Adaptive Weighting Scheme for Multi-Objective Optimization in Metasurface Antenna Design

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***Abstract*—We present a novel adaptive weighting scheme suited to the current-based optimization of metasurface antennas. The problem is formulated in terms of a weighted sum of individual objective functions corresponding to different constraints. A non-linear conjugate gradient optimization algorithm is combined with an hyperplane adaptive weighting scheme to improve the convergence properties. The proposed approach is inspired by a geometrical interpretation of the properties of the Pareto front, and guarantees a balancing of the individual goals. The procedure has been applied to the design of broadside-radiating metasurface antenna working at 23 GHz, and demonstrated its effectiveness in improving both the speed of convergence and quality of the solution with respect to existing algorithms.**

I. Introduction

In the last decade, the automated design of metasurface (MTS) antennas has been the subject of extensive research, both from a theoretical and a practical standpoint. The intrinsic multiscale nature of these antennas, with small scattering elements arranged in a periodic lattice over large distances, has prompted researchers to develop increasingly efficient methods to tackle the complexity of this problem.

Established methods rely on analytic approximations of the metasurface to alleviate the computational burden, with limitations on the shape of the radiating domain [1]. More recent approaches are based on a fully numerical modelling of the problem, with the associated solution found via direct methods [2] or optimization algorithms [3], [4].

Among the latter, the current-based approach [5] has been recently introduced as a promising method to tackle the design of very large antennas. However, as all non-linear, non-convex optimization problems, it suffers from typical drawbacks such as slow convergence or getting stuck in local minima.

In this work, we propose a novel adaptive weighting scheme for the current-based optimization of metasurface antennas. The approach is based on a single objective function, but takes into account the multi-objective nature of the problem by adaptively changing the weights assigned to each goal. It is based on a geometric interpretation of the weights as the component of the vector normal to the hyperplane passing through the points of the Pareto front which minimize each objective individually. This improves the robustness of the original algorithm.

II. Design of Metasurface Antennas

A. Multi-objective optimization

The design of metasurface antennas is a *multi-objective optimization problem*, i.e., a set of constraints have to be satisfied simultaneously (passivity, losslessness, pattern masks, etc.). Due to the local nature of these properties, the number of constraints becomes very large (on the order of $10^5 - 10^6$ for typical antenna sizes). In order to reduce its numerical complexity, the problem is usually formulated as a single objective minimization with the *weighted sum method* [6]:

$$f_{\text{tot}}(\mathsf{x}) = \sum_{i=1}^{K} w_i f_i(\mathsf{x}) \tag{1}$$

where f_i are the individual objective functions, w_i are the weights and $\mathsf{x} \in \mathbb{C}^N$ is the optimization variable.

As the name suggests, the weights assigned to each objective function reflect the importance of a specific goal. However, in general it is not possible to identify which goals should be prioritized over the others. In multi-objective optimization, the concept of *Pareto front*, i.e., is the set of solutions which cannot improve one objective without simultaneously worsening at least another one, is used to assess the optimality of a solution.

An a-priori, heuristic choice of weights often results in sub-optimal solutions and premature convergence to local minima of the objective function. Moreover, when the components of the objective function can vary over different orders of magnitude, as is the case for MTS antenna design, an effective choice of weights is difficult. Considering that a numerically efficient formulation requires a single objective function, a possible solution is to adaptively modify the weights during the optimization.

B. Hyperplane Adaptive Weighting Scheme

To overcome the aforementioned shortcomings of the fixed-weight approach, a new adaptive weighting scheme has been devised, based on a geometrical interpretation of the weights introduced in [7].

Before going into the details of the algorithm, an overview of the current-based optimization of MTS antennas will be given. A comprehensive description can be found in [5] and will not be repeated here. The variable is the surface current, expanded in Rao-Wilton-Glisson (RWG) basis functions, and the electromagnetic problem is modeled with an Electric Field Integral Equation (EFIE). Each of the objective functions arising from the constraints, namely *passivity*, *impedance bounds*, *scalarity* and *pattern masks*, has the following form:

$$f_i(\mathsf{x}) = (\mathsf{x}^{\mathsf{H}} \mathbf{A}_i \mathsf{x} + \mathsf{x}^{\mathsf{H}} \mathsf{b}_i + c_i)^2 \tag{2}$$

where $\mathbf{A}_i \in \mathbb{C}^{N \times N}$ is hermitian, $\mathsf{b} \in \mathbb{C}^N$, and $c_i \in \mathbb{C}$.

The minimization of the entire objective function is carried out with the *Non-Linear Conjugate Gradient algorithm* (NLCG) [8], which involves a *line search* procedure at each iteration. This procedure implies the minimization of the function of a single variable $f(\alpha) = f(\mathsf{x}_0 + \alpha\,\mathsf{p})$, where x_0 is the starting point, p is the update direction and α is the coefficient to be found. To this end, each f_i in (2) must be made explicit with respect to α:

$$f_i(\mathsf{x}_0 + \alpha\,\mathsf{p}) = \left(Q_i^{(0)} + \alpha\,Q_i^{(1)} + \alpha^2\,Q_i^{(2)}\right)^2 \tag{3}$$

where

$$Q_i^{(0)} = \mathsf{x}_0^{\mathsf{H}}\mathbf{A}_i\mathsf{x}_0 + \mathsf{x}_0^{\mathsf{H}}\mathsf{b}_i + c_i \tag{4}$$

$$Q_i^{(1)} = \mathsf{x}_0^{\mathsf{H}}\mathbf{A}_i\mathsf{p} + \mathsf{p}^{\mathsf{H}}\mathbf{A}_i\mathsf{x}_0 + \mathsf{p}^{\mathsf{H}}\mathsf{b}_i \tag{5}$$

$$Q_i^{(2)} = \mathsf{p}^{\mathsf{H}}\mathbf{A}_i\mathsf{p} \tag{6}$$

where the apex refers to the order of the coefficient. Functions of the type (3) are 4^{th}-order polynomials in the coefficient x. Therefore, as outlined in [5], they can be minimized efficiently by exploiting this polynomial structure.

Here, the same procedure is employed to find the *anchor points* of the Pareto front, which will then be used to construct the hyperplane passing through them. Each f_i is minimized individually along the search direction p, obtaining a vector of coefficients

$$\alpha_i^\star = \arg\min f_i(\alpha), \quad i = 1, \dots, K \tag{7}$$

In this way, K different K-dimensional points can be found:

$$\mathsf{f}_i^\star = [f_1(\alpha_i^\star), f_2(\alpha_i^\star), \dots, f_K(\alpha_i^\star)]^{\mathsf{T}}, \quad i = 1, \dots, K \tag{8}$$

A depiction of the anchor points in the case $K = 2$ is represented graphically in Fig. 1. These points, in addition to being computationally straightforward to calculate, ensure that if the Pareto front has a convex shape, the *knee point* is located between them. Therefore, the normal to the hyperplane constructed from anchor points will head toward the knee point. Starting from the definition of the weighted sum function (1), and of a generic hyperplane in the objective functions space,

$$\mathsf{n}^{\mathsf{T}}\mathsf{f} = d, \qquad d \in \mathbb{R} \tag{9}$$

where n is the normal vector, we can immediately identify the coefficient of the vector normal to the hyperplane passing through the anchor points as the weights of a new objective function that has a minimum which is guaranteed to lie between the anchor points.

The normal to the hyperplane can be found by solving with respect to $\mathsf{w} = [w_1, w_2, \dots, w_n]^{\mathsf{T}}$ the following linear system:

$$\mathsf{w}^{\mathsf{T}}\mathsf{f}_i^\star = 1, \qquad i = 1, ..., K \tag{10}$$

With this choice, at each weight update, a balancing of the different objective functions is obtained. Lastly, to avoid scaling issues, the set of weights is normalized with respect to the norm of the vector of weights.

During the optimization, in the numerical solution of (10), two or more anchor points could be very close to each other. This means that the resulting linear system will be badly conditioned. However, this problem is easily solved by removing rows and columns corresponding to these points, and setting their weights equal to each other, in order for the system to have a unique solution. This concludes the weights update procedure.

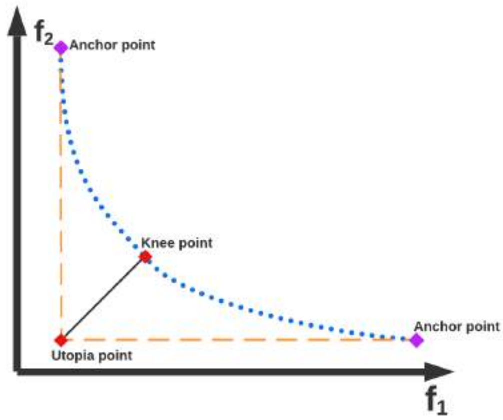


Fig. 1: Graphical representation of the Pareto front for the case $K = 2$. The two anchor points are indicated, together with the utopia point (which minimizes all function simultaneously, but is not feasible) and knee point (minimum distance to the utopia point on the Pareto front).

From this point on, the algorithm proceeds with the usual line-search minimization, with the updated set of weights. In particular, the polynomial coefficients are updated as follows:

$$Q_i^{(0)} \leftarrow w_i\,Q_i^{(0)}, \quad i = 1, ..., K \tag{11}$$

$$Q_i^{(1)} \leftarrow w_i\,Q_i^{(1)}, \quad i = 1, ..., K \tag{12}$$

$$Q_i^{(2)} \leftarrow w_i\,Q_i^{(2)}, \quad i = 1, ..., K \tag{13}$$

The complete NLCG-HAW algorithm is reported in Algorithm 1.

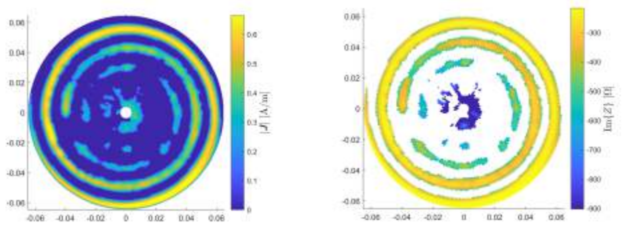
III. Results

A. Broadside beam with circular polarization

To evaluate the performance of the proposed algorithm, results have been compared to the ones obtained with the classical NLCG algorithm for the same number of iterations. In particular, the antenna is a circular MTS with a diameter of $10\lambda_0$ at the frequency of $23\,\text{GHz}$. The design specifications were for a broadside beam with circular polarization. The impedance range was set between $-1000\,\Omega$ and $-100\,\Omega$, in accordance with the values achievable with the employed substrate and unit cell shape. For both algorithms, the maximum number of iterations has been set to 400 (a higher number of iterations would have rendered the comparison less conclusive, as both algorithm would eventually converge to a similar result, albeit with different speed). For the NCLG-HAW case, the weights are updated every 100 iterations. It is known from the literature that such a problem leads to a spiral patterning of the surface impedance. By comparing the

Algorithm 1 NLCG-HAW Algorithm

Input: x_0
Output: $\mathsf{x}^\star$
  Compute $\nabla f(\mathsf{x}_0)$
  $\mathsf{p}_0 \leftarrow -\nabla f(\mathsf{x}_0)$
  $i \leftarrow 0$
  while $i < N_{\max}$ **do**
    Compute $\mathsf{Q}^{(0)}, \mathsf{Q}^{(1)}, \mathsf{Q}^{(2)}$
    if weight update **then**
      $\mathsf{w} \leftarrow \mathrm{HAW}(\mathsf{Q}^{(0)}, \mathsf{Q}^{(1)}, \mathsf{Q}^{(2)})$
      $\mathsf{Q}^{(0)} \leftarrow \mathsf{w} \odot \mathsf{Q}^{(0)}$
      $\mathsf{Q}^{(1)} \leftarrow \mathsf{w} \odot \mathsf{Q}^{(1)}$
      $\mathsf{Q}^{(2)} \leftarrow \mathsf{w} \odot \mathsf{Q}^{(2)}$
    end if
    Compute α_i by minimizing $f(\mathsf{x}_i + \alpha_i \mathsf{p}_i)$
    $\mathsf{x}_{i+1} \leftarrow \mathsf{x}_i + \alpha_i \mathsf{p}_i$
    Compute $\nabla f(\mathsf{x}_{i+1})$
    Compute β_i
    $\mathsf{p}_{i+1} \leftarrow -\nabla f(\mathsf{x}_{i+1}) + \beta_i \mathsf{p}_i$
    $i \leftarrow i + 1$
  end while
  $\mathsf{x}^\star \leftarrow \mathsf{x}_{N_{\max}}$



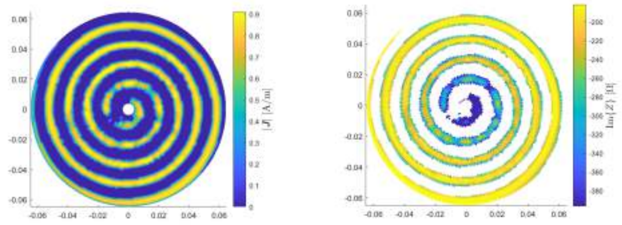
(a) (b)

Fig. 2: Circular MTS antenna ($D = 10\lambda$) with circularly polarized broadside beam, optimized running 400 iterations of the NLCG algorithm: (a) Optimized surface current, (b) corresponding surface impedance.

current and impedance for the two cases (Figs. 2 and 3), one can immediately see that the proposed solution is able to reach a satisfactory result, both in terms of impedance range and smoothness, within a limited number of iterations, whereas the classical solution is still far from convergence.

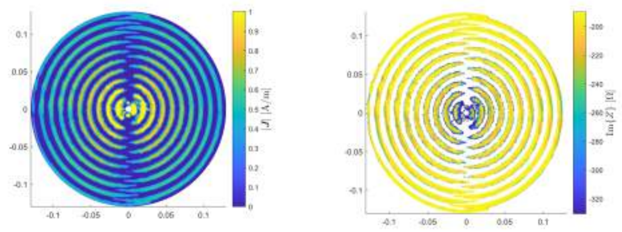
B. Broadside beam with linear polarization

The second analyzed case is a circular MTS antenna, working at 23 GHz, and with a $20\lambda_0$ diameter. The impedance range, which depends only on the substrate properties and unit cell shape, is the same as in the previous case. Fig. 4a shows the resulting current after running the NLCG-HAW algorithm for 800 iterations, with a weight update each 100 iterations. The corresponding impedance (Fig. 4b is clearly within the prescribed range, and exhibits a smooth behaviour. The radiated pattern, shown in Fig. 5, compares very well with similar results in literature. In fact, there is a 1 dB increase in



(a) (b)

Fig. 3: Circular MTS antenna ($D = 10\lambda$) with circularly polarized broadside beam, optimized running 400 iterations of the NLCG-HAW algorithm: (a) Optimized surface current, (b) corresponding surface impedance.



(a) (b)

Fig. 4: Circular MTS antenna ($D = 20\lambda$) with linearly polarized broadside beam, optimized with the NLCG-HAW algorithm: (a) Optimized surface current, (b) corresponding surface impedance.

directivity with respect to the same design obtained with the classical NLCG algorithm [5].

IV. Conclusions

A novel adaptive weighting scheme has been introduced, which allows to speed up the convergence of existing, current-based optimization algorithms for the design of metasurface antennas. The geometrical interpretation of the hyperplane passing through the anchor points of the Pareto front allows a robust choice of weights. Problem remain in cases in which the Pareto front is not convex, which is still not possible to determine in advance. Numerical results showed a performance increase in terms of speed of convergence and optimality of the solution.

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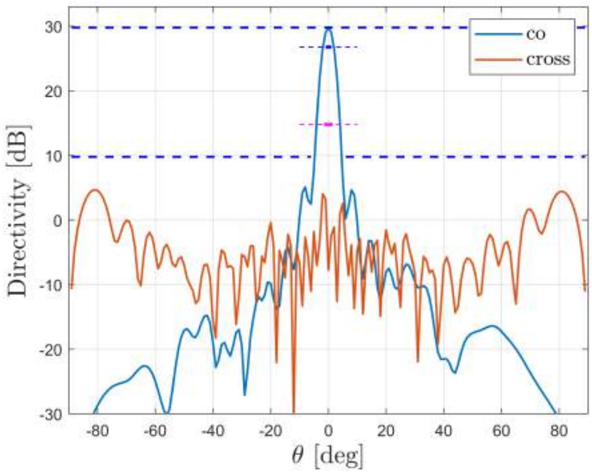

Fig. 5: Directivity (co- and cross-polarization) in the $\phi = 90°$ plane cut for the Circular MTS antenna ($D = 20\lambda$) with linearly polarized broadside beam, optimized with the NLCG-HAW algorithm.

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