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A CONJECTURE FROM THE PAPER: 'TIME, IRREVERSIBILITY AND COSMOLOGICAL THROTTLING, *ATTI ACCAD. PELORIT. PERICOL. CL. SCI. FIS. MAT. NAT.* 103(2), A2 (2025)'

UMBERTO LUCIA *

ABSTRACT. We have recently presented a thermodynamic viewpoint that explores the concept of a potential cosmological Joule-Thomson effect, which is supported by the Universe's considerable entropy, primarily stemming from black-body radiation. In an open system, the evolution of entropy is influenced by interactions with the environment and is characterized by two components: entropy flow and production of internal entropy. To fully comprehend these dynamics, it is essential to understand irreversible processes, as they bridge the gap between order and disorder. This new perspective shifts our understanding of thermodynamics, suggesting that the emergence of order is related to non-equilibrium conditions, whereas disorder is generally associated with stable equilibrium. This insight necessitates a re-evaluation of how we analyse natural systems, emphasizing the intricate relationships between order, disorder, and the nature of time. This short communication aims to further develop this approach by investigating the pressure associated with irreversibility in the context of space-time interpretation and proposing a conjecture regarding the geometric consequences of this thermodynamic concept.

The Nobel Prize-winning physicist Richard Feynman pointed out that "in modern physics, we lack an understanding of what energy truly is" (Feynman, Leighton, and Sands 2005). Nonetheless, energy is a crucial quantity that is intimately connected to every process in nature. Energy changes drive all modifications, effects, and phenomena we encounter daily. Furthermore, variations in energy are closely linked to the progression of time. Every system possesses a power, and the change in energy occurs through the application of power over time. Thus, power is the essential characteristic of physical systems: any system can produce energy by generating power over a specific duration of any given process, which is the lifespan of that process.

In his Nobel Lecture, delivered on 8 December 1977, Ilya Prigogine emphasised that time is intimately connected to the second law of thermodynamics in the fields of physics and chemistry (Prigogine 1977). Therefore, time is a crucial element in any process and has been interpreted in various ways throughout history in physics; it functions as a parameter in both the Galilean and Newtonian frameworks (Borghi 2016), while in Einstein's formulation, it becomes a part of the space-time continuum (Borghi 2016). The definition and comprehension of the essence of time should be emphasised as challenging, since any process is typically characterised within the context of time flow. In Newtonian physics, time is treated as a mathematical variable; however, it is also considered to be something real (Borghi 2016). Both simultaneity and the duration of events are seen as absolute: duration is an abstract characteristic of the whole. In contrast, Einstein's theory of Relativity presents a radically different perspective on time, making moments and durations contingent upon the observer. The physical universe is conceived as a space-time continuum, a mathematical realm continuously occupied by ideal clocks (Borghi 2016). These clocks are all synchronised to a specific observer, who gauges the duration of an event using two clocks positioned at the start and end points of the phenomenon. Therefore, a different inertial observer, who is in motion relative to the first, will not agree on the space-time coordinates of the same events. As a result, the same phenomenon will have a varying duration depending on the observer. This distinction leads to the necessity for defining physical time (Borghi 2016).

A key question remains about how time interacts with physical quantities, especially concerning thermodynamic variables. All processes involve dissipation and irreversibility, which give rise to the concept of the arrow of time. Recently, we have introduced a definition of time (Lucia and Grisolia 2019, 2020) related to the analysis of irreversibility (Lucia 2016) in open quantum systems, simplified in the study of interaction between electromagnetic waves and atomic electrons in an Hydrogen-like atom (Lucia 2016, 2018, 2023). In 2011, Perlmutter, Schmidt, and Reiss made the groundbreaking discovery that the Universe's expansion is accelerating, which contradicts the expectation that gravity would slow this process. This finding leads to essential inquiries about dark energy, which makes up roughly three-quarters of the Universe's mass-energy and drives this acceleration. Recently, we have introduced a thermodynamic perspective that considers a potential cosmological Joule-Thomson effect (Lucia 2025), supported by the Universe's significant entropy, mainly resulting from black-body radiation. In an open system, entropy evolution involves interactions with the surroundings and is characterized by two elements: entropy flow and internal entropy production. To understand these dynamics, it is crucial to grasp irreversible processes, as they connect order and disorder. This perspective alters our view of thermodynamics, indicating that the formation of order is linked to non-equilibrium conditions, while disorder is typically associated with stable equilibrium. This new understanding calls for a reassessment of how natural systems are analysed, focusing on the complex interplay between order, disorder, and the nature of time. This short communication seeks to enhance this approach by examining the pressure resulting from irreversibility in relation to space-time interpretation and proposing a conjecture concerning the geometric implications of this thermodynamic hypothesis.

The large-scale world exhibits irreversibility (O'Byrne *et al.* 2022), which aligns with our daily experiences. A clear demonstration is that no one can reverse time. Conversely, all fundamental physical laws exhibit symmetry concerning the reversal of the time arrow. Ludwig Boltzmann was the first scientist to offer a convincing explanation for this paradox, recognised for two primary contributions to physics: the physical and mathematical interpretation of entropy as a measure of what we commonly refer to today as atomic disorder, and the equation known as the Boltzmann equation (Cercignani 1998). This equation describes the statistical characteristics of a gas composed of molecules, representing the initial effort to govern the time evolution of probabilities (Cercignani 1988).

Boltzmann's perspective on entropy behaviour revolves around his definition of statistical entropy, which does not take molecular correlations into account, a concept later introduced by Paul and Tatjana Pavlovna Ehrenfest as molecular chaos (P. Ehrenfest and T. Ehrenfest 1959). The time progression of a closed system starting from an initially highly ordered state results in a reduction of the initial macroscopic and microscopic free energy, manifesting as molecular correlations. Mathematically and physically, this translates to an increase in Boltzmann entropy. However, the microscopic laws are articulated by deterministic and reversible differential equations grounded in Newton's classical dynamics or quantum mechanical motion equations, leading to time reversibility for conservative systems described by a real Hamiltonian (Doyle 2016). Gibbs expanded on this by including molecular correlations among particles in his treatment of statistical thermodynamics, demonstrating that Gibbs entropy preserves the path probability encoded in molecular correlations (Gibbs 1960).

A significant challenge in current statistical mechanics is understanding how indeterministic macroscopic irreversibility can emerge from deterministic and reversible microscopic motions (Lebowitz 1999; Gollub and Pine 2006). In this section, we present our proposed space-time changes attributed to quantum irreversibility. To do this, we first summarise our earlier findings regarding the origin of irreversibility (Lucia 2016, 2023), which we explained through the continuous electromagnetic interaction between atoms and their environment due to thermal disequilibrium.

To achieve this, we have analysed a Hydrogen-like atom interacting with an electromagnetic wave (Lucia 2016, 2023). The electromagnetic wave can be understood as a stream of photons. These photons enter and exit the atoms. At the atomic scale, photons may be absorbed by electrons in atoms or molecules, leading to electronic transitions between two stationary energy states of the atom. Subsequently, excited electrons can emit photons as they return to their original energy levels. While the atom's energy remains unchanged, there is a shift in the electronic transition. Conversely, there is a change in the kinetic energy of the atom's center of mass (E. Condon 1926; E. U. Condon 1928; Alonso and Finn 1968); however, this change is minor compared to the energy variation in electronic transitions, and its time duration (10^{-13} s) exceeds the duration of electronic transitions (10^{-15} s) (Alonso and Finn 1968). Nevertheless, this contribution to energy balance becomes significant when we analyse a large number of interactions, as is common in macroscopic systems. Any atomic stationary state is fully defined by its energy level, which is represented by the principal quantum number. Each electronic transition, occurring between two identified energy levels marked as f for the final state and i for the initial state, adheres to the quantum selection rule: $\Delta n = n_f - n_i = \pm 1$. We continue our investigation by focusing on a Hydrogen-like atom with atomic number Z as it interacts with an external electromagnetic wave. For this type of atom, utilising the methods found in spectroscopy, the external orbital electron can be characterised by the following parameters:

- the apparent atomic radius (Alonso and Finn 1968):

$$r_n = \frac{4\pi\varepsilon_0\hbar^2}{m_e Z e^2}\, n^2 \tag{1}$$

where ε_0 is the electric permittivity, $\hbar = h/2\pi$ with h the Planck constant, m_e is the mass of the electron, e is the elementary charge, $n = 1, 2, 3, \ldots$, is the principal quantum number, always an integer;

- the energy of the atomic level (Alonso and Finn 1968):

$$E_n = \frac{m_e}{2}\left(\frac{Ze^2}{4\pi\varepsilon\hbar}\right)^2 \frac{1}{n^2} \tag{2}$$

- the Sommerfeld-Wilson rule (Alonso and Finn 1968):

$$\oint p_e\, dr_n = n\hbar \tag{3}$$

where $p_e = m_e v_e$ is the electronic momentum, with v_e its mean velocity inside the orbital, $p_e \cdot r_n$ is the angular momentum of the electron.

Maintaining always generality, we selected a geometric reference system centred at the nucleus's center of mass, placing the atom at rest with zero momentum ($\mathbf{p}_{atm}$) in its initial state, before the electromagnetic interaction. The corresponding Schrödinger equation is given by (Slater 1968):

$$\left[-\frac{\hbar^2}{2m_N}\nabla^2_{\mathbf{r}_N} - \frac{\hbar^2}{2m_e}\nabla^2_{\mathbf{r}_e} + V\left(\mathbf{r}_e - \mathbf{r}_N\right)\right]\psi = E_{tot}\,\psi \tag{4}$$

where m_N is the mass of the nucleus, m_e is the mass of the electron, $\mathbf{r}_N$ is the nucleus coordinate, $\mathbf{r}_e$ is the electron coordinate, $V\left(\mathbf{r}_e - \mathbf{r}_N\right)$ is the electrostatic potential, E_{tot} is the total energy and ψ is the wave function. Now, following the usual quantum mechanical approach (Alonso and Finn 1968; Slater 1968) we introduce the quantities useful for the analysis of the two bodies problem:

- the relative coordinates $\mathbf{r} = \mathbf{r}_e - \mathbf{r}_N$;
- the total mass $M = m_N + m_e$;
- the coordinates of the center of mass $\mathbf{R} = (m_e\,\mathbf{r}_e + m_N\,\mathbf{r}_N)/M$;
- the reduced mass $\mu = \left(m_e^{-1} + m_N^{-1}\right)^{-1}$;
- the momentum of the center of mass $\mathbf{P} = M\dot{\mathbf{R}} = -i\hbar\nabla_{\mathbf{R}}$;
- the momentum of the reduced mass particle $\mathbf{p} = \mu\,\dot{\mathbf{r}} = -i\hbar\nabla_{\mathbf{r}}$;
- the wave function $\psi(\mathbf{r}, \mathbf{R}) = \varphi(\mathbf{r}) \cdot \vartheta(\mathbf{R})$;
- the energy $E_{CM} = \mathbf{P}^2/2M$ of the free particle center of mass;
- the energy E_μ of the bound particle of reduced mass, such that $E_{tot} = E_{CM} + E_\mu$;
- the electrostatic potential $V(\mathbf{r}) = -Ze^2/r$.

Consequently, the Schrödinger equation results in:

$$\left[-\frac{\hbar^2}{2M}\nabla^2_{\mathbf{R}} - \frac{\hbar^2}{\mu}\nabla^2_{\mathbf{r}} + V\left(\mathbf{r}\right)\right]\psi(\mathbf{r}, \mathbf{R}) = E_{tot}\,\psi(\mathbf{r}, \mathbf{R}) \tag{5}$$

that brings to the following two differential equations:

$$
\begin{aligned}
&-\frac{\hbar^2}{2M}\nabla^2_{\mathbf{R}}\,\vartheta(\mathbf{R}) = E_{CM}\,\vartheta(\mathbf{R})\\
&\left(-\frac{\hbar^2}{\mu}\nabla^2_{\mathbf{r}} + V\big(\mathbf{r}\big)\right)\varphi(\mathbf{r}) = E_{\mu}\,\varphi(\mathbf{r})
\end{aligned}
\tag{6}
$$

We now introduce the approach to interaction between the atomic electron with an external electromagnetic wave. Electromagnetic radiation can be described as a stream of photons, with the following properties (Alonso and Finn 1968):

- the energy E_γ:

$$E_\gamma = h\,\nu \tag{7}$$

  where ν is the frequency of the electromagnetic wave;
- the momentum $\mathbf{p}_\gamma$:

$$\mathbf{p}_\gamma = \frac{h\nu}{c}\,\hat{\mathbf{u}}_c \tag{8}$$

  where c is the speed of light and $\hat{\mathbf{u}}_c = \mathbf{c}/c$ is the direction of the photon.

Considering an atom with a principal quantum number n, we define our control volume as a sphere with a radius of $n+1$, centred at the atom's center of mass. Thus, the interaction between electromagnetic radiation and the hydrogen-like atom can be studied as the exchange between the photon flux and an open system (the atom with principal quantum number n) across the boundary of the control volume defined by the sphere of that radius (Alonso and Finn 1968):

$$r_{n+1} = \frac{4\pi\varepsilon_0\hbar^2}{m_e Z e^2}\,(n+1)^2 \tag{9}$$

with center in the center of the atomic nucleus. The atomic electron absorbs the incoming photon only if the photon frequency ν is the resonant frequency required by the transition between the initial E_i and final E_f energy levels (Alonso and Finn 1968), corresponding to the quantised energy:

$$\nu = \frac{E_f - E_i}{h} \tag{10}$$

Considering the absorption process, at the initial state we use a geometric reference system centred on the center of mass of the nucleus, so the atom results at rest with null initial momentum $\mathbf{p}_{atm}$. When the atomic electron absorbs the incoming photon, the atomic momentum becomes (Alonso and Finn 1968; Lucia 2016):

$$\mathbf{p}_{atm} = -\mathbf{p}_\gamma = \frac{h\nu}{c}\,\hat{\mathbf{u}}_c \tag{11}$$

Due to the absorption of the incoming photon, the electron experiences a transition between energy levels, moving from the initial energy state E_i to the final energy state E_f. The atom's final energy can be determined through energy balance analysis, leading to the following evaluation (Alonso and Finn 1968; Lucia 2016):

$$E_f = E_i + h\nu - \frac{p_{atm}^2}{2M} = E_i + h\nu - \frac{(h\nu)^2}{2Mc^2} \tag{12}$$

where $p_{atm}^2/2M = (h\nu)^2/2Mc^2$ is the kinetic energy gained by the atom, with M the mass of the atom. Thus, it follows (Alonso and Finn 1968; Lucia 2016):

$$h\nu = \frac{E_f - E_i}{1 - \dfrac{h\nu}{2Mc^2}} \tag{13}$$

Both the conservation of momentum and the conservation of energy laws apply as a result of the photon's absorption:

- the relative velocity electron-nucleus $\dot{\mathbf{r}}_{ab} = \dot{\mathbf{r}}_{e,n+1} - \dot{\mathbf{r}}_N = \mathbf{p}_{e,n+1}/m_e + \mathbf{p}_N/m_N$;
- the velocity of the center of mass $\dot{\mathbf{R}}_{ab} = (m_e\,\dot{\mathbf{r}}_{e,n+1} + m_N\,\dot{\mathbf{r}}_N)/M = (\mathbf{p}_{e,ab} + \mathbf{p}_N)/M$;
- the momentum of the center of mass $\mathbf{P}_{ab} = M\,\dot{\mathbf{R}}_{ab}$;
- the momentum of the reduced mass particle $\mathbf{p}_{ab} = \mu\,\dot{\mathbf{r}}_{ab}$;
- the wave function $\psi(\mathbf{r}_{ab}, \mathbf{R}_{ab}) = \varphi(\mathbf{r}_{ab}) \cdot \vartheta(\mathbf{R}_{ab})$;
- the energy $E_{CM} = \mathbf{P}_{ab}^2/2M$ of the free particle center of mass;
- the coordinates of the center of mass $\mathbf{R}_{ab} = \int \mathbf{P}_{ab}\,dt/M$,

where $\mathbf{p}_N$ is the momentum of the nucleus and $\mathbf{p}_e$ is the momentum of the electron.

The emission of this photon results in the reverse process. Based on the same approach of the absorption, but now for the emission of a photon, we can obtain (Alonso and Finn 1968; Lucia 2016):

$$h\nu = \frac{E_f - E_i}{1 + \dfrac{h\nu}{2Mc^2}} \tag{14}$$

and

- the relative velocity electron-nucleus $\dot{\mathbf{r}} = \dot{\mathbf{r}}_{e,n} - \dot{\mathbf{r}}_N = \mathbf{p}_{e,n}/m_e + \mathbf{p}_N/m_N$;
- the velocity of the center of mass $\dot{\mathbf{R}} = (m_e\,\dot{\mathbf{r}}_{e,n} + m_N\,\dot{\mathbf{r}}_N)/M = (\mathbf{p}_e + \mathbf{p}_N)/M$;
- the momentum of the center of mass $\mathbf{P} = M\,\dot{\mathbf{R}}$;
- the momentum of the reduced mass particle $\mathbf{p} = \mu\,\dot{\mathbf{r}}$;
- the energy $E_{CM} = \mathbf{P}^2/2M$ of the free particle center of mass;
- the coordinates of the center of mass $\mathbf{R} = \int \mathbf{P}\,dt/M$,

But, considering Equations (13) and (14), it follows that after the whole process of absorption and emission there is an energy footprint (Lucia 2016, 2023):

$$E_{ftp} = \frac{m_e}{M}\,h\nu \tag{15}$$

Now, a paradox emerges. Indeed, from the analysis of the two processes individually, it emerges that the variation in kinetic energy of the nucleus should be zero, as well as the variation in momentum, while the study of the energy variation of the photon obtains an energy footprint. In our previous papers on the origin of irreversibility (Lucia 2016, 2018), we have suggested that this footprint is the cause of irreversibility. Moreover, we have also suggested that this footprint is the origin of time, that we have defined by using the entropy production (Lucia and Grisolia 2019, 2020). Indeed, following the dimensional analysis in thermodynamics (Langhaar 1951), we have proposed the definition of time, τ, as follows (Lucia and Grisolia 2019, 2020):

$$\tau = \frac{s_i}{\sigma} \tag{16}$$

where, the entropy production was defined as (Lucia and Grisolia 2019):

$$s_i = \frac{1}{V\,T} E_{ftp} \tag{17}$$

with V the control volume (the volume of the Hydrogen-like atom considered, or of the systems of atoms for a macroscopic approach), and the entropy production rate density σ was written in relation to the electromagnetic waves in the Universe, by considering the Gouy-Stodola theorem (Bejan 2006), as follows (Beretta and Gyftopoulos 2015):

$$T\,\sigma = \frac{A}{2V}\varepsilon_0 c E_{el}^2 + \frac{A}{2\mu_0 V} c B_m^2 \tag{18}$$

where E_{el} is the mean value of the electric field, B_m is the mean value of the magnetic field, ε_0 is the electric permittivity in vacuum and μ_0 is the magnetic permeability in vacuum.

From these results we can state that time is energy, lost by Universe during any electromagnetic interaction between electromagnetic waves (photons) and atomic electrons. Moreover, the apparent paradox can be solved if we consider that E_{ftp} can be rewritten as:

$$E_{ftp} = \frac{\mathbf{P}_{CM,final}^2 - \mathbf{P}_{CM,final}^2}{2M} \Rightarrow \dot{\mathbf{R}}_{final} = \frac{1}{M}\sqrt{2\,m_e\,h\nu + \mathbf{P}_{CM}^2} \tag{19}$$

where the subscript '*final*' refers to the whole process of absorption-emission. Consequently, we can obtain the new position of the center of mass:

$$\mathbf{R}_{final} = \int \dot{\mathbf{R}}_{final}\,dt \tag{20}$$

In this approach the wave function after the absorption-emission process could be $\psi(\mathbf{r},\mathbf{R}_{ab}) = \varphi(\mathbf{r})\cdot\vartheta(\mathbf{R}_{ab})$, while if we evaluate it by using the Schrödinger approach (Slater 1968) it results $\psi(\mathbf{r},\mathbf{R}_{ab}) = \varphi(\mathbf{r})\cdot\vartheta(\mathbf{R})$. Thus, we propose to consider that the energy lost which we conjecture that causes time variation, could also be able to change locally the geometry of space-time from $\mathbf{R}$ to $\mathbf{R}_{ab}$, leaving invariant the analytical expression of the Schrödinger equations and solutions.

Conflicts of interest

The author declares that he has no conflicts of interest.

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* Politecnico di Torino,
Dipartimento Energia "Galileo Ferraris"
Corso Duca degli Abruzzi 24, 10129 Turin, Italy

Email: umberto.lucia@polito.it

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