The Fredholm Factorization Method Directly Applied to Generalized Wiener-Hopf Equations for Wedge Diffraction Problems in Complex Media

The Fredholm Factorization Method Directly Applied to Generalized Wiener-Hopf Equations for Wedge Diffraction Problems in Complex Media

Vito Daniele[(1)], Guido Lombardi[(1)]
(1) Politecnico di Torino, Italy, e-mail: vito.daniele@polito.it, guido.lombardi@polito.it

In this work we present a new and comprehensive theory for the solution of electromagnetic diffraction problems involving impenetrable wedges in arbitrary linear media. This theory utilizes Bresler-Marcuvitz transverse equations, characteristic Green's function procedure, and the Wiener-Hopf technique to address complex scattering problems [1]. The effectiveness of this technique has already been demonstrated in the analysis of wedge problems in isotropic media [2], and now we have extended its application to more general cases [3-4]. We derive Generalized Wiener-Hopf equations (GWHEs) from spectral functional equations in angular regions filled with arbitrary linear media with the application of boundary conditions as solution of transverse equations. These equations can be systematically interpreted with network formalism. We recall that GWHEs have plus and minus unknowns that are defined into different complex planes but related together. We believe that this mathematical technique significantly expands the possibilities for spectral analysis of electromagnetic problems involving angular regions filled with complex arbitrary linear media. We observe that the GWHEs in arbitrary linear media usually report physical unknowns defined into multiple complex planes (more than 2) as the physical problem usually contains more than one propagation constant. Traditional spectral methods such as Sommerfeld-Malyuzhinets (SM) method, the Kontorovich-Lebedev (KL) transform method, and the Wiener-Hopf (WH) method are fundamental and complementary in studying diffraction problems in presence of sharp discontinuities immersed in isotropic region. The primary benefit in utilizing the SM and KL techniques is the application of the spectral complex angular plane derived from the Sommerfeld integral theory, which has also been successfully applied in the Wiener-Hopf framework through the concept of rotating waves in isotropic angular region, see for instance [2] and references therein. However, the definition of this complex plane is possible in problem with one propagation constant. One of the most important result obtained in presence of anisotropic media is the exact solution obtained by Felsen in the case of the scattering by a PEC wedge immersed in uniaxial medium illuminated by plane waves at normal incidence [5], by applying a generalized version of separation of variables. In the present work, we apply for the first times direct Fredholm factorization [6] to GWHEs avoiding introduction of spectral mapping to resort to CWHEs. In particular the method is effective for problems with more than one propagation constat where the spectral mapping cannot be introduced, and other techniques are ineffective. The fundamental tool in direct Fredholm factorization is the application of Cauchy integral representation to GWHEs that allows to represent unknowns in one complex plane starting from multiple complex planes. With this tool we allow all unknows of GWH problem to be represented using integral equations in one complex plane that can be regularized with Fredholm factorization procedure, see for instance [6,7]. The proposed methodology is validated through examples, beginning with the examination of diffraction by a PEC wedge in anisotropic media**.**

This work was supported by Next Generation EU within PNRR M4C2, Inv 1.4-Avv n.3138 16/12/2021-CN00000013 National Centre for HPC, Big Data and Quantum Computing(HPC).

1. V. Daniele, G. Lombardi, "The Generalized Wiener-Hopf Equations for wave motion in angular regions: electromagnetic application," *Proc. R. Soc. A*, 477:20210040, n.2252, pp. 1-27, 2021, doi:10.1098/rspa.2021.0040

2. V. Daniele, G. Lombardi, *Scattering and Diffraction by Wedges 2: The Wiener-Hopf Solution - Advanced Applications*, Hoboken, NJ: John Wiley & Sons, Inc. London, UK: ISTE, 2020

3. V. Daniele, G. Lombardi, "Radiation and Scattering of an Arbitrarily Flanged Dielectric-Loaded Waveguide," *IEEE Trans. Antennas Propag.,* vol. 67, no. 12, pp. 7569-7584, Dec. 2019

4. V. Daniele, G. Lombardi, V. Daniele, G. Lombardi and R. S. Zich, "Physical and Spectral Analysis of a Semi-Infinite Grounded Slab Illuminated by Plane Waves," *IEEE Trans. Antennas Propag.,* vol. 70, no. 12, pp. 12104-12119, Dec. 2022

5. L. B. Felsen, "Propagation and diffraction in uniaxially anisotropic regions. i—theory. ii—applications," *Proc. Inst. Elect. Eng.*, vol. 111, no.3, pp. 445–464, Mar. 1964

6. V. Daniele, G. Lombardi, "Spectral Analysis of Electromagnetic Diffraction Phenomena in Angular Regions Filled by Arbitrary Linear Media," *Applied Science*, to be submitted, 2024

7. V. Daniele, G. Lombardi, "Fredholm factorization of Wiener-Hopf scalar and matrix kernels," *Radio Science*, 42(06), pp. 1-19, 2007.