cannot emerge when private information is held by policyholders only. In Internet Appendix C, we extend our baseline model to encompass two-sided asymmetric information (i.e., a scenario in which insurers have private information about the environment and policyholders have private information about their own characteristics), comparing our results to those of Rothschild and Stiglitz (1976).

To explain why it must be that $\beta \in [\underline{\beta}, \bar{\beta}]$, observe that a sufficiently large degree of the policyholder's risk aversion ($\beta \geq \underline{\beta}$) is needed to guarantee the existence of gains from trade, even when the risk assessment of the policyholder is smaller than that of the $\hat{d}$ insurer. At the same time, since the lower $\beta$ is, the lower is the policyholder's willingness to pay for insurance, a sufficiently small degree of risk aversion ($\beta \leq \bar{\beta}$) is needed for no insurers to have an incentive to deviate regardless of their assessment of risk. In fact, for $\beta \leq \bar{\beta}$, the deviations that are acceptable by a sufficiently optimistic policyholder (as established by Lemma 1) are such that the expected profits of both $\hat{d}$ and $\hat{s}$ insurers are lower than those they obtain in equilibrium. To understand the role of $\alpha$ and $p_s$ in guaranteeing that $\bar{\beta} > \underline{\beta}$, note that when $\alpha$ is large the loss probability $p_{\hat{s}}$ assessed by an $s$ insurer is close to $p_s$. If the latter is small, the acceptable deviations by an optimistic policyholder entail small profits and can, therefore, be ruled out even for high levels of risk aversion.

Corollary 1 shows how the critical thresholds of the policyholder's degree of risk aversion are affected by the level of insurers' information precision $\alpha$ and by the relative probability of dangerous versus safe environments $q$. We focus on the situation in which the loss is large, which makes for a more relevant case to study the effects of risk aversion.

**Corollary 1.** If the policyholder's loss $L$ is sufficiently large, $\underline{\beta}$ increases in $\alpha$ and $q$, while $\bar{\beta}$ increases in $\alpha$ and decreases in $q$.

Higher values of $\alpha$ and $q$ imply a higher and more precise assessment of risk by a $\hat{d}$ insurer, increasing $p_{\hat{d}}$. This determines an increase in the insurance premiums needed to ensure the participation of $\hat{d}$. As only a policyholder characterized by a greater degree of risk aversion is willing to pay more for insurance, $\underline{\beta}$ increases. Moreover, an increase in $\alpha$ or a decrease in $q$ induce a smaller and more precise assessment of risk $p_{\hat{s}}$ by a $\hat{s}$ insurer, so that an optimistic policyholder, that is, one that thinks that a deviation comes from a $\hat{s}$ insurer, is less willing to pay for insurance. This reduces insurers' incentives to deviate, implying an increase of $\bar{\beta}$.

A careful analysis of the conditions that need to be satisfied for noninformative equilibria to exist reveals interesting properties of such equilibria. First, one can show that there can be a multiplicity of equilibrium contracts $c^e \in \mathcal{E}^*$. The reason this is the case is best illustrated through Figure 1, which represents geometrically the key ingredients of our baseline model. All contracts in the shaded area can be noninformative equilibrium outcomes because they meet the

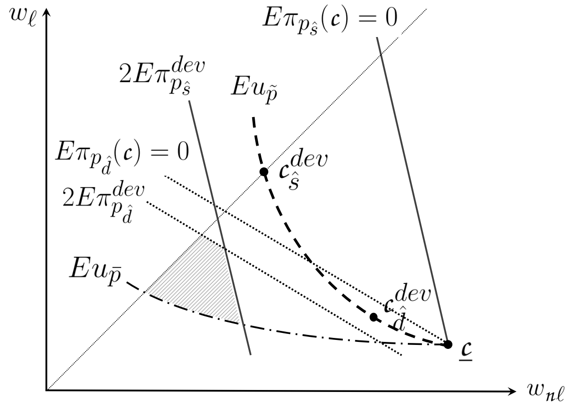


**Figure 1**
**Set of noninformative equilibria.**
**Alt text:** Graphical representation of the set of noninformative equilibria. The figure illustrates the insurers' zero isoprofit lines, the policyholder's binding participation constraint, and the insurers' no-deviation conditions for $\hat{s}$ and $\hat{d}$ defined in (6) taken with the equality. The set of noninformative equilibria lies in the area defined by the intersection between the policyholder's *ex ante* participation and the insurers' no-deviation conditions.

participation constraints of both the policyholder (represented by the indifference curve $Eu_{\bar{p}}$) and the insurers (lying below the zero isoprofit lines of both the $\hat{d}$ and $\hat{s}$ insurers), as well as the no-deviation conditions (6) (the lines $2E\pi^{dev}_{p_{\hat{s}}}$ and $2E\pi^{dev}_{p_{\hat{d}}}$).[16] Second, noninformative equilibria are always strictly profitable for $\hat{s}$ insurers, that is, $E\pi_{p_{\hat{s}}}(c^e)>0$ for all $c^e \in \mathcal{E}^*$. If a contract is profitable for a $\hat{d}$ insurer, then it is profitable for a $\hat{s}$ insurer as well, given the latter's lower expectation of risk. Moreover, also $\hat{d}$ insurers obtain strictly positive profits when the right-hand side of Condition (6) is strictly positive.[17]

Finally, noninformative equilibrium contracts $c^e \in \mathcal{E}^*$ may entail full insurance and, therefore, induce *ex ante* Pareto efficient allocations. Intuitively, higher levels of insurance are associated with higher profits, and thus are more likely to satisfy Condition (6). Differently from what is argued by most of the pertinent literature, this builds a case in favor of

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[16] In Figure 1, the policyholder's out-of-equilibrium participation constraint is represented by the indifference curve $Eu_{\tilde{p}}$.

[17] In Internet Appendix A.1, we show that there exist profitable noninformative equilibria that are robust to the intuitive criterion and the D1 criterion refinements. Intuitively, the deviations that, regardless of beliefs, would fail the refinements because they are profitable for $\hat{d}$, but not for $\hat{s}$ insurers entail a significant amount of underinsurance, due to the higher risk assessment of $\hat{d}$. Hence, in equilibrium, a risk-averse policyholder may be willing to pay higher premiums than the actuarially fair ones to increase coverage. From this perspective, the refinements rule out the most inefficient equilibria.

the efficiency of noninformative equilibria (i.e., risk pooling) in insurance markets.[18]

## 4. Informative Equilibria

We now investigate a situation in which each insurer's menu conveys information about his private signal. Then, the offers of the two types of insurers must differ, implying separating equilibria in the first stage. $\mathcal{C}^e_{\hat{s}}$ and $\mathcal{C}^e_{\hat{d}}$, $\mathcal{C}^e_{\hat{s}} \neq \mathcal{C}^e_{\hat{d}}$, denote the menus of informative equilibrium contracts offered by $\hat{s}$ and $\hat{d}$, respectively, and let $\mathcal{E}^{**}$ be the set of informative equilibrium allocations. By observing the two menus $\mathcal{C}^e_1$ and $\mathcal{C}^e_2$ (where $\mathcal{C}^e_i \in \left\{\mathcal{C}^e_{\hat{\theta}_i}\right\}_{\hat{\theta}_i \in \{\hat{s},\hat{d}\}}$, $i \in \{1,2\}$), the policyholder infers the signal profile $\sigma \in \Sigma = \{0,1,2\}$, assesses the corresponding loss probability $p_\sigma$, and uses this market information to select the contract. Market information is not available to insurers when they make their offers, as they are unaware of their competitor's type. In this respect, the problem is analogous to a screening problem with better informed agents, where the market information about the signal profile unambiguously identifies the policyholder's type. Insurers thus offer a menu of contracts, with one contract for each possible signal profile.[19] Given that $\sigma \in \Sigma = \{0,1,2\}$ denotes the number of insurers who receive signal $\hat{\theta}_i = \hat{s}$, the set of possible signal profiles when an insurer makes an offer depends on his signal. Hence, for a $\hat{s}$ insurer, the signal profile can only belong to the set $\Sigma_{\hat{s}} = \{1,2\} \subset \Sigma$. Conversely, for a $\hat{d}$ insurer, the set of possible signal profiles is $\Sigma_{\hat{d}} = \{0,1\} \subset \Sigma$. Any informative equilibrium allocation $(\mathcal{C}^e_{\hat{s}}, \mathcal{C}^e_{\hat{d}}) \in \mathcal{E}^{**}$ must entail a vector of contracts $\mathcal{C}^e_{\hat{\theta}_i} = \{c^e_{\hat{\theta}_i,\sigma}\}_{\sigma \in \Sigma_{\hat{\theta}_i}} \in \Omega^2$ for all $i \in \{1,2\}$ and $\hat{\theta}_i \in \{\hat{s},\hat{d}\}$. It follows that informative equilibria imply separation, both with respect to the insurers' types (through differences in the equilibrium menus) and with respect to the policyholder's types (through the offer of a menu of contracts, with one contract for each signal profile).

### 4.1 Characterization

To effectively address the screening problem, the menus $\mathcal{C}^e = (\mathcal{C}^e_{\hat{s}}, \mathcal{C}^e_{\hat{d}}) \in \mathcal{E}^{**}$ offered in any informative equilibrium must be incentive compatible for both the insurers and the policyholder, given equilibrium beliefs $p_{\mu^e(\mathcal{C}^e)} = p_\sigma$. This implies that the menu $\mathcal{C}^e_{\hat{\theta}_i} = \{c^e_{\hat{\theta}_i,\sigma}\}_{\sigma \in \Sigma_{\hat{\theta}_i}} \in \mathcal{C}^e$ is incentive compatible for the policyholder ($c^e_{\hat{\theta}_i,\sigma} = \operatorname{argmax}_{c \in \mathcal{C}^e_{\hat{\theta}_i}} Eu_{p_\sigma}(c)$)

[18] For a notable exception pointing in the same direction as our result, see Diamond (1992) or, in a much more general framework, Hirshleifer (1966).

[19] With a slight abuse of notation, we indicate with $\sigma$ both the policyholder's type and the signal profile.

and that the menus $\mathcal{C}^e = (\mathcal{C}^e_{\hat{s}}, \mathcal{C}^e_{\hat{d}})$ are incentive compatible for the insurers (truthful telling, $\mathcal{C}^e_{\hat{\theta}_i} = \arg\max_{\mathcal{C}^e_i \in \mathcal{C}^e} E\pi_{p_{\hat{\theta}_i}}(\mathcal{C}^e_i)$) for a given equilibrium belief system and for all $\hat{\theta}_i$. Proposition 2 lists the key characteristics of informative equilibria.[20]

**Proposition 2.** Any informative equilibrium is a vector of contract menus $\mathcal{C}^e = (\mathcal{C}^e_{\hat{s}}, \mathcal{C}^e_{\hat{d}}) \in \mathcal{E}^{**}$, with $\mathcal{C}^e_{\hat{s}} \neq \mathcal{C}^e_{\hat{d}}$, such that accepted contracts are (a) fully separating in each signal profile, and (b) incentive compatible both for the insurers and for the policyholder across all signal profiles. (c) In $\sigma = 1$, contract $c^e_{\hat{s},1}$ is accepted with probability one, it entails full insurance, strictly positive profits, and the same expected utility as $c^e_{\hat{d},1}$. (d) In $\sigma = 2$, all equilibrium contracts are of underinsurance and actuarially fair. (e) Beliefs are fully optimistic.

To convey the key insights of Proposition 2, we rely on Figure 2. In the equilibrium illustrated in the figure, type $\hat{s}$ insurers offer a menu that includes two contracts $c^e_{\hat{s},1}$ and $c^e_{\hat{s},2}$, that are fully separating, incentive compatible across states, and such that $c^e_{\hat{s},2}$ is actuarially fair. Type $\hat{d}$ insurers offer a menu including contracts $c^e_{\hat{d},0}$ and $c^e_{\hat{d},1}$, such that the policyholder is indifferent between $c^e_{\hat{s},1}$ and $c^e_{\hat{d},1}$. In $\sigma = 0$, both insurers offer $\mathcal{C}^e_{\hat{d}} = (c^e_{\hat{d},0}, c^e_{\hat{d},1})$, inducing the policyholder with belief $p_0$ to choose $c^e_{\hat{d},0}$ based on incentive compatibility. Analogously, in $\sigma = 2$ both insurers offer $\mathcal{C}^e_{\hat{s}} = (c^e_{\hat{s},1}, c^e_{\hat{s},2})$ and the policyholder chooses $c^e_{\hat{s},2}$ given beliefs $p_2$. In $\sigma = 1$, they offer $(\mathcal{C}^e_{\hat{d}}, \mathcal{C}^e_{\hat{s}})$, with the policyholder choosing $c^e_{\hat{s},1}$ given beliefs $p_1$.

Competition between insurers plays an important role in explaining Conditions (a)-(d) in the proposition. When both insurers receive signal $\hat{s}$ (i.e., $\sigma = 2$), their profits in contract $c^e_{\hat{s},2}$ are driven to zero by competition, as undercutting strategies would be accepted even by a fully optimistic policyholder. Furthermore, $c^e_{\hat{s},2}$ guarantees separation and binding incentive compatibility with $c^e_{\hat{s},1}$, which implies that informative equilibria must entail an inefficient outcome.[21] Matters become trickier for $\sigma = 1$, when both types of insurers are present. In this case, the contract $c^e_{\hat{d},1} \in \mathcal{C}^e_{\hat{d}}$ offered by insurer $\hat{d}$ is constrained by incentive compatibility with $c^e_{\hat{d},0}$ to be an underinsurance contract. This requirement does not apply to $\hat{s}$ insurers, as they are not in the market when $\sigma = 0$. Then, the only possible equilibrium contract $c^e_{\hat{s},1}$

[20] Refer to Proposition B.1 in Internet Appendix B for a formal characterization of informative equilibria in the presence of a generic number of insurance firms.

[21] In Figure 2, contract $c^e_{\hat{s},2}$ lies on the zero isoprofit line for $\sigma = 2$, and on the same indifference curve as $c^e_{\hat{s},1}$ for $\tilde{p} = p_1$. It is immediate to see that $c^e_{\hat{s},2}$ entails underinsurance.

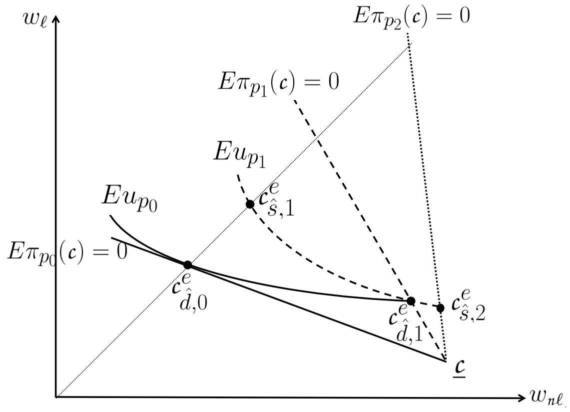


**Figure 2**
**Informative equilibrium allocation.**
**Alt text:** Graphical representation of an informative equilibrium allocation. The figure illustrates the zero isoprofit lines in signal profiles 0, 1 and 2, the relevant incentive compatibility constraints taken with the equality, and the equilibrium menus offered by the two types of insurer. Insurer $\hat{d}$ offers the incentive compatible menu $\mathcal{C}^e_{\hat{d}}=(c^e_{\hat{d},0},c^e_{\hat{d},1})$, in which contract $c^e_{\hat{d},0}$ is actuarially fair and of full insurance given $p_0$, and $c^e_{\hat{d},1}$ is actuarially fair and of underinsurance. Insurer $\hat{s}$ offers the incentive compatible menu $\mathcal{C}^e_{\hat{s}}=(c^e_{\hat{s},1},c^e_{\hat{s},2})$, where $c^e_{\hat{s},1}$ entails full insurance and is weakly preferred to $c^e_{\hat{d},1}$ given $p_1$, while $c^e_{\hat{s},2}$ is actuarially fair and of underinsurance.

maximizes $\hat{s}$'s profit in $\sigma=1$, conditional on being preferred to $c^e_{\hat{d},1}$, that is, $c^e_{\hat{s},1}=\arg\max_{c\in\Omega} E\pi_{p_1}(c)$ s.t. $Eu_{p_1}(c)\geq Eu_{p_1}(c^e_{\hat{d},1})$. This contract entails full insurance, it is strictly profitable, and it is associated with the same level of expected utility as $c^e_{\hat{d},1}\in\mathcal{C}^e_{\hat{d}}$ in $\sigma=1$, as illustrated in Figure 2. Furthermore, it is accepted with probability 1; that is, contract $c^e_{\hat{d},1}$ remains latent. The strict profitability of equilibrium outcomes in $\sigma=1$ is because $\hat{s}$ insurers have a competitive edge over $\hat{d}$ types. Such an advantage does not come from a more favorable estimation of risk than that of $\hat{d}$ insurers,[22] but rather from the requirement that the contract offered by a $\hat{d}$ insurer for $\sigma=1$ is incentive compatible with the one for $\sigma=0$. Given that $c^e_{\hat{d},1}$ is never accepted, $c^e_{\hat{d},0}$ entails nonnegative profits in $\sigma=0$. This implies that informative equilibria require different outcomes for the different signal profiles (i.e., they are fully separating). Finally, to explain why beliefs must be fully optimistic, note that, if this were not the case, a $\hat{s}$ insurer could deviate by offering a contract entailing a higher premium than $c^e_{\hat{s},1}$ in Figure 2. Such deviation would be accepted in $\sigma=1$ given the belief $p_{\tilde{\mu}}>p_1$.[23] Fully optimistic beliefs

[22] Under signal profile $\sigma=1$ all insurers estimate risk by using $p_1$.

[23] Noninformative equilibria exist for a larger set of out-of-equilibrium beliefs than informative ones. Indeed, the former simply requires that policyholders' beliefs are

imply that competition might not be effective for $\hat{d}$ insurers, because the policyholder's systematic underestimation of risk prevents insurer $\hat{d}$'s deviations. Then, $c^e_{\hat{d},0}$ may entail positive profits in $\sigma=0$.

### 4.2 Existence

When insurers' contract menus convey information about their private signals, the contract design problem has both a signaling and a screening component. Hence, the existence of informative equilibria requires that no insurer has an incentive to misreport his private signal (truthful telling) and to deviate by offering contract menus entailing cross-subsidization. Note that a $\hat{s}$ insurer may have an incentive to mimic a $\hat{d}$ insurer offering menu $\mathcal{C}^e_{\hat{d}}=(c^e_{\hat{d},0},c^e_{\hat{d},1})$ to obtain a benefit in $\sigma=1$. By doing so, the $\hat{s}$ insurer would sell with probability 1/2 contract $c^e_{\hat{d},0}$ that, being designed for a riskier signal profile, would be more profitable. Conversely, by adhering to his candidate equilibrium strategy, he would sell the less profitable contract $c^e_{\hat{s},1}$ with probability one. The truthful telling condition $\frac{1}{2}E\pi_{p_1}(c^e_{\hat{d},0})\leq E\pi_{p_1}(c^e_{\hat{s},1})$ guarantees that such deviation does not occur.

Given the nonconvexities embedded in the relevant constraints, a general existence result is difficult to prove. Proposition 3 below provides necessary and sufficient conditions, under the CARA specification (1), guaranteeing the existence of a specific equilibrium $\bar{\bar{C}}^e$ such that the $\hat{d}$ insurer offers the actuarially fair, full insurance contract in signal profile zero, and an underinsurance incentive compatible contract that satisfies with equality the policyholder's participation constraint in signal profile 1.[24] This equilibrium shares important similarities with that considered by Rothschild and Stiglitz (1976). Hence, it is most suitable to highlight the implications of competition between heterogeneously informed insurers for the profitability of equilibria.

**Proposition 3.** Under the CARA specification (1) of the policyholder's utility, an informative equilibrium $\bar{\bar{C}}^e$ exists if and only if there

sufficiently optimistic, while the latter entails fully optimistic beliefs. The more restrictive set of beliefs supporting informative equilibria might explain why insurers screen in life insurance markets, but not in annuity markets. Annuity and life insurance insure opposite mortality risks. From the perspective of an insurance company, a lower risk annuitant is one who has a lower chance of a long life, as opposed to a lower risk policyholder in life insurance. If the policyholder has optimistic beliefs about her life expectancy, our model predicts that informative equilibria may emerge in life insurance markets, but not in annuity markets.

[24] By incentive compatibility, the considered equilibrium implies no insurance in signal profile 2 and is a special instance of the class of equilibria considered in Figure 2. We show in Internet Appendix A.1 that this equilibrium is robust to the intuitive criterion, provided that $\alpha$ and $q$ are large enough. These conditions guarantee that the probability of $\sigma=1$ occurring is too low to make cross-subsidy deviations appealing for a $\hat{d}$ insurer. Ruling out any profitable deviation by $\hat{d}$ supports optimistic beliefs.

exist $\underline{\underline{\beta}}$ and $\bar{\bar{\beta}}$ such that $\underline{\underline{\beta}} \le \bar{\bar{\beta}}$, and $\beta \in \left[\underline{\underline{\beta}}, \bar{\bar{\beta}}\right]$. The set $\left[\underline{\underline{\beta}}, \bar{\bar{\beta}}\right]$ is non empty if $\alpha$ and $q$ are large.

Analogous to noninformative equilibria, the threshold $\underline{\underline{\beta}}$ is determined by truthful telling, while the upper bound is $\bar{\bar{\beta}} = \min\{\bar{\bar{\beta}}_{\hat{s}}, \bar{\bar{\beta}}_{\hat{d}}\}$, where $\bar{\bar{\beta}}_{\hat{s}}$ and $\bar{\bar{\beta}}_{\hat{d}}$ are the unique solutions of the relevant no deviation constraints for insurers $\hat{s}$ and $\hat{d}$, respectively, taken with equality.[25] The lower the degree of the policyholder's risk aversion is, the lower her willingness to pay for insurance and, therefore, the lower the equilibrium profit that the $\hat{s}$ insurer obtains when $\sigma = 1$. A smaller equilibrium profit, in turn, increases the $\hat{s}$ insurer's incentive to deviate by offering the contract that $\hat{d}$ would offer in signal profile 0, which explains the presence of a lower bound on $\beta$ for equilibrium existence. To understand why there also exists an upper bound on $\beta$, recall that a deviation in cross-subsidies increases profits in safer states, but it decreases them in riskier ones.[26] Hence, the $\hat{s}$ insurer may have an incentive to deviate from the equilibrium by offering more profitable contracts when $\sigma = 2$ and less profitable ones when $\sigma = 1$. For such deviation to be profitable, the increase in profits when $\sigma = 2$ should more than compensate the decrease in profits when $\sigma = 1$. As $\beta$ increases, the policyholder's willingness to pay increases, which in turn increases profits when $\sigma = 2$. Hence, $\beta$ should be sufficiently small for cross-subsidies deviations not to become profitable, which explains the existence of the upper bound $\bar{\bar{\beta}}$. Finally, the critical thresholds $\underline{\underline{\beta}}$ and $\bar{\bar{\beta}}$ for informative equilibria (as for noninformative equilibria) hinge upon the levels of signal precision $\alpha$ and of the probability of the $d$ environment, $q$, which play a crucial role in guaranteeing that $\bar{\bar{\beta}} > \underline{\underline{\beta}}$. If the probability of the $d$ environment is high and the signal is very precise, the policyholder's willingness to pay in $\sigma = 1$ is almost the same as that in $\sigma = 0$. In this case, the $\hat{s}$ insurer has an incentive to truthfully reveal his signal in $\sigma = 1$ for all $\beta$, ensuring that $\underline{\underline{\beta}}$ is zero. At the same time, when $q$ is large, the probability of $\sigma = 2$ (a state in which $\hat{s}$ may obtain profits from cross-subsidy deviations) is small, which ensures that $\bar{\bar{\beta}} > 0$.

## 5. The Role of Signal Precision

The level of signal precision affects the policyholder and insurers' estimation of risk, so that changes in $\alpha$ affect insurance premiums

[25] The full expressions of the truthful telling and no-deviation constraints are not particularly revealing and are relegated to the proof of the proposition in Internet Appendix D.

[26] The notion of a cross-subsidizing deviation generalizes that of a pooling deviation in the framework of Rothschild and Stiglitz (1976).

and profitability in equilibrium. While *ex ante*, that is, before the insurers receive their signals, the assessment of the loss probability $\bar{p}$ depends on the distribution of the underlying environment, but not on $\alpha$, the precision of the signal changes insurers' *ad interim* and *ex post* estimation of risk. Intuitively, when $\alpha$ increases, an insurer who receives signal $\hat{s}$ (resp. $\hat{d}$) reduces (resp. increases) his *ad interim* estimation of risk $p_{\hat{s}}$ (resp. $p_{\hat{d}}$). Furthermore, *ex post*, a more precise signal reduces the loss probability estimated when both insurers receive signal $\hat{s}$ (i.e., in signal profile 2), it increases it when they both receive signal $\hat{d}$ (i.e., in signal profile 0), and it has no effect on the loss probability when the two insurers receive opposite signals (i.e., in signal profile 1).

Although the effects of signal precision on loss probabilities are clearly determined, the existence of multiple equilibria makes the analysis of the impact of changes in information precision on equilibrium premiums and profits far more difficult. The key problem is that different sets of equilibria emerge for different values of $\alpha$, as the latter affects both agents' participation constraints and firms' profitability. Proposition 4 summarizes the effects of signal precision on the (maximum) levels of premiums for (a) noninformative and (b) informative equilibria, focusing on the equilibrium contract $c^{max}$ and on the menu of contracts $(\mathcal{C}_{\hat{s}}^{max},\mathcal{C}_{\hat{d}}^{max})$, respectively, that maximize *ex ante* profits. Note that, both for noninformative and for informative equilibria, the contracts guaranteeing the highest level of *ex ante* profits to insurers are also those entailing the largest premiums in all states in which profits are strictly positive (see the proof of Proposition 4 in Internet Appendix D).

**Proposition 4.** (a) Under the conditions guaranteeing that $c^{max}\in\mathcal{E}^*$ is a noninformative equilibrium, an increase in $\alpha$ does not affect the maximum level of equilibrium premiums.

(b) Under the conditions guaranteeing that $(\mathcal{C}_{\hat{s}}^{max},\mathcal{C}_{\hat{d}}^{max})\in\mathcal{E}^{**}$ is an informative equilibrium, an increase in $\alpha$ increases the maximum level of equilibrium premiums in signal profile 0, while it does not change it for the other signal profiles.

To understand part (a) of the proposition, observe that, in the noninformative equilibrium case, $c^{max}$ is the full insurance contract at which the policyholder's *ex ante* participation constraint is binding. Since this constraint depends on the *ex ante* loss probability, which is unaffected by $\alpha$, it follows that the precision of the signal has no effect on the maximum level of equilibrium premiums.

For part (b), note that $\mathcal{C}_{\hat{d}}^{max}\equiv(c_{\hat{d},0}^{max},\underline{c})$ and $\mathcal{C}_{\hat{s}}^{max}\equiv(c_{\hat{s},1}^{max},\underline{c})$. $c_{\hat{d},0}^{max}$ is the full insurance contract at which the policyholder's participation constraint in signal profile 0 is binding and that maximizes the $\hat{d}$ insurer's profit in that signal profile. Analogously, $c^{max}_{\hat{s},1}$ is the full insurance contract at which the policyholder's participation constraint in signal profile 1 is binding and that maximizes the $\hat{s}$ insurer's profit in that signal profile.[27] Finally, the autarky contract $\underline{c}$ – offered by $\hat{d}$ in signal profile 1 and by $\hat{s}$ in signal profile 2 – serves the purpose of guaranteeing that the relevant incentive compatibility constraints are met (see the proof of the proposition in Internet Appendix D). It is then easy to see that, in an informative equilibrium, a higher $\alpha$ increases the loss probability estimated in signal profile 0. This increases the premium that the policyholder is willing to pay in that signal profile (i.e., the maximum premium that can be charged by insurers consistently with the policyholder's participation). In signal profile 1, instead, the precision of the signal does not affect the estimation of the loss probability $p_1$. Then, the maximum premium that can be charged to the policyholder does not depend on $\alpha$. Finally, in signal profile 2, $\alpha$ has no effect on the equilibrium outcome as the latter coincides with autarky.

The effects of $\alpha$ on equilibrium profits are studied in the following proposition.

**Proposition 5.** (a) Under the conditions guaranteeing that $c^{max} \in \mathcal{E}^*$ is a noninformative equilibrium, an increase in $\alpha$ increases (resp. decreases) the maximum level of the $\hat{s}$ (resp. $\hat{d}$) insurer's equilibrium profit.

(b) Under the conditions guaranteeing that the menu of contracts $(\mathcal{C}^{max}_{\hat{s}}, \mathcal{C}^{max}_{\hat{d}}) \in \mathcal{E}^{**}$ is an informative equilibrium, an increase in $\alpha$ decreases (resp. increases) the maximum equilibrium profit for the $\hat{s}$ (resp. $\hat{d}$) insurer.

Proposition 5 states that the precision of the signal has opposite effects on the maximum level of profits achievable by $\hat{s}$ and $\hat{d}$ insurers, with the sign of these effects depending on whether one focuses on noninformative or informative equilibria. Proposition 4 shows that in noninformative equilibria $\alpha$ does not affect the policyholder's maximum premium. Nonetheless, a more precise signal $\alpha$ reduces the risk assessment for a $\hat{s}$ insurer, thus increasing his profit in equilibrium. Conversely, a higher $\alpha$ induces the $\hat{d}$ insurer to assess a higher level of risk, thus decreasing the maximum expected profit. An opposite result holds for informative equilibria, in which a change of $\alpha$ also affects the probability of each signal profile occurring. On the one hand, when information is more

[27] Since in equilibrium only signal profiles 0 and 1 can be profitable, if an equilibrium menu of contracts contains the contracts described above, then such a menu maximizes *ex ante* profits.

precise, it becomes more unlikely that the two firms receive different signals (i.e., that signal profile 1 occurs). Since in signal profile 1 a change of $\alpha$ does not affect the policyholder's maximum premium (by Proposition 4), an increase of $\alpha$ does not affect the *ex post* profit of $\hat{s}$. However, it decreases the expected profit of a $\hat{s}$ insurer by reducing the probability that signal profile 1 occurs, given that the latter is the only signal profile in which insurer $\hat{s}$ obtains positive profits. On the other hand, a higher signal precision increases the probability of signal profile 0 – the only profitable one for a $\hat{d}$ insurer – occurring, as well as the maximum level of premiums in this signal profile (see again Proposition 4). Both effects increase the $\hat{d}$ insurer's profits when $\alpha$ increases.

A case of special interest is that in which the precision of insurers' private information converges to one, allowing for a direct comparison of our results with those of Villeneuve (2005), who assumes perfectly informed insurers. The following corollary of Proposition 2 shows that informative equilibria entail an inefficient outcome also in this limit case.

**Corollary 2.** The equilibrium contract implemented in signal profile 2, $c^e_{\hat{s},2} \in \mathcal{C}^e$ with $\mathcal{C}^e \in \mathcal{E}^{**}$, does not converge to full insurance for $\alpha \to 1$.

In the limit for $\alpha$ converging to 1, the relevant loss probabilities are $\bar{p}$ for signal profile 1 and $p_s$ for signal profile 2. Since $p_s$ does not converge to $\bar{p}$, and $c^e_{\hat{s},2}$ must be incentive compatible with $c^e_{\hat{s},1}$ by Proposition 2, $c^e_{\hat{s},2}$ must be inefficient. While Villeneuve (2005) finds that with identical, perfectly informed insurers, informative equilibria can support efficient allocations, Corollary 2 shows that the screening problem that arises when heterogeneously informed insurers compete among themselves does not allow for fully efficient informative outcomes.

## 6. Extensions

### 6.1 Withdrawable contracts

The model of Section 2 assumes that insurers commit to their offers and cannot withdraw them based on their competitor's contract offers. We now extend the analysis by allowing withdrawable contracts in the spirit of Wilson (1977) and Hellwig (1987). This requires a change in the timing of the baseline model, such that insurer $i$ can withdraw one or more contracts from the menu $\mathcal{C}_i$ after having observed his competitor's offers.[28] $\mathcal{C}'_i$ denotes the menu of remaining contracts. The

[28] This timing is consistent with that proposed by Mimra and Wambach (2019), while it differs from that adopted by Hellwig (1987), who assumes that insurers can withdraw a contract after having observed the policyholder's choice, which reveals her type.

policyholder updates her beliefs and selects one contract $c \in \mathcal{C}' = (\mathcal{C}'_1, \mathcal{C}'_2)$ in the updated menus, or she remains uninsured by receiving contract $\underline{c}$. The following proposition shows how our results extend to the augmented setup with withdrawable contracts.

**Proposition 6.** Assume that contracts are withdrawable.

(1) (Noninformative equilibria). All noninformative equilibria $c^e \in \mathcal{E}^*$ under nonwithdrawable contracts are equilibria also under withdrawable contracts.

(2) (Informative equilibria). If $\beta \in [\underline{\underline{\beta}}, \bar{\bar{\beta}}_{\hat{d}}]$, an informative equilibrium exists, (a) it entails fully efficient outcomes in both signal profiles 1 and 2, but not necessarily in signal profile 0, (b) it is actuarially fair in signal profile 2, (c) it is strictly profitable for $\hat{s}$, and (d) it may be profitable for $\hat{d}$.

The equilibria in Part (1) of Proposition 6 entail that both insurer types offer the same contract $c^e$.[29] Under such equilibria, all insurers adopt the same withdrawal strategy, requiring that only contracts entailing negative profits be withdrawn (see the proof of Proposition 6). Given that insurers adopt the same strategy both at the offer stage and at the withdrawal stage, the policyholder cannot infer information about their signals. In this respect, the possibility of withdrawal does not affect the equilibria we focus on in our baseline model. This follows from the fact that despite the presence of an additional stage, the policyholder has exactly the same information in both setups. The existence of only one type of policyholder prevents the possibility of cream-skimming. This is important because it marks a clear difference with respect to the insurance models with adverse selection in which there are different types of policyholders (e.g., Hellwig 1987; Wilson 1977). In these papers, the possibility of contract withdrawal works as a threat against the possibility of "cream-skimming" deviations. This is what sustains the existence of pooling equilibria, which however always involve zero *ex ante* profits due to the competitive mechanism. Conversely, in our setup noninformative equilibria are sustained by the fact that the policyholder holds optimistic beliefs off the equilibrium path, which determines a

Nonetheless, in our framework, as in Hellwig (1987), insurers can withdraw their contracts after observing all initial offers and thus share the same assessment of the policyholder's risk.

[29] Under the assumption that a contract is withdrawn only when it entails strictly negative profits, it can be shown that those characterized in the proposition are the only noninformative equilibria. Not surprisingly, this is not the case when the two insurers' types may choose to behave differently when indifferent between withdrawing a contract or not, as this would signal their type and affect the policyholder's beliefs.

failure of the competitive mechanism and the possibility of positive profits in equilibrium, as highlighted in our baseline model.

To understand Part (2) of the proposition, note that the withdrawal of a contract occurs after insurers observe all offers in the initial stage and, hence, can learn the policyholder's type. This implies that insurers may initially offer a menu that includes several contracts and then withdraw loss-making contracts based on their updated assessment of the policyholder's risk. Accordingly, incentive compatibility does not need to hold in $\sigma = 2$, where Bertrand competition results in fully efficient, actuarially fair outcomes that maximize the policyholder's expected utility. Note that the efficiency of equilibrium outcomes in $\sigma = 2$ prevents cross-subsidy deviations by $\hat{s}$, which explains why the existence of informative equilibria under withdrawable contracts is independent of $\bar{\bar{\beta}}_{\hat{s}}$. Conversely, the role of out-of-equilibrium beliefs in hindering competition and the existence of a competitive edge for type $\hat{s}$, respectively, explain why the equilibrium outcomes in signal profiles 0 and 1 under withdrawable contracts are analogous to those derived under nonwithdrawable contracts.

Overall, the key insight of Proposition 6 is that under noninformative equilibria observing competitors' offers does not convey any information, so that no incentives to withdraw contracts arise. This is not the case for informative equilibria, for which withdrawing contracts improves the efficiency of equilibrium outcomes. This implies that the outcomes in Proposition 2 can no longer be supported as informative equilibria under the same conditions established in Proposition 3 for nonwithdrawable contracts.

### 6.2 Market concentration

We now extend the analysis of the duopolistic model in Section 2 to an oligopolistic industry with $N$ firms, $N > 2$, and investigate how market concentration affects our results.[30]

Focusing first on noninformative equilibria, the *ex ante* and *ad interim* loss probabilities do not depend on the number of firms. Hence, $N$ does not affect the participation constraints of the insurers or of the policyholder, although it makes the no-deviation constraint more stringent. Denoting with $\mathcal{E}_N^*$ the set of noninformative equilibria with $N$ firms, since each firm sells the equilibrium contract $c^e \in \mathcal{E}_N^*$ with probability $1/N$, it is immediate to write a condition analogous to (6) for the two-firm case in the $N-$firm framework, that is,

$$\frac{1}{N} E\pi_{p_{\hat{\theta}}}(c^e) \geq E\pi_{p_{\hat{\theta}}}\left(c_{\hat{\theta}_i}^{dev}\right), \tag{7}$$

[30] All technical details on the characterization of the equilibria for the $N$-firm case are in Internet Appendix B.

for all $\hat{\theta}_i \in \left\{\hat{s},\hat{d}\right\}$, where $c^{dev}_{\hat{\theta}_i}$ is the solution of Problem (5). Condition (7) implicitly defines an upper bound to the number of insurers consistent with the existence of a noninformative symmetric equilibrium $c^e$, which we denote as $\bar{N}(c^e)$ and which is specific to the equilibrium outcome $c^e$. By inspection of (7), it is also easy to see that the set of noninformative equilibria shrinks as the industry becomes more dispersed, given that the less profitable contracts are no longer sustainable as equilibria.[31] Since noninformative equilibria are not unique, we focus on the equilibrium entailing the highest possible number of firms $\bar{N}$ among all possible noninformative equilibria, that is, $\bar{N} = \max_{c^e \in \mathcal{E}^*_N} \bar{N}(c^e)$. From (7), it follows that the larger are the expected profits associated with the equilibrium contract, the larger the upper bound to the number of firms operating in the industry is, which immediately yields the following proposition.

**Proposition 7.** There exists $\bar{N} \in \mathbb{R}_+$ such that, for all $N > \bar{N}$, $\mathcal{E}^*_N$ is empty. For all $N < \bar{N}$, the minimum expected profit in the set of contracts $c^e \in \mathcal{E}^*_N$ increases with $N$.

Proposition 7 establishes the possibility of a nonstandard negative relationship between insurance profits and market concentration. Since the set of noninformative equilibrium outcomes grows when the number of firms decreases, an increase in market concentration is not necessarily welfare detrimental for customers.

Additionally, the characterization of informative equilibria closely follows the discussion of the two-firm case, although increasing the number of firms also increases the number of signal profiles (see Proposition B.1 in Internet Appendix B). The conditions characterizing informative equilibria imply the existence of an upper bound to the number of firms consistent with the equilibrium, as shown in the following proposition, in which $\mathcal{E}^{**}_N$ denotes the set of informative equilibria.[32]

**Proposition 8.** There exists $\bar{\bar{N}} \in \mathbb{R}_+$ such that $\mathcal{E}^{**}_N$ is empty for all $N > \bar{\bar{N}}$.

When the number of firms is large, signal profiles in which the majority of firms received the safe signal entail that the safe environment

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[31] The characterization of the contract $c^{dev}_{\hat{\theta}_i}$ in (7) depends on the loss probabilities $p_{\hat{\theta}}$ and $p_{\mu^e(c)}$, defined in Problem (5), which are unaffected by $N$.

[32] One can show that the maximum number of firms that is consistent with an informative equilibrium must also have a lower bound to induce insurers to truthfully disclose their information.

is likely. This favors cream-skimming deviations aimed at attracting the policyholder to the safest signal profiles, in the spirit of the pooling deviations arising in the Rothschild and Stiglitz environment.

Jointly taken, Propositions 7 and 8 indicate that equilibria fail to exist when the number of firms in the industry becomes too large, which is consistent with the evidence of concentrated insurance markets. Although market concentration may be due to several factors, among which bankruptcy constraints or risk pooling are probably the most relevant, our results highlight how the number of firms must be bounded even in the absence of such frictions.

### 6.3 Policyholder's private information

In Internet Appendix C, we extend the model of Section 2 by assuming that risk depends both on the general riskiness of the environment ($d$ or $s$) – on the assessment of which insurers have an imperfect informational advantage – and on the specific policyholder's personal characteristics – that are perfectly known by the policyholder but unobserved by the insurers. Proposition C.1 in the Internet Appendix shows that, under appropriate restrictions, there exist noninformative equilibria entailing risk pooling among the policyholder types when the precision of the insurers' signals converges to one. This specification allows for a direct comparison of our setup with that of Rothschild and Stiglitz (1976). Because of the presence of a second layer of asymmetric information, that is, the policyholder's private information on the idiosyncratic components of her own risk, one could expect the emergence of cream-skimming deviations, analogous to Rothschild and Stiglitz (1976). We show that this is not necessarily the case. Consistent with signaling games, the choice of out-of-equilibrium beliefs, by allowing sufficient degrees of freedom, may sustain pooling equilibria, preventing such deviations.

A special case worth considering is that in which insurers' private signals are *homogeneous* and the policyholder has private information on her idiosyncratic risk (see Internet Appendix C.2 for analytical details). In this case, both a signaling and a screening problem are present, similar to our baseline model, although the signal profile can only be $\sigma = 0$ or $\sigma = 2$. Proposition C.2 shows that under appropriate conditions, there exist both noninformative equilibria entailing risk pooling among insurers' types, and informative equilibria in which $\hat{s}$ insurers offer the actuarially fair Rothschild-Stiglitz menu.[33] When insurers receive homogeneous signals, a profitable noninformative contract pooling all type profiles can be supported in equilibrium. As in the baseline framework with

[33] These results extend to a two-sided information framework the results obtained by Villeneuve (2005) in a setup with privately and symmetrically informed insurers.

heterogeneously informed insurers of Section 2, the result follows from the role of optimistic beliefs in hindering competition and making undercutting deviations unacceptable. However, having symmetrically informed insurers implies that competition works effectively under informative equilibria, leading to an actuarially fair outcome for $\hat{s}$ insurers. Because of the policyholder's private information, such an outcome corresponds to the equilibrium menu of Rothschild and Stiglitz (1976). Because of the presence of bilateral asymmetric information, principals face a trade-off, which has already been highlighted in a monopolistic setting by Brunnermeier, Lamba, and Segura-Rodriguez (2020). By giving up their informational advantage, insurers can increase discrimination through a richer set of contracts. However, under competition, the revelation of insurers' information erodes their informational rent when their information is homogeneous. Only the heterogeneity of insurers' signals allows them to earn a rent from their private information by making them unable to undercut each other without violating incentive compatibility.

## 7. Concluding Remarks

Improvements in data collection and the rise of computing power in recent years have significantly affected the insurance industry, allowing insurers to estimate risk more precisely than in the past. From this perspective, the traditional information asymmetry affecting the insurance sector may flip over to the other side of the market. Because of their expertise and access to relevant statistics, insurers are better equipped than policyholders to accurately assess the level of risk associated with a specific environment. Nevertheless, a precise assessment of risk is not straightforward even for practitioners, and some heterogeneity in insurers' evaluations should be expected, possibly related to access to different data warehouses or the availability of alternative predictive algorithms.

We contribute to the signaling literature on competitive insurance markets by showing that, in a setup in which insurers have an imperfect informational advantage over policyholders, equilibrium contracts are always strictly profitable, at least for some insurers. Furthermore, even in a competitive environment, equilibrium does not necessarily imply full disclosure of insurers' information and there is an upper bound to the number of firms that is consistent with the existence of an equilibrium. Although our analysis is purely theoretical, these results are consistent with the available empirical evidence, which highlights the abundance of unused observables in the definition of insurance contracts, as well as the profitability and concentration of insurance markets. Nevertheless, the conditions under which an equilibrium exists in our setup are fairly restrictive, as is often the case in the pertinent literature on competitive insurance models. Hence, a deeper understanding of the matter of equilibrium existence remains needed.

Existing models of insurance markets focus on either insurers or policyholders who hold perfect information about risk. A more realistic representation should allow both parties to hold private information. Our analysis, by combining features of pure-signaling and pure-screening models in a competitive framework, is only a first step in this direction, but it is enough to suggest novel insights could be uncovered. By looking at the effects of the precision of insurers' information, we find that more precise signals affect equilibrium premiums and profits in nontrivial ways, with effects that depend on whether insurers' information is revealed through their offers. Nonetheless, an important issue that remains to be addressed is related to insurers' incentives to invest in the precision of their information. Further research is needed to fully assess the optimality of information acquisition by insurers and its implications for market outcomes.

**Code Availability:** No new code was generated in support of this research.

**References**


Abraham, I., S. Athey, M. Babaioff, and M. D. Grubb. 2020. Peaches, lemons, and cookies: Designing auction markets with dispersed information. *Games and Economic Behavior* 124:454–77.

Attar, A., and A. Chassagnon. 2009. On moral hazard and nonexclusive contracts. *Journal of Mathematical Economics* 45:511–25.

Attar, A., T. Mariotti, and F. Salanié. 2011. Nonexclusive competition in the market for lemons. *Econometrica* 79:1869–918.

Balkenborg, D., and M. Makris. 2015. An undominated mechanism for a class of informed principal problems with common values. *Journal of Economic Theory* 157:918–58.

Banks, J. S., and J. Sobel. 1987. Equilibrium selection in signaling games. *Econometrica* 55:647–61.

Bisin, A., and D. Guaitoli. 2004. Moral hazard and non-exclusive contracts. *RAND Journal of Economics* 2:306–28.

Bouvard, M. 2014. Real option financing under asymmetric information. *Review of Financial Studies* 27:180–210.

Brunnermeier, M. K., R. Lamba, and C. Segura-Rodriguez. 2020. Inverse selection. Mimeo, Princeton University.

Cabral, M., M. Geruso, and N. Mahoney. 2018. Do larger health insurance subsidies benefit patients or producers? Evidence from Medicare advantage. *American Economic Review* 108:2048–87.

Cawley, J., and T. Philipson. 1999. An empirical examination of information barriers to trade in insurance. *American Economic Review* 89:827–46.

Chiappori, P.-A., and B. Salanié. 2000. Testing for asymmetric information in insurance markets. *Journal of Political Economy* 108:56–78.

Cho, I.-K., and D. M. Kreps. 1987. Signaling games and stable equilibria. *Quarterly Journal of Economics* 102:179–221.

Dafny, L., M. Duggan, and S. Ramanarayanan. 2012. Paying a premium on your premium? Consolidation in the US health insurance industry. *American Economic Review* 102:1161–85.

Dauda, S. 2018. Hospital and health insurance markets concentration and inpatient hospital transaction prices in the US health care market. *Health Services Research* 53:1203–26.

DeMarzo, P. M., and D. M. Frankel. 2022. Mechanism design with an informed principal: Extensions and generalizations. Mimeo.

DeMarzo, P. M., D. M. Frankel, and Y. Jin. 2021. Portfolio liquidity and security design with private information. *Review of Financial Studies* 34:5841–85.

Diamond, P. 1992. Organizing the health insurance market. *Econometrica* 60:1233–54.

Dosis, A. 2022. On the informed principal model with common values. *RAND Journal of Economics* 53:792–825.

Economist. 2017. The coming revolution in insurance. `https://www.economist.com/finance-and-economics/2017/03/09/the-coming-revolution-in-insurance`. March 9.

Emons, W. 1997. Credence goods monopolists. *Journal of Industrial Economics* 45:345–54.

Epstein, L. G., and M. Peters. 1999. A revelation principle for competing mechanisms. *Journal of Economic Theory* 88:119–60.

Finkelstein, A., and J. Poterba. 2014. Testing for asymmetric information using unused observables in insurance markets: Evidence from the U.K. annuity market. *Journal Risk and Insurance* 81:709–34.

Fombaron, N. 1997. No-commitment and dynamic contracts in competitive insurance markets with adverse selection. Working Paper, THEMA.

Galperti, S. 2015. Common agency with informed principals: Menus and signals. *Journal of Economic Theory* 157:648–67.

Gorbenko, A. S., and A. Malenko. 2011. Competition among sellers in securities auctions. *Journal of Finance* 66:1729–70.

Guttman, I., O. Kadan, and E. Kandel. 2010. Dividend stickiness and strategic pooling. *Review of Financial Studies* 23:4455–95.

Hartman-Glaser, B., and B. Hebert. 2020. The insurance is the lemon: Failing to index contracts. *Journal of Finance* 75:463–506.

Hellwig, M. 1987. Some recent developments in the theory of competition in markets with adverse selection. *European Economic Review* 31:319–25.

Hertzendorf, T., and P. B. Overgaard. 2001. Monopoly pricing of experience goods. *International Journal of Industrial Organization* 19:1083–100.

Hirshleifer, J. 1966. Investment decision under uncertainty: Applications of the state-preference approach. *Quarterly Journal of Economics* 80:252–77.

Hortaçsu, A., and C. Syverson. 2004. Product differentiation, search costs, and competition in the mutual fund industry: A case study of S&P 500 index funds. *Quarterly Journal of Economics* 119:403–56.

Hyman, D. A., and W. E. Kovacic. 2004. Monopoly, monopsony, and market definition: An antitrust perspective on market concentration among health insurers. *Health Affairs* 23:25–8.

Inderst, R. 2005. Matching markets with adverse selection. *Journal of Economic Theory* 121:145–66.

Koessler, F., and V. Skreta. 2016. Informed seller with taste heterogeneity. *Journal of Economic Theory* 165:456–71.

———. 2019. Selling with evidence. *Theoretical Economics* 14:345–71.

Koijen, R., and M. Yogo. 2015. The cost of financial frictions for life insurers. *American Economic Review* 105:445–75.

———. 2016. Shadow insurance. *Econometrica* 84:1265–87.

Martimort, D., and W. Sand-Zantman. 2006. Signalling and the design of delegated management contracts for public utilities. *RAND Journal of Economics* 37:763–82.

Martimort, J., and L. Stole. 2002. The revelation and delegation principles in common agency games. *Econometrica* 70:1659–73.

Maskin, E., and J. Tirole. 1990. The principal-agent relationship with an informed principal, I: Private values. *Econometrica* 58:379–410.

———. 1992. The principal-agent relationship with an informed principal, II: Common values. *Econometrica* 60:1–42.

Milgrom, P. R., and R. J. Weber. 1982. A theory of auctions and competitive bidding. *Econometrica* 50:1089–122.

Mimra, W., and A. Wambach. 2019. Contract withdrawals and equilibrium in competitive markets with adverse selection. *Economic Theory* 67:875–907.

Morellec, E., and N. Schurhoff. 2011. Corporate investment and financing under asymmetric information. *Journal of Financial Economics* 99:262–88.

Myerson, R. 1983. Mechanism design by an informed principal. *Econometrica* 51:1767–98.

Ottaviani, M., and A. Prat. 2001. The value of public information in monopoly. *Econometrica* 69:1673–83.

Philippon, T., and V. Skreta. 2012. Optimal interventions in markets with adverse selection. *American Economic Review* 102:1–28.

Robinson, J. 2004. Consolidation and the transformation of competition in health insurance. *Health Affairs* 23:11–24.

Rothschild, M., and J. E. Stiglitz. 1976. Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. *Quarterly Journal of Economics* 90:629–49.

Seog, S. 2009. Insurance markets with differential information. *Journal of Risk and Insurance* 76:279–94.

Severinov, S. 2008. An efficient solution to the informed principal problem. *Journal of Economic Theory* 141:114–33.

Sommer, D. 2017. The impact of firm risk on property-liability insurance prices. *Journal of Risk and Insurance* 63:501–14.

Villeneuve, B. 2005. Competition between insurers with superior information. *European Economic Review* 49:321–40.

Vives, X. 2011. Strategic supply function competition with private information. *Econometrica* 79:1919–66.

Wilson, C. 1977. A model of insurance markets with incomplete information. *Journal of Economic Theory* 16:167–207.

Wolinsky, A. 1995. Competition in a market for informed experts' services. *RAND Journal of Economics* 26:380–98.