Forecasting Network Traffic: A Survey and Tutorial with Open-Source Comparative Evaluation

Thermal degradation in Carrara marbles as the cause of deformation of cladding slabs

A. Spagnoli
Dipartimento di Ingegneria Civile, dell'Ambiente del Territorio e Architettura, Università di Parma
spagnoli@unipr.it

M. Migliazza, M. Zucali
Dipartimento di Scienze della Terra "Ardito Desio", Università di Milano

A.M. Ferrero
Dipartimento di Scienze della Terra, Università di Torino

ABSTRACT. Marble slabs, typically used as façade panels to externally cover buildings, might permanently deform after a certain time of environmental exposure. This phenomenon, called bowing, is generally accompanied by a reduction of strength which increases with increasing degree of bowing. In the present paper, a theoretical model to calculate the progressive bowing of marble slabs submitted to temperature cycles is briefly recalled and applied to a specific Carrara marble sample. The marble is investigated by a microscopic analysis of thin sections cut along three orthogonal directions. The digital photographs are treated by an image analysis code which is capable of extracting grain size and shape distributions. In this way the anisotropic microstructure of the marble is quantified and taken into account in the numerical analyses. The influence of size distribution of grains as well as of their distribution of optic axis orientation on the slab bowing is discussed with the attempt of offering a quantitative tool for a better understanding of in situ bowing measurements.

KEYWORDS. Bowing; Crack propagation; Fracture mechanics; Marble; Thermal cycles.

INTRODUCTION

Marble slabs are often used as façade panels to externally cover buildings. They are subjected to different actions that deteriorate the material, including: temperature (daily and seasonal excursions, through-thickness gradient), mechanical loads (wind, self-weight), chemical attacks (acid rain), humidity changes. Temperature may induce stresses due to thermal expansion (restraint effects of the anchorage system, nonlinear temperature fields, nonuniform thermal expansion). One visible phenomenon linked to deterioration of marble is bowing, which is characterised by permanent out-of-plane deflections. Bowing is generally accompanied by an overall reduction of strength which increases with increasing degree of bowing, while at the material microstructural level the bowing is characterized by a decohesion of calcite grains.

In order to understand the phenomenon of bowing in marble slabs, several experimental and theoretical studies [1-8] have been carried out, starting with the pioneering work of Rayleigh [9]. The results of these studies show that the strength of

marble after environmental exposition decreases due to grain decohesion. For instance, Royer-Carfagni [3] showed that thermal action produces self-equilibrated stress states at calcite grain (whose size ranges typically between 100 and 500 μm) interfaces, which are responsible of progressive damage in the material leading to initiation and propagation of intergranular cracks.
In the present paper, following some recent works by the authors [10-11], a theoretical model to estimate the progressive bowing and the thermal fatigue of marble slabs submitted to temperature cycles is briefly recalled and applied to a Carrara marble sample whose microstructure is experimentally analysed in details. The model, developed within the framework of LEFM, takes into account the mechanical microstructural characteristics of the marble as well as the actual cyclic temperature field in the material. The slabs are subjected to a thermal gradient along their thickness as well as to thermal fluctuation on the two sides of the slab due to daily and seasonal temperature excursions. This thermal action causes a stress field which can locally determine microcracks due to decohesion of grains. Stress intensification near the cracks occurs and leads to crack propagation in the slab. Such crack propagation under thermal actions is evaluated and the corresponding deflection (bowing) is calculated. A Monte Carlo simulation, where the orientation distribution of grain optic axis along the slab thickness is varied randomly, is performed in order to quantify the scatter of bowing evolution.

THE MECHANICS OF THERMAL DEGRADATION

A one-dimensional theoretical model of a marble slab is considered. The thickness of the slab is h, while x and z are the through-thickness and longitudinal axis, respectively (x = 0 corresponds to the inner surface of the slab). The kinematic assumptions are those of beam theory. The material is mechanically linear elastic, homogeneous and isotropic, while the thermal expansion is heterogeneous along the panel thickness. The resulting longitudinal normal stress is

$$\sigma_z(x,t) = \frac{(1-\nu)E}{(1-2\nu)(1+\nu)}\varepsilon_z(x,t) - \frac{\alpha_z(x)E}{(1-2\nu)}\Delta T(x,t) \tag{1}$$

where ΔT(x, t) is the temperature function of time t and space x. Considering the relevant case of a slab with hinged ends, we have $\varepsilon_z(x,t) = A(t)x + B(t)$ where A and B, functions of time, are determined by the condition of axial force and bending moment being equal to zero at the slab ends.
As has been mentioned in the Introduction section, the thermal expansion of calcite grains is anisotropic. In the present model, where thermal expansion is assumed to be heterogeneous and hence the coefficient α_z is a function of the through-thickness coordinate x, the thermal expansion heterogeneity is linked to the aforementioned thermal anisotropy of calcite grains. Now let us assume that the thermal expansion of calcite grains is orthotropic (1-2 are the material thermal expansion axes, characterized by the coefficients α_1 and α_2, respectively), the longitudinal thermal expansion coefficient along the z-axis α_z is obtained from

$$\alpha_z(x) = \alpha_1 \cos^4 \beta(x) + \alpha_2 \sin^4 \beta(x) \tag{2}$$

where β is the angle formed by the material thermal expansion axes with the longitudinal axis z.
In the light of the above, the thermal expansion heterogeneity (see $\alpha_z(x)$) is due to the different orientation (see $\beta(x)$) of the material thermal expansion axes of each grain. Therefore $\alpha_z(x)$ is hereafter assumed to be a stepwise varying function where a jump in such a function occurs at each calcite grain boundary. Assuming a geometrically simple grain arrangement for our model, we might have a stack of layers with different values of α_z in each layer, where the layer thickness might be taken as the calcite grain mean dimension *d*. In the following, we consider a random distribution of thermal axis orientation in each grain with some specific statistical characteristics.
Under environmental exposure a diffuse cracking, mainly developing at the calcite grain boundaries, can take place at the external surface of the marble slab. Such a diffuse cracking can be incorporated in the present model to calculate the consequent convex bowing of the slab due to thermal loading. Intergranular cracking due to decohesion of calcite grains are treated as equivalent multiple edge cracks. To this end a crack density parameter *n*, corresponding to a number of equivalent external edge cracks, can be defined. Assuming hence that intergranular cracking occurs, the crack density

parameter can be correlated with the specific surface S_s of the grains (i.e. the total surface area of the grains per unit volume of material) as follows for a slab of length L (Fig. 1)

$$n = \frac{S_s \cdot L}{2} \tag{3}$$

Consider now an external edge crack of depth a submitted to Mode I (opening) load due to the thermal stress $\sigma_z(x,t)$ of Eq. 1 ($\sigma_z(x,t)$ is the stress acting in the solid slab along the line where the crack is assumed to be located). The Mode I Stress Intensity Factor (SIF) of the crack can be calculated as follows [12]

$$K_I(a,t) = \frac{2}{\sqrt{\pi a}} \frac{1}{\left(1-\frac{a}{h}\right)^{3/2}} \int_0^a \sigma_z(a',t) \frac{G\left(\frac{a'}{a},\frac{a}{h}\right)}{\sqrt{1-\left(\frac{a'}{a}\right)^2}} da' \tag{4}$$

where $G\left(\frac{a'}{a},\frac{a}{h}\right)$ is a dimensionless function of the crack geometry (a' = h–x).

At this point, in might be worth considering the influence of calcite grain shape on the mechanical performances of marble (marble microstructures do present sometimes different geometric features ranging from the extreme cases of xenoblastic textures, characterized by wavy contours, and homoblastic textures, characterized by gently curving boundaries). Generally speaking, xenoblastic marbles tend to exhibit a higher resistance to mechanical actions [3]. In order to consider such an aspect, for two-dimensional models it might be appropriate to use a shape parameter like the parameter ξ equal to area/(perimeter)^2, which has an upper limit corresponding to the value for a smooth circle ($\xi_{max} = 1/(4\pi) = 0.080$) and tends to zero ($\xi_{min} = 0$) for extremely rough boundaries. Typically, homoblastic marbles are characterized by $\xi >\approx 0.045$ and xenoblastic marbles by $\xi <\approx 0.030$.

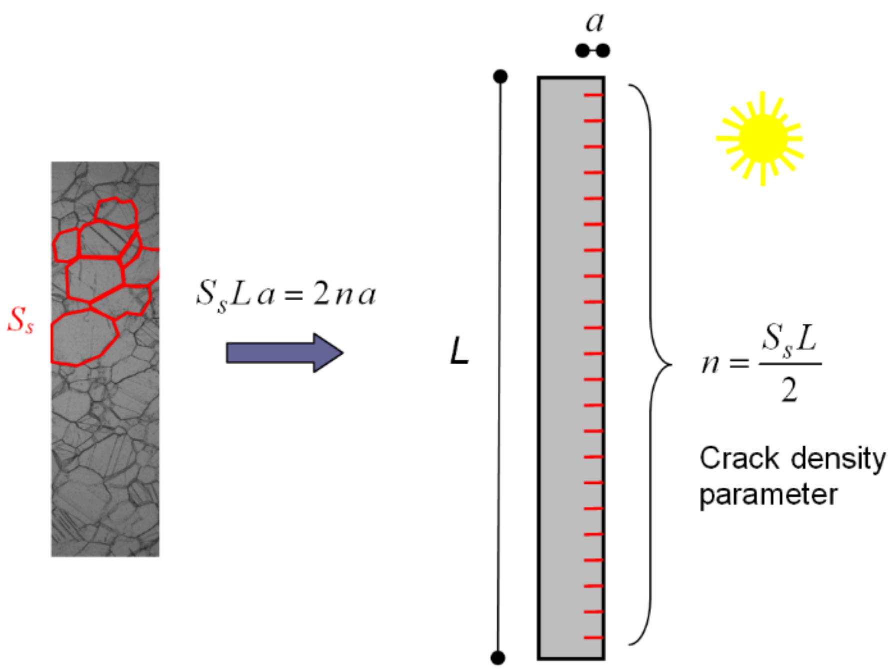

Figure 1: Schematics on the intergranular equivalent cracking.

The degree of roughness of grain boundaries tends to enhance the interlocking resistance of intergranular cracks. To include such an interlacing effect in the model, it is believed that the driving force can somehow be reduced as a function of fracture surface roughness. The concept of roughness-induced crack closure, largely used for fatigue crack propagation problems (e.g. see [13]), can be borrowed for the present context. The following empirical linear relation is proposed to reduce the effective SIF with respect to the actual SIF [11]:

$$K_{Ieff} = K_I \frac{\xi - \xi_{\min}}{\xi_{\max} - \xi_{\min}} \qquad (5)$$

The crack-induced elastic deflection of marble slab can be determined from energy considerations. Accordingly, the central deflection f_{1crack} in the slab due to the presence of a single external edge crack is [10]

$$f_{1crack}(a,t) = \frac{3L}{Eh^2}\left(\frac{\xi - \xi_{\min}}{\xi_{\max} - \xi_{\min}}\right)^2 \int_0^a K_I(a',t)\sqrt{\pi a'}\,F\left(\frac{a'}{h}\right)da' \qquad (6)$$

where the SIF $K_I(a,t)$ is that of Eq. 4 related to the thermal stress field, $F(a/h)$ is a dimensionless function of the crack geometry dependent on the SIF due to an outward point force applied to the centre of the slab [12], and the corrective factor of the effective SIFs (see Eq. 5) is considered.

In the case of multiple equally spaced cracks, Eq. 5 becomes

$$f_{ncracks}(a,t) = \frac{3nL}{2Eh^2}\left(\frac{\xi - \xi_{\min}}{\xi_{\max} - \xi_{\min}}\right)^2 \int_0^a K_I(a',t)\sqrt{\pi a'}\,F\left(\frac{a'}{h}\right)da' \qquad (7)$$

Finally, the maximum value during a temperature cycle of the crack-induced deflection for a crack of length a is given by $f(a) = \max_t f_{ncracks}(a,t)$.

It is well known that under cyclic loading conditions, stable propagation of cracks might occur. According to the Paris law [14], the crack growth rate (expressed as the derivative da/dN of crack length a with respect to the number of cycles N) is a function of the range of the effective SIF (Eq. 5) in a loading cycle $\Delta K_{Ieff}(a)$ (where $\Delta K_{Ieff}(a) = K_{Ieff,\max}(a) - K_{Ieff,\min}(a)$), that is $da/dN = C\left(\Delta K_{Ieff}\right)^m$ where C and m are material constants.

Propagation of microcracks is an irreversible phenomenon, so that it is deemed to be reasonable to correlate the increment of slab deflection due to crack propagation to the slab bowing, which is characterized by permanent deflections. In other words, bowing $b(N)$ after a given number of cycles N is taken to be equal to $f[a(N)] - f(a_0)$.

The experimental observation of marble microstructure

In the following, numerical simulations of bowing using the present theoretical model are illustrated. As a reference, a specific marble used in the vertical cladding slabs (of thickness 30mm and span L corresponding to the vertical distance between anchorages equal to 0.6 m) of a building located in the central part of Italy [15] is considered. The anchorages are modeled as hinges. The applied thermal cycles on the external and internal surfaces have a variation range of ±11 °C and ±9 °C, respectively. These temperature ranges are characteristic of diurnal temperature excursions in the marble claddings under consideration [15]. In the present simulations, the initial cracking depth is taken as equal to the mean value of the average grain size d (see below), namely a_0 = 125 μm.

The values of material parameters obtained from experimental tests conducted by the authors are [16]: E = 52 GPa, ν = 0.16, m = 4, C = 3x10^{-4} (for da/dN expressed in m/cycle and ΔK_I in MPam$^{0.5}$) K_{IC} = 1.35 MPam$^{0.5}$ (incidentally, it is intriguing to report that the experimental fatigue crack growth results of Ref. [16] have been exploited in [17] to interpret the process of regolith generation on small asteroids). As for thermal expansion, we assume α_1 = 25 μm/m/°C, α_2 = -6 μm/m/°C [9].

Three orthogonal thin sections are cut from the marble slab of the building under consideration [15], Fig. 2. From a qualitative viewpoint, the three photographs show common features of calcite grains with granoblastic texture and anhedral grain shape. The grain size distribution is characterized by the prevalence of large grains (200-300μm), followed by smaller grains (30-50μm). Grain boundaries are typically ranging from polygonal shape to interlobate one in both large and small crystals. A third group of calcite grains is detected as fine grained aggregates (<10μm) commonly rimming large porphyroblasts.

In order to quantify the grain-size distribution and preferred orientation of calcite we perform two sets of microstructural analyses, devoted to i) define the grain edges in order to quantify the distributions of some grain geometric parameters and ii) quantify the optic preferred orientation by individuating three main extinction directions (A, B, C) and quantifying the number of grains falling in each direction class.

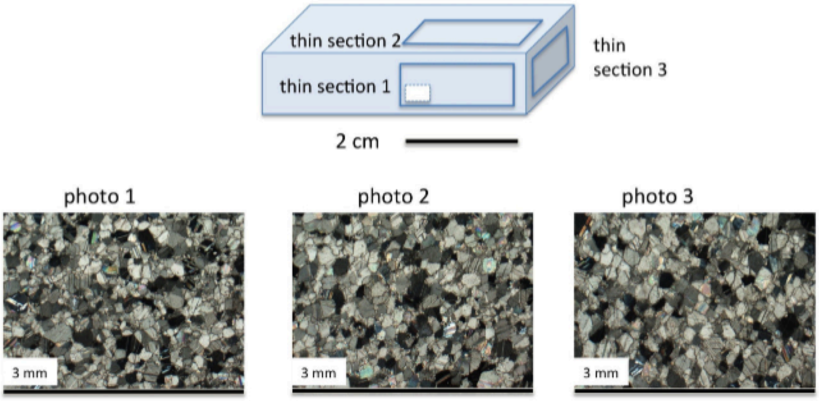

Figure 2: View of the three orthogonal thin sections being analysed

The pictures of the three thin sections are image processed. Automatic processing functions available in the freeware software Gwyddion [18] are used to detect grain edges and to calculate the min/max size, area and perimeter of each grain. The frequency distributions of some geometrical parameters of the automatically detected grains are analysed. From the observation of the distribution of min and max grain sizes for each section, it can be noticed that the size distribution of grains is quasi-isotropic as there is little differences between the three sections. The results indicate a Gaussian-like distribution of grain sizes. The mean value and standard deviation of the max grain size d_{max} distribution are equal to 157 and 77 μm, respectively. For the min grain size d_{min} distribution such parameters are equal to 93 and 46 μm, respectively. The mean value of the average grain size d_{ave} ($d_{ave} = (d_{max} + d_{min})/2$) is 125 μm. Such a mean value of grain size is adopted in the numerical simulations for grain size d and initial crack length a_0.

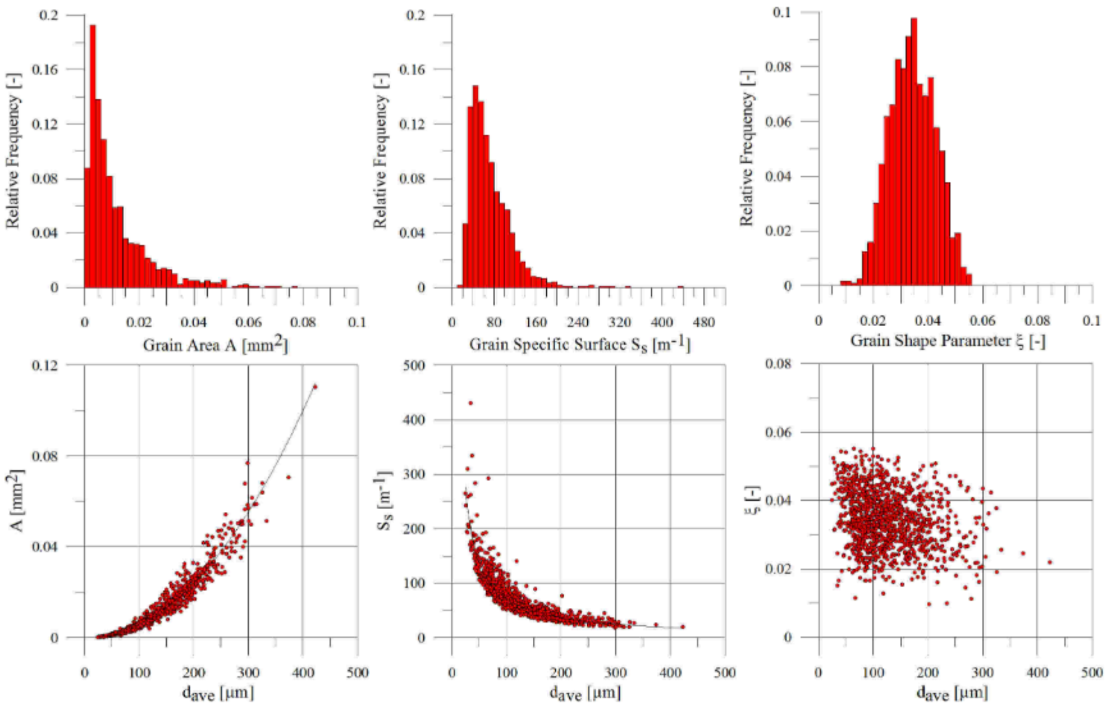

Figure 3: Frequency distributions and distribution as a function of average grain size d_{ave} for the three thin sections of the following grain geometric parameters: area A, specific surface S_s and shape parameter ξ.

Fig. 3 depicts the distributions of three geometric parameters of calcite grains: area A, specific surface S_s (equal to perimeter-to-area ratio) and the shape parameters ξ defined above, as a function of the average grain size d_{ave}. The mean values of S_s and ξ are equal to 72.7 mm^{-1} and 0.034, respectively. Such values are used to work out (see Eq. 3) n = 21810 and to calculate the effective SIF (see Eq. 5 where the reduction factor is equal to $(0.034-0)/(0.08-0)=0.425$).

Some lattice/crystallographic preferred orientations may be inferred from microstructural analysis by separating groups of crystals on the basis of the orientation of twin/cleavage planes and of similar birefringence/extinction directions. In particular three groups of birefringence of large crystals may be identified: class A with orientation angle in the range 0-60°, class B with orientation 60-120° and class C with orientation 120-180°. The frequency distribution of grain orientation for the three classes is reported elsewhere [11].

As has been discussed above, calcite grains exhibit anisotropic thermal expansion whose principal axes are linked to the optic orientation of grains. The present simulations concentrate on the random distribution of grain orientation in the material, whilst the distributions of grain size and shape are disregarded here by considering their mean values. Although the present experimental findings highlights some non-uniform distribution of grain orientation [11], for the sake of simplicity in the following a uniform distribution is considered. More in details, a uniform Probability Density Function (PDF) for the orientation of thermal expansion axes of calcite grains with respect to the longitudinal axis z of the slab is assumed, that is $p(\beta)=1/\pi$ with $0\le\beta\le\pi$.

Monte Carlo simulation

In the following simulations, as far as thermal expansion is concerned, we analyse a random case where $\alpha_z(x)=\alpha_1\cos^4\beta(x)+\alpha_2\sin^4\beta(x)$ and $\beta(x)$ distribution follows a uniform PDF. Random values of $\beta(x)$ are generated, so that their sequences along the slab thickness can be regarded as a stochastic process. A Monte Carlo simulation is performed by considering a relatively large number (say 20) of realizations of this stochastic process. In this way statistical distribution of the number of thermal cycles producing a given bowing level can be worked out. More in details, the PDF of N for $b(N)/L$ (being $b(N)=f[a(N)]-f(a_0)$) equal to e.g. 0.1% (1mm/m) can be analysed.

Fig. 4 shows the relative bowing $b(N)/L$ of slab against the number of thermal cycles N for 20 different random distributions of grain orientation. In general, the curves are characterized by an initial stage where the bowing rate is either accelerating or decelerating, followed by some retardation stage (see the plateaus in the curves) where the rate is nearly null; some phases of strong bowing acceleration (see the steps in the curves) are also evident. The final stage leading to failure is characterized by vertical slopes of the curves, indicating a large scatter in the number of cycles to failure (ranging from 10^5 to 10^8 cycles). Such a three order of magnitude scatter of the number of cycles to failure points out the dramatic variability of bowing evolution of slabs despite the relatively homogeneous nature of Carrara marble.

A useful tool that might be applied to interpret in-situ measurements is given in Fig. 5 where the logarithmic frequency distribution of the number of cycles required to attain a relative bowing $b(N)/L$ of 0.1%, according to the 20 simulations of grain orientations being performed, is reported. In the plot, the dispersion of the number of cycles leading to a certain bowing level can clearly be observed, offering a direct indication of the data scatter that can be obtain in an in-situ measurement campaign.

Conclusions

The paper analyses the influence of the material microstructure on the bowing of cladding marble slabs applying a theoretical model developed by the authors and taking into account the results of microscopic image processing of thin sections in terms of grain geometrical features and grain optic orientation.

The model is based on LEFM concepts applied to marble slabs where grain decohesion due to surface damage can occur. The model is able to estimate the stress intensification near the crack tip and to compute the stress which leads to crack propagation in the slab. Such crack propagation under thermal actions is evaluated and the corresponding bowing is calculated. Some examples, where a random distribution between calcite grains of the thermal expansion coefficient is considered (this being a reasonable description of the actual anisotropic thermal expansion of grains), have been presented to show the strong influence of material microstructure on the degree of bowing.

The obtained results can be utilised as an interpretation tool for in-situ bowing measurements which often show a wide dispersion even under nominally equal environmental condition. The proposed methodology based on standard microscopic analysis of marble microstructure and on the application of an image processing code can be suggested as a first attempt for a better understanding and forecasting of the bowing evolution of monitored slabs in building façades.

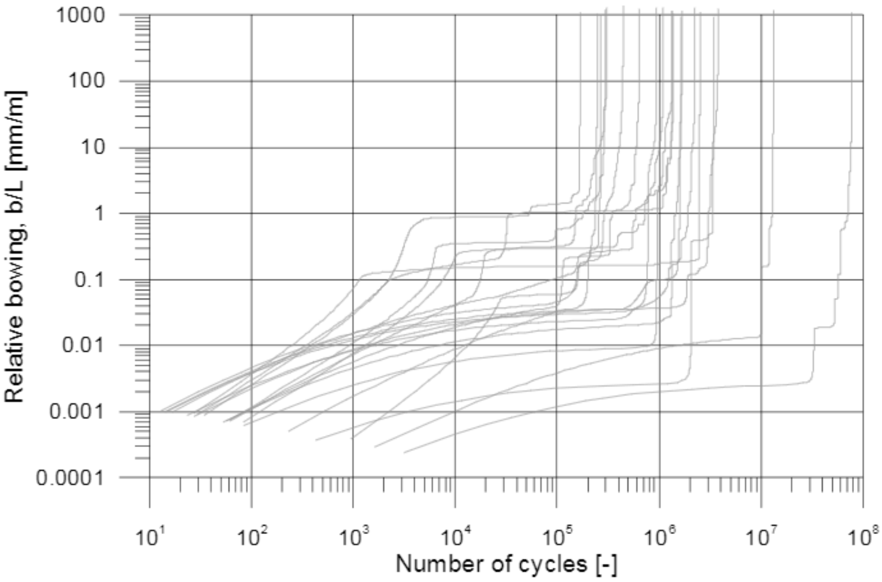

Figure 4: Relative bowing b/L vs number of cycles for 20 different realizations of grain orientation distribution along the slab thickness.

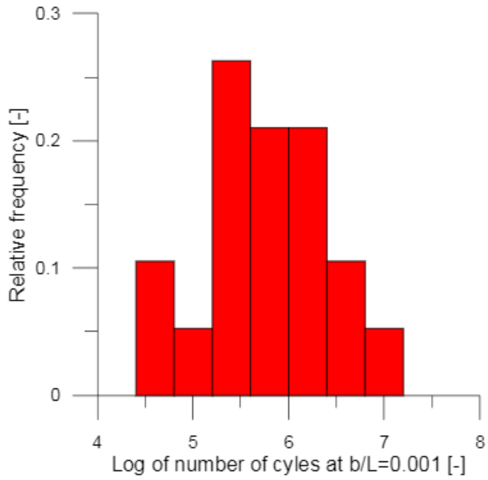

Figure 5: Frequency distribution of the number of cycles (log scale) required to attain a relative bowing b/L of 0.1%, according to the 20 simulations of grain orientations being performed.

REFERENCES

[1] Ferrero, A.M., Marini, P., Experimental studies on the mechanical behaviour of two thermal cracked marbles, Rock Mechanics and Rock Engineering, 34 (2001) 57-66.
[2] Leiss, B., Weiss, T., Fabric anisotropy and its influence on physical weathering of different types of Carrara marbles, Journal of Structural Geology, 22 (2000) 1737-1745.
[3] Royer-Carfagni, G., On the thermal degradation of marble, International Journal of Rock Mechanics and Mining Science, 36 (1999) 119-126.

[4] Widhalm, C., Tschegg, E., Eppensteiner, W., Anisotropic thermal expansion causes deformation of marble claddings, ASCE Journal of Performance of Constructed Facilities, 10 (1996) 5-10.
[5] Saylor, D.M., Fuller, E.R., Weiss, T., Thermal-elastic response of marble polycrystals: Influence of grain orientation configuration, International Journal of Materials Research, 98 (2007) 1256-1263.
[6] Weiss, T., Siegesmund, S., Fuller, E.R., Thermal degradation of marble: indications from finite-element modeling, Building and Environment, 38 (2003) 1251–1260.
[7] Siegesmund, S., Ullemeyer, K., Weiss, T., Tschegg, E.K., Physical weathering of marbles by anisotropic thermal expansion, International Journal of Earth Sciences, 89 (2000) 170-182.
[8] Chau, K.T., Shao, J.F., Subcritical crack growth of edge and center cracks in façade rock panels subject to periodic surface temperature variations, International Journal of Solids and Structures, 43 (2006) 807-827.
[9] Raileigh, A., The bending of marble, Proceeding Royal Society of London, 19 (1934) 266-279.
[10] Spagnoli, A., Ferrero, A.M., Migliazza, M., A micromechanical model to describe thermal fatigue and bowing of marble, International Journal of Solids and Structures, 48 (2011) 2557-2564.
[11] Ferrero, A.M., Migliazza, M., Spagnoli, A., Zucali, M., Micromechanics of intergranular cracking due to anisotropic thermal expansion in calcite marbles, Engineering Fracture Mechanics, Article in Press (2014) http://www.scopus.com/inward/record.url?eid=2-s2.0-84893189538&partnerID=40&md5=200ab6486acc34bdc94057648918bf89
[12] Tada, H., Paris, P.C., Irwin, G.R., The Stress Analysis of Crack Handbook. Del Research Corporation, St.Louis (1985).
[13] Suresh, S., Ritchie, R.O., A geometric model for fatigue crack closure induced by fracture surface roughness, Metallurgic Transactions A, 13 (1982) 1627-1631.
[14] Paris, P.C., Gomez, M.P., Anderson, W.P., A rational analytic theory of fatigue, The Trend in Engineering, 13 (1961) 9-14.
[15] Ferrero, A.M., Migliazza, M., Spagnoli, A., Theoretical modelling of bowing in cracked marble slabs under cyclic thermal loading, Construction and Building Materials, 23 (2009) 2151-2159.
[16] Migliazza, M., Ferrero, A.M., Spagnoli, A., Experimental investigation on crack propagation in Carrara marble submitted to cyclic loads, International Journal of Rock Mechanics and Mining, 48 (2011) 1038-1044.
[17] Delbo, M., Libourel, G., Wilkerson, J., Murdoch, N., Michel, P., Ramesh, K.T., Ganino, C., Verati, C., Marchi, S., Thermal fatigue as the origin of regolith on small asteroids. Nature, 508 (2014) 233.
[18] Nečas, D., Klapetek, P., Gwyddion: an open-source software for SPM data analysis, Cent. Eur. J. Phys., 10 (2012) 181-188.