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Duplex LCF-VHCF P-S-N design curves: a methodology based on the Maximum Likelihood Principle

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Abstract

Probabilistic-S-N (P-S-N) curves are commonly employed to model the fatigue response of specimens and components and are generally obtained by assuming the statistical distribution of the fatigue life. In industrial applications, starting from the P-S-N curves, the so-called "design curves" or "lower bound S-N curves" are defined for the design of components, in order to ensure a safety margin against fatigue failures. Due to the growing interest in the Very High Cycle Fatigue (VHCF) response of materials, methodologies for the estimation of the design curves should not be limited to the Low Cycle Fatigue (LCF)-High Cycle Fatigue (HCF) life range but should also cover the VHCF region.

In this work, the Maximum Likelihood Principle is exploited for the assessment of the design curves of datasets obtained through tests in the LCF-VHCF range, with the P-S-N curves showing a duplex trend. First, the model for the P-S-N curves with duplex trend is defined. Then the proposed methodology for the design curves is described and finally validated with literature datasets.

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1. Introduction

Probabilistic-S-N (P-S-N) curves are commonly employed to model the fatigue response of specimens and components (Stephens et. al. (2000), BS ISO 12107:2003, ASTM E739-10, Tridello et al. (2022)). They are generally obtained by assuming the statistical distribution of the fatigue life, in order to take into account the randomness that is intrinsically associated with the fatigue phenomenon and the scatter of the experimental data (Lee et al. (2005)). In industrial applications, starting from the P-S-N curves, the so-called "design curves" or "lower bound S-N curves" are defined for the design of components, in order to ensure a safety margin against possible fatigue failures (Lee et al. (2005), Tridello et al. (2022)). The proper estimation of the design curves, especially for experimental datasets with a limited number of available data, is therefore of fundamental importance to prevent possible unexpected fatigue failures, whose effects can be catastrophic.

In the literature, different solutions and strategies are employed to estimate the design curves. For example, the lower 3 sigma (median S-N curve shifted by a factor equal to three times the standard deviation associated with the experimental dataset), the approximate Owen one-side tolerance limit and the double-sided confidence intervals approach (Lee et al. (2005)) are commonly adopted. If the experimental data end with an asymptotic trend (i.e., a fatigue limit), the stair-case method is generally employed. According to the industrial practice, the choice for a specific estimation strategy of the design curve is arbitrary, depending on the internal safety policy and on the specific application. However, no methodology has been proposed for modelling the design curves in case of failures in the range Low Cycle Fatigue (LCF) - Very High Cycle Fatigue (VHCF), with the experimental data showing a so-called duplex trend, i.e., a first decreasing trend, a plateau and a second decreasing trend ending with asymptote in the VHCF region. However, many structural components employed in industrial applications (automotive, aerospace, energy production) are prone to failures in the VHCF life region (Bathias et al. (2005)) and the assessment of the design P-S-N curves even in the VHCF life region become fundamental to guarantee their structural integrity.

In this work, the Maximum Likelihood Principle is exploited for the assessment of the design curves of datasets obtained through tests in the LCF-VHCF range, with the P-S-N curves showing a duplex trend. First, the model for the P-S-N curves with duplex trend is defined. Then the proposed methodology for the design curves is described and finally validated with experimental literature datasets.

Nomenclature

$\mu_{Y,surf}(x)$, $\sigma_{Y,surf}$, μ_{X_t}, σ_{X_t}, $\mu_{Y,int}(x)$, $\sigma_{Y,int}$, μ_{X_l}, σ_{X_l} : parameters of the statistical distribution
x, y: logarithm of the applied stress amplitude, s_a, and of number of cycles to failure, n_f
a_0, a_1, b_0, b_1: parameters to be estimated from the experimental data
x_R, x_C: reliability and confidence levels
$PL[\theta_1]$: Profile Likelihood
$L[\cdot]$: Maximum Likelihood function

2. Methodology

In Section 2.1, the general statistical model for P-S-N curves showing a duplex trend is described. In Section 2.2, the adoption of conventional methods for the estimation of the P-S-N design curves from datasets in the LCF-VHCF life range is discussed. In Section 2.3 and Section 2.4, the analytical formulation and the procedure developed for implementing the proposed methodology are described.

It must be noted that a design P-S-N curve generally corresponds to the lower bound, at a specific confidence level, of a high-reliability quantile P-S-N curve. The notation $\mathrm{Rx}_R\mathrm{Cx}_C$ P-S-N curve, i.e, the $(1-\mathrm{x}_C/100)$ confidence bound of the $(1-\mathrm{x}_R/100)$ quantile of the P-S-N curve will be used in the following, according to (Lee et al. (2005), Tridello et al. (2022)).

2.1. General statistical model for the duplex P-S-N curves

The P-S-N curves with a duplex trend are characterized by a first linear decreasing trend in the LCF – HCF life range, with failures generally originating from the specimen surface, an almost horizontal trend, the so-called transition stress, and a second decreasing trend, with failures starting from internal defects in the VHCF region, with a final asymptotic behaviour, corresponding to the VHCF fatigue limit (Shiozawa et al. (2001), NishiJima et al. (1999), Sakai et al. (2010), Mughrabi (2006), Tridello et al. (2022)). The transition stress corresponds to the conventional fatigue limit at a number of cycles above $2 \cdot 10^6$ and discriminates between failures from surface and from internal defects, typical of the VHCF region. Fig. 1 shows a representative example of a duplex S-N curve, with the above-described life ranges.

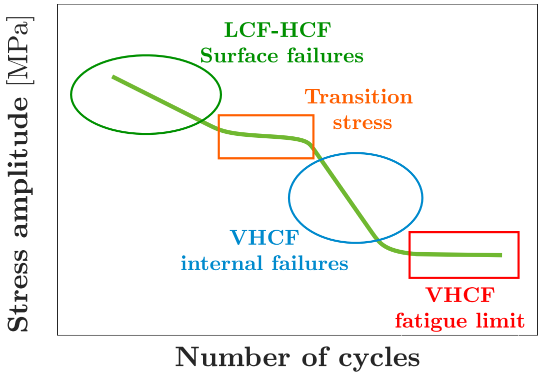


Fig. 1. Representative image of a duplex trend on the S-N plot.

Several models have been proposed in the literature for the duplex S-N curves. The reader is referred to Tridello et al. (2022) for an extensive literature review. The statistical model in Eq. 1 has been proposed by the Authors in Paolino et al. (2013):

$$F_Y(y;x) = \Phi_{surf}\left(\frac{y-\mu_{Y,surf}(x)}{\sigma_{Y,surf}}\right)\Phi_{Xt}\left(\frac{x-\mu_{X_t}}{\sigma_{X_t}}\right) + \left(1-\Phi_{Xt}\left(\frac{x-\mu_{X_t}}{\sigma_{X_t}}\right)\right)\Phi_{int}\left(\frac{y-\mu_{Y,int}(x)}{\sigma_{Y,int}}\right)\Phi_{Xl}\left(\frac{x-\mu_{X_l}}{\sigma_{X_l}}\right), \quad (1)$$

being $F_Y(y;x)$ the probability of failure for an applied stress amplitude s_a ($x = \log_{10}(s_a)$) and a number of cycles to failure n_f (($y = \log_{10}(n_f)$), $\Phi(\cdot)$ the standardized cumulative distribution function (cdf) of a Normal distribution, i.e., Φ_{surf} for the LCF-HCF life region characterized by surface failures, Φ_{Xt} for the transition stress, Φ_{int} for internal failures in the VHCF life region and Φ_{Xl} for the VHCF limit. According to Eq. 1, the fatigue life is assumed to follow a Normal distribution for all the considered life regions. In particular, the mean value of the fatigue life for the LCF-HCF life region with surface failures is linearly dependent on x (i.e., $\mu_{Y,surf}(x) = a_0 + a_1 \cdot x$, being a_0, a_1 material parameters to be estimated from the experimental data) and constant standard deviation $\sigma_{Y,surf}$. The same approach has been considered for modelling the linear decreasing trend of the fatigue life in the VHCF life region: the mean of $\Phi_{int}(\cdot)$ is linearly dependent on x (i.e., $\mu_{Y,int}(x) = b_0 + b_1 \cdot x$, being b_0, b_1 material parameters to be estimated from the experimental data), whereas the standard deviation $\sigma_{Y,int}$ is constant. On the other hand, the transition stress and the fatigue limit are normally distributed with constant mean and standard deviation (μ_{X_t} and σ_{X_t} for the transition stress, respectively, and μ_{X_l} and σ_{X_l} for the fatigue limit). With the proposed general model, the duplex trend is described by 10 material parameters that must be properly estimated from the experimental data. The α-quantile of the fatigue strength for a selected y can be easily obtained by substituting $F_Y(y;x)$ in Equation 1 with α and by solving it numerically with respect to x. By repeating this procedure for the range of y of interest, the α-quantile P-S-N curve can be built point by point.

2.2. Estimation of the design curves with conventional methods

Generally, for the LCF-HCF life region with experimental data showing a linear trend and a final asymptote (the conventional fatigue limit), the design curves are obtained by shifting the median curve by a conservative factor multiplied by the estimated standard deviation. For example, according to Lee et al. (2005), the median curve can be shifted by a conservative factor corresponding to 3 times the standard deviation. Similarly, methodologies analyzed in Lee et al. (2005) and based on the approximate Owen method are also widely adopted, with the shifting factor tabulated as a function of the reliability and confidence levels and the specimen numerosity. The shifting factor is computed separately for the finite life range, i.e., for the linear decreasing trend, and for the horizontal asymptote, i.e., the fatigue limit.

These procedures can be at first considered and adapted for the LCF-VHCF life range and for curves showing a duplex trend. The design curves for each considered life region, i.e., LCF-VHCF life range with surface failures, transition region, VHCF life region with failures from internal defects and infinite VHCF life region can be assessed separately following the procedure described for the LCF-HCF life range. Accordingly, for each life range, a shifting factor can be assessed. However, with this approach, the continuity of the design curve may not be guaranteed, especially if the scatter in the different life regions vary significantly, with considerably different shifting factors. Moreover, the shifting factor for the transition stress can be hardly estimated, since it depends on the arbitrary choice of surface and internal failures to be considered for this region. Similarly, the scatter associated with the VHCF fatigue limit can be hardly assessed, since staircase data in the VHCF life region are generally not available.

Due to these main drawbacks, a procedure that considers together all the life ranges and that models the experimental scatter in the transition region without an arbitrary subdivision of the experimental data would permit a more reliable estimation of the design P-S-N curves with duplex trend.

2.3. Lower confidence bound for the quantile of the duplex P-S-N curve

In order to overcome the criticalities highlighted in Section 2.2, the Likelihood Ratio Confidence Intervals (LRCIs) are exploited for the estimation of the lower bound of a specific quantile of the P-S-N curve. By selecting the appropriate confidence and reliability levels, the design curves can be estimated.

First of all, the 10 material parameters in Eq. 1 must be estimated. The Maximum Likelihood Principle is exploited for the estimation, so that both failure and runout data can be considered. The set of estimated parameters, i.e., those that maximize the Likelihood function, is called in the following $\tilde{\theta}$ (i.e., $\tilde{\theta} = \left(a_0, a_1, \sigma_{Y,surf}, \mu_{X_t}, \sigma_{X_t}, b_0, b_1, \sigma_{Y,int}, \mu_{X_l}, \sigma_{X_l}\right)$.

Thereafter, the so-called Profile Likelihood $PL[\theta_1]$, defined according to Eq. 2, should be computed:

$$PL[\theta_1] = \frac{\max_{\boldsymbol{\theta_2}}[L[\theta_1, \boldsymbol{\theta_2}]]}{L[\tilde{\boldsymbol{\theta}}]} \geq e^{\frac{\chi^2(1;1-\beta)}{2}}, \tag{2}$$

where θ_1 is the investigated parameter, corresponding to the quantile of the fatigue strength, s_α, $\boldsymbol{\theta_2}$ is the subset of the other material parameters involved in the model, $L[\tilde{\boldsymbol{\theta}}]$ is the value of the Maximum Likelihood function computed for the set of parameters $\tilde{\boldsymbol{\theta}}$, $\chi^2(1; 1-\beta)$ is the $(1-\beta)$-th quantile of a Chi-square distribution with 1 degree of freedom. The solution to Eq. 2 provides an estimate of the $\mathrm{Rx}_R\mathrm{Cx}_\mathrm{C}$ fatigue strength at a specific n_f: the x_R value corresponds to the $(1-\alpha)\%$ quantile of the P-S-N curve, whereas $(1-\beta_{th})$ is equal to $(2 \cdot \mathrm{x}_\mathrm{C} - 1)$ (one side confidence interval). Namely, the s_α value that solves Eq. 2 corresponds to the lower bound of the investigated α quantile of the fatigue strength at the selected number of cycles n_f. By iteratively repeating this procedure for the fatigue life range of interest, the design curve can be built point by point.

However, the solution to Eq. 2 can be obtained only if the Profile Likelihood $PL[\theta_1]$ is expressed as a function of s_α. Accordingly, Eq. 1 must be rewritten as a function of the α quantile of the fatigue strength. This can be achieved following two steps:

1. By substituting $F_Y(y;x)$ with α, i.e., in order to obtain the α-quantile of the fatigue strength, according to Eq. 3 (being $x_\alpha = \log_{10}(s_\alpha)$ and $y_\alpha = \log_{10}(n_{f,\alpha})$, with $n_{f,\alpha}$ the number of cycles to failure computed at s_α):

$$\alpha = \Phi_{surf}\left(\frac{y_\alpha - \mu_{Y,surf}(x_\alpha)}{\sigma_{Y,surf}}\right)\Phi_{Xt}\left(\frac{x_\alpha - \mu_{X_t}}{\sigma_{X_t}}\right) + \left(1 - \Phi_{Xt}\left(\frac{x_\alpha - \mu_{X_t}}{\sigma_{X_t}}\right)\right)\Phi_{int}\left(\frac{y_\alpha - \mu_{Y,int}(x_\alpha)}{\sigma_{Y,int}}\right)\Phi_{Xl}\left(\frac{x_\alpha - \mu_{X_l}}{\sigma_{X_l}}\right), \tag{3}$$

2. By rearranging Eq. 3, the expression of one of the parameters in the model can be obtained as a function of s_α. For example, the coefficient a_0 can be expressed as:

$$a_0 = y_\alpha - \Phi^{-1}\left(\frac{\left(\alpha - \left(1 - \Phi_{Xt}\left(\frac{x_\alpha - \mu_{X_t}}{\sigma_{X_t}}\right)\right)\Phi_{int}\left(\frac{y - \mu_{Y,int}(x_\alpha)}{\sigma_{Y,int}}\right)\Phi_{Xl}\left(\frac{x_\alpha - \mu_{X_l}}{\sigma_{X_l}}\right)\right)}{\Phi_{Xt}\left(\frac{x_\alpha - \mu_{X_t}}{\sigma_{X_t}}\right)}\right)\cdot \sigma_{Y,surf} - a_1 \cdot x_\alpha, \tag{4}$$

Alternatively, the mean value of the fatigue limit μ_{X_l} can be obtained:

$$\mu_{X_l} = x_\alpha - \Phi^{-1}\left(\frac{\left(\alpha - \Phi_{surf}\left(\frac{y_\alpha - \mu_{Y,surf}(x_\alpha)}{\sigma_{Y,surf}}\right)\Phi_{Xt}\left(\frac{x_\alpha - \mu_{X_t}}{\sigma_{X_t}}\right)\right)}{\left(1 - \Phi_{Xt}\left(\frac{x_\alpha - \mu_{X_t}}{\sigma_{X_t}}\right)\right)\Phi_{int}\left(\frac{y_\alpha - \mu_{Y,int}(x_\alpha)}{\sigma_{Y,int}}\right)}\right)\cdot \sigma_{X_l}, \tag{5}$$

By substituting Eq. 4 or Eq. 5 in Eq. 1, the model for the fatigue life of datasets showing a duplex trend can be expressed as a function of s_α and, accordingly, the Profile Likelihood can be computed. If Eq. 4 is considered, $\theta_1 = s_\alpha$ and $\theta_2 = (a_1, \sigma_{Y,surf}, \mu_{X_t}, \sigma_{X_t}, b_0, b_1, \sigma_{Y,int}, \mu_{X_l}, \sigma_{X_l})$. Similarly, if Eq. 5 is considered, $\theta_1 = s_\alpha$ and $\theta_2 = (a_0, a_1, \sigma_{Y,surf}, \mu_{X_t}, \sigma_{X_t}, b_0, b_1, \sigma_{Y,int}, \sigma_{X_l})$. It must be noted that Eq. 4 and Eq. 5 may provide infinite values. For example, if $\Phi^{-1}(\text{arg})$, with $\text{arg} > 1$, a_0 becomes infinite and the solution to Eq. 2 cannot be found. In this case, Eq. 5 can be considered or, alternatively, one of the other ten material parameters can be expressed as a function of s_α and substituted in Eq.1 according to the above-described procedure, till finite values for $PL[\theta_1]$ are found.

2.4. Implemented procedure for LRCI estimation

In this Section, the procedure developed for implementing the methodology proposed in Section 2.3 is described. Fig. 2 shows the four steps followed to obtain the lower bound of the α quantile of the fatigue strength for a specific n_f. In particular, in Fig. 2 $PL[s_\alpha]$ as a function of s_α is shown.

According to Fig. 2 and to the procedure described in previous section, s_α should be iteratively varied in order to assess the $PL(s_\alpha)$ variation with respect to s_α. In Fig. 2, $s_{\alpha,i-th}$ refers to the $i-th$ value of s_α for which the Profile Likelihood has been computed. According to Fig. 2a, the first considered s_α value ($s_{\alpha,1}$) corresponds to the s_α obtained from Eq. 3 and by considering $\theta = \tilde{\theta}$ ($PL(s_{\alpha,\tilde{\theta}})$ in Fig. 2a). For $s_{\alpha,\tilde{\theta}}$, $PL(s_{\alpha,\tilde{\theta}})$ must be equal to 1, providing the first point of the implemented procedure. Then, $s_{\alpha,i-th}$ is iteratively decreased with steps of 1 MPa (or smaller, depending on the tested material) and, for each $s_{\alpha,i-th}$, $PL(s_{\alpha,i-th})$ is computed. This iterative procedure is stopped when the computed $PL(s_{\alpha,i-th})$ is below a threshold value, set equal to 10^{-3}. Thereafter, the estimated $PL(s_{\alpha,i-th})$ points with respect to $s_{\alpha,i-th}$ are interpolated with Piecewise Cubic Hermite Interpolating Polynomial (PCHIP), as shown in Fig. 2c. Finally, the s_α value for which the interpolating PCHIP function equals $e^{-\frac{\chi^2(1;1-\beta)}{2}}$, i.e., the value of s_α that solves Eq. 2, corresponds to the lower bound of the α quantile of the fatigue strength for the investigated n_f. By repeating this procedure for the range of n_f of interest, the design curve can be built point by point.

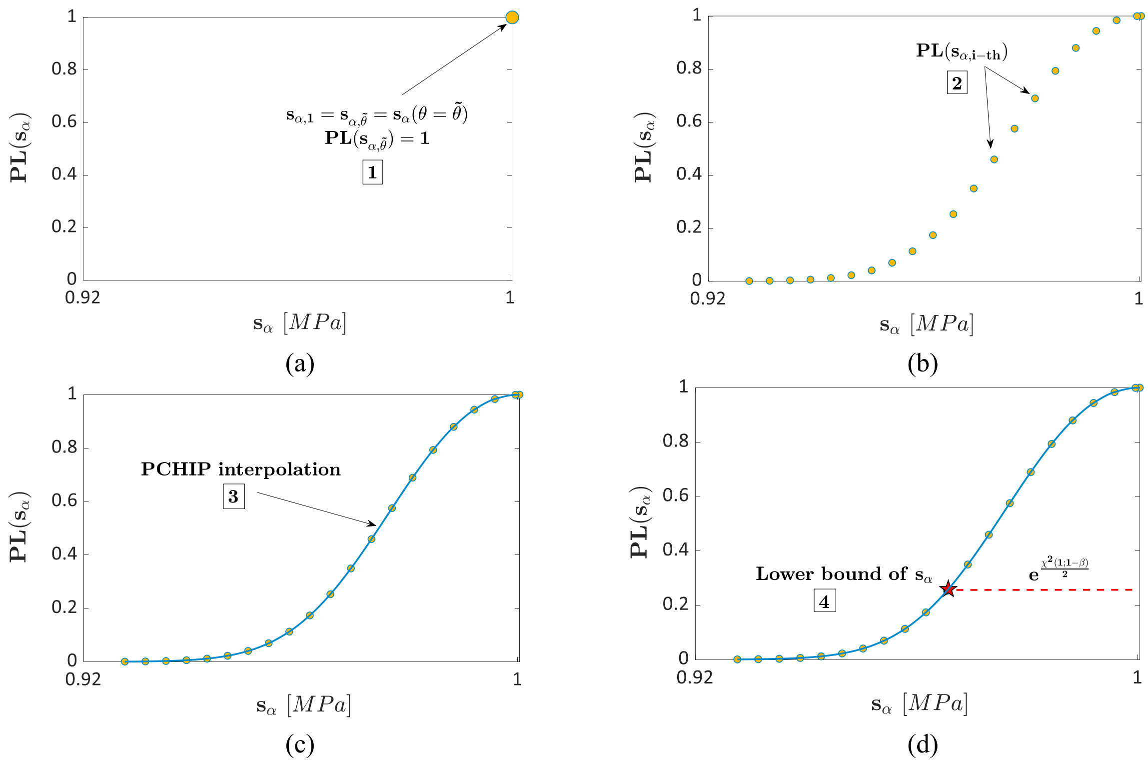


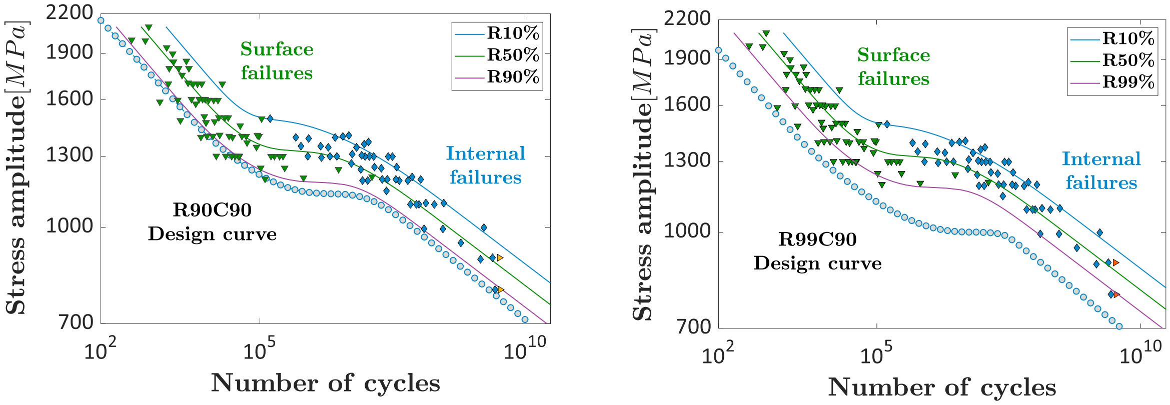
Fig. 2. Procedure implemented for the assessment of the lower bound of the α quantile of the fatigue strength.

3. Literature validation

In this Section, the procedure developed in Section 2 has been validated on literature datasets (Sakai et al. (2010), NIMS (2005)). According to Section 2.4, the developed methodology requires several optimization processes to estimate the lower bound of s_α for a specific n_f (Fig. 2). This procedure should be repeated for the range of n_f values of interest. A Matlab script that automatically computes the design curve has been developed. The *fmincon* algorithm has been used for the optimizations. The iterative procedure described in Fig. 2 has been implemented with a *while* loop, with an *if* condition checking that a sufficient number of points is available for the estimation of the $PL(s_\alpha)$ trend with the PCHIP interpolating function. If this condition is not met, the step of 1 MPa is decreased.

In the following, the experimental datasets considered for the validation have been obtained by digitizing the data points from the S-N plot images in the original papers with the software *Engauge*.

The literature dataset experimentally obtained in (Sakai et al. (2010)) by testing a SUJ2 steel from the LCF to the VHCF life range has been at first considered for the validation. Failures in the LCF-HCF life range originated from the specimen surface, whereas failures in the VHCF life range originated from internal defects with a fish-eye morphology. The experimental data do not show a VHCF fatigue limit, as discussed by the authors and suggested by failure data close to runout data. Accordingly, the original model in Eq. 1 has been modified to model a linear decreasing trend in the VHCF region (i.e., $\Phi_{Xl}\left(\frac{x-\mu_{X_l}}{\sigma_{X_l}}\right) = 1$). This has proven the adaptability of the proposed general model, which is capable to fit datasets that do not show the complete duplex trend described in Fig.1, provided that the initial model is properly modified. Fig. 3 shows the experimental data together with the estimated curves. In particular, in Fig. 3a and 3b the median, the R90 ($\alpha = 10\%$) and the R10 ($\alpha = 90\%$) P-S-N curves are shown. In Fig. 3a, the R90C90 design curve is shown, whereas in Fig. 3b the R99C90 design curve is plotted.



(a) (b)

Fig. 3. Validation of the methodology described in Section 2 on the literature dataset obtained in (Sakai et al. (2010)): a) R90C90 design curve; b) R99C90 design curve.

According to Fig. 2a, the estimated design curve follows the same trend of the median curve and is below the R90 P-S-N curve, according to the definition of design curve. Due to quite large number of experimental data (more than 65 data for surface failures and 54 data for internal failures), the estimated design curve is close to the R90 P-S-N curve. The difference increases in the transition stress region. 7 data out of 65 are below the R90C90 design curve. This means that a design curve with larger reliability or confidence levels should be considered to ensure a larger safety margin with respect to both surface and internal failures, as proved by the R99C90 design curve in Fig. 3b. Indeed, the R99C90 design curve is below all the experimental failures. The difference between the R90 P-S-N curve and the R99C90 curve is larger in the transition stress region, where the random occurrence of surface and internal failures tends to increase the experimental scatter, whereas it is quite constant in the LCF-HCF and in the VHCF life range.

A second validation has been carried out by considering the experimental dataset available in NIMS (2005) and obtained by testing a Ti6Al4V alloy. Fig. 4 shows the experimental data, with surface failures in the LCF-HCF life region and internal failures in the VHCF life region. The median, the R10 and the R90 P-S-N curves are shown, together with the R90C90 design curve.

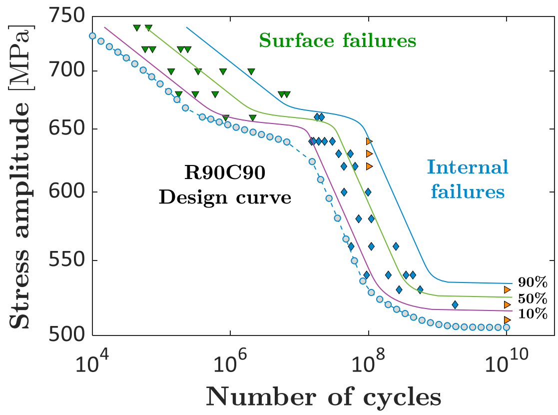


Fig. 4. Validation of the methodology described in Section 2: R90C90 design curves for the literature dataset obtained in NIMS (2005).

Fig. 4 further validated the proposed approach for experimental data showing a duplex trend. Indeed, it allows estimating the design curve even for datasets showing a duplex trend ending with a VHCF fatigue limit. The trend of the R90C90 design curve is the same of the median curve and it below all the experimental failures. The proposed methodology based on the Likelihood Ratio Confidence Intervals has proven effective for the estimation of the design curves of datasets showing a duplex trend, filling a literature gap of knowledge. The shape of the design curve varies depending on the number of data available in the different regions, thus adapting to the experimental data.

4. Conclusions

The present paper focuses on the so-called duplex P-S-N curves, i.e., the fatigue curves that cover the Low Cycle Fatigue (LCF)-Very High Cycle Fatigue (VHCF) life range with a first slope in the Low Cycle Fatigue (LCF)- High Cycle Fatigue (HCF) region and failures generally originating from the specimen surface, a transition stress, a second decreasing trend in the VHCF life range characterized by failures from internal defects and ending with an asymptote, i.e., the VHCF fatigue limit. In particular, a procedure for the estimation of the lower confidence bound of a specific quantile of the fatigue strength has been proposed. By selecting the proper high-reliability quantile and confidence level, the design curve of P-S-N curves showing a duplex trend can be attained. In the literature, no methodology has been proposed for the design curves in the VHCF life region and for curves showing a duplex trend. The proposed methodology is based on the Likelihood Ratio Confidence Intervals (LRCI). The analytical formulation and the procedure, implemented in a Matlab script since it involves several iterative optimizations, have been described. Finally, the developed methodology has been validated on literature datasets, showing its effectiveness and its capability to adapt to the experimental data for a reliable design of components subjected to loads in the wide LCF-VHCF life region.

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