

# Appendix for “Identifying Imbalance Thresholds in Input Data to Achieve Desired Levels of Algorithmic Fairness”

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## **APPENDIX A: predictors and targets**

For each dataset that we analyzed in our study, we provide predictors and target variables employed in the different classification tasks.

### **Credit card default**

*Predictors:*

- LIMIT\_BAL: amount of given credit
- PAY\_0: repayment status in September, 2005 (-1=pay duly, 1=payment delay for one month, ..., 8=payment delay for eight months, 9=payment delay for nine months and above)
- BILL\_ATM1: amount of bill statement in September, 2005
- PAY\_AMT1: amount of previous payment in September, 2005

*Target:*

- default.payment.next.month.f: default payment (yes=1, no=0)

### **Statlog**

*Predictors:*

- Purpose: purpose of the credit (car, furniture, business, etc.)
- Duration: duration in month
- Credit\_history: credit history
- Credit\_amount: credit amount

- Savings: savings account/bonds
- Employment\_since: present employment since
- Installment\_rate: installment rate in percentage of disposable income
- Other\_Debtors\_Guarantors: other debtors/guarantors
- Property: type of properties
- Housing: type of housing
- Residence\_since: present residence since
- Other\_installment\_plans
- Existing\_credits: number of existing credits in this bank
- Job: type of job
- People\_liable\_to\_provide\_maintenance\_for: number of people being liable to provide maintenance for
- Telephone: present or not

*Target:*

- costMatrix: good customer = 0, bad customer = 1

## **Census income**

*Predictors:*

- workclass: employment status
- occupation: type of occupation
- capital.gain: gains
- capital.loss: losses
- fnlwgt: final weight, based on the demographics
- education.num: level of education
- hours.per.week: hours of work per week
- marital.status: status of the marriage
- relationship: type of relationship

*Target:*

- test\_income: under 50k = 0, over 50k = 1

## Student (Mathematics and Portuguese)

### *Predictors:*

- school: student's school
- address: student's home address type
- famsize: family size
- Pstatus: parent's cohabitation status
- reason: reason to choose this school
- nursery: attended nursery school
- internet: internet access at home
- studytime: weekly study time
- failures: number of past class failures
- paid: extra paid classes within the course subject
- activities: extra-curricular activities
- nursery: attended nursery school
- higher: wants to take higher education
- freetime: free time after school
- goout: going out with friends
- Dalc: workday alcohol consumption
- Walc: weekend alcohol consumption
- health: current health status
- absences: number of school absences
- guardian: student's guardian
- traveltime: home to school travel time
- famsup: family educational support
- romantic: with a romantic relationship
- famrel: quality of family relationships

### *Target:*

- G3\_target: final grade (less than or equal to 9 = 0, higher than 9 = 1)

## Drug consumption (Cannabis)

*Predictors:*

- Nscore: NEO-FFI-R Neuroticism
- Escore: NEO-FFI-R Extraversion
- Oscore: NEO-FFI-R Openness to experience
- Ascore: NEO-FFI-R Agreeableness
- Cscore: NEO-FFI-R Conscientiousness
- SS: sensation seeking measured by ImpSS

*Target:*

- Cannabis\_target: cannabis consumption (never used / used over a decade ago = 0, used less than a decade ago = 1)

## Drug consumption (Impulsive)

*Predictors:*

- Alcohol: consumption of alcohol
- Amphet: consumption of amphetamines
- Amyl: consumption of amyl nitrite
- Benzos: consumption of benzodiazepine
- Caff: consumption of caffeine
- Cannabis: consumption of cannabis
- Choc: consumption of chocolate
- Coke: consumption of cocaine
- Crank: consumption of crack
- Ecstasy: consumption of ecstasy
- Heroin: consumption of heroine
- Ketamine: nconsumption of ketamine
- Legalh: consumption of legal highs
- LSD: consumption of lsd
- Meth: consumption of methadone

- Mushrooms: consumption of magic mushrooms
- Nicotine: consumption of nicotine
- VSA: consumption of volatile substances
- Semer: consumption of the fictious drug “Semeron”

*Target:*

- Impulsive: impulsivess rating (less than or equal to average = 0, higher=1)

## **Heart disease**

*Predictors:*

- cp: chest pain type
- trestbps: resting blood pressure
- chol: serum cholestorol
- fbs: fasting blood sugar
- restecg: relieved after rest
- thalach: maximum heart rate achieved
- exang: exercise induced angina
- oldpeak: ST depression induced by exercise relative to rest
- slope: the slope of the peak exercise ST segment
- ca: number of major vessels colored by flourosopy
- thal: state of blood disorder called thalassemia

*Target:*

- Diagnosis: absence of disease = 0, presence = 1

## APPENDIX B: configurations of the thresholds

In this appendix we provide the five configurations of thresholds that we defined during the procedure of Identification of Risk Thresholds. The configurations have been built so as to distribute the values of  $f$  evenly in the range with the highest concentration of unfairness values, which is approximately between the minimum and the mean of the distribution (around the first quartile). Hence, for each configuration we specify the two theoretical values of unfairness thresholds that we chose a priori,  $f1\_base$  and  $f2\_base$ , or  $f\_base$  if we are in the case of only one threshold defined a priori. In the five figures, we report a violin plot that represents the compact display of the (continuous) distribution of the values of the Separation criterion in the case of the True Positive rate; indeed, this kind of plot allows to show the probability density of the data at different values. Thus, we took as reference the Sep\_TP criterion as it is the one with the largest range of values with respect to the other fairness criteria (which presented very high probability density in correspondence of very low values). In the figures, we color  $f1\_base$  and  $f2\_base$  with gray and highlight their mean  $f$  in red color (or  $f\_base$  in the case of the definition of only one threshold), in a way such that the red line is progressively moved to the left (where we observe the highest concentration of unfairness values). In particular: configurations 1, 2 and 4 belong to the general case of the two thresholds  $f1\_base$  and  $f2\_base$  defined a priori, whereas configurations 3 and 5 belong to the case with only one  $f\_base$  threshold.

### Configuration 1

- $f1\_base$ : *1st quartile*
- $f2\_base$ : *mean*

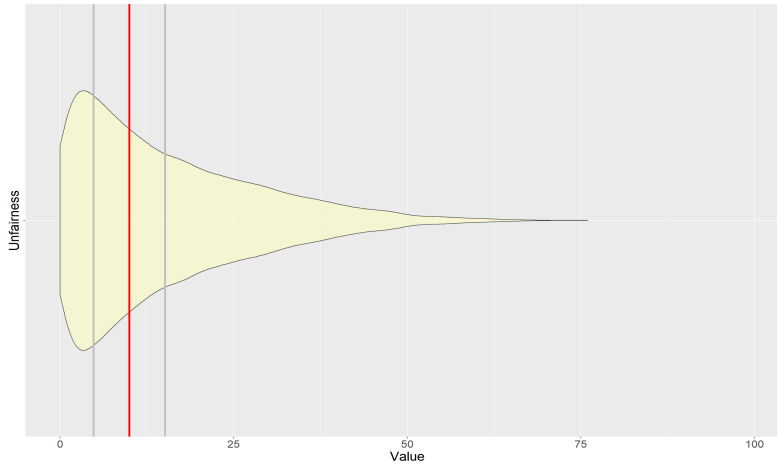


Figure 1: Thresholds Configuration 1.

## Configuration 2

- f1\_base: mean between *minimum* and *1st quartile*
- f2\_base: mean between *1st quartile* and *mean*

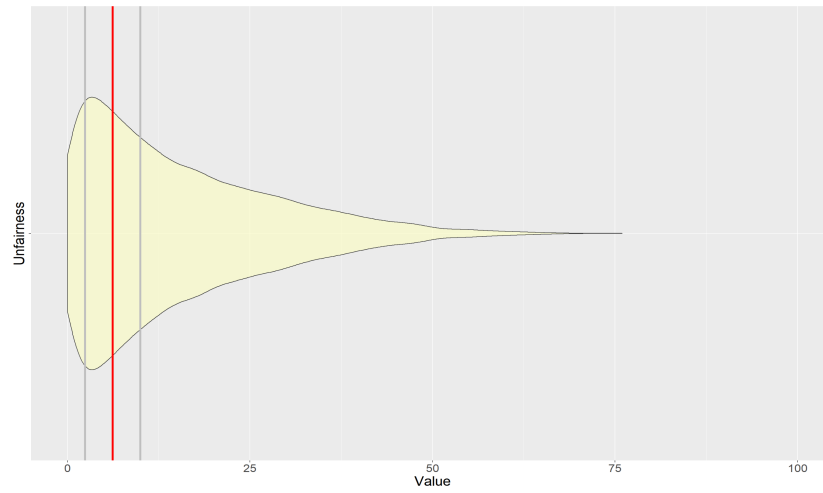


Figure 2: Thresholds Configuration 2.

## Configuration 3

- f\_base: *1st quartile*

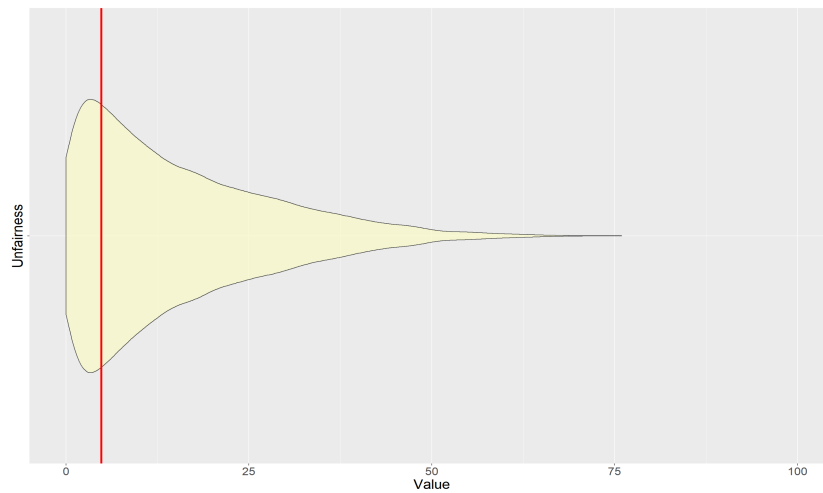


Figure 3: Thresholds Configuration 3.

### Configuration 4

- f1\_base: mean between *minimum* and *1st quartile*
- f2\_base: *1st quartile*

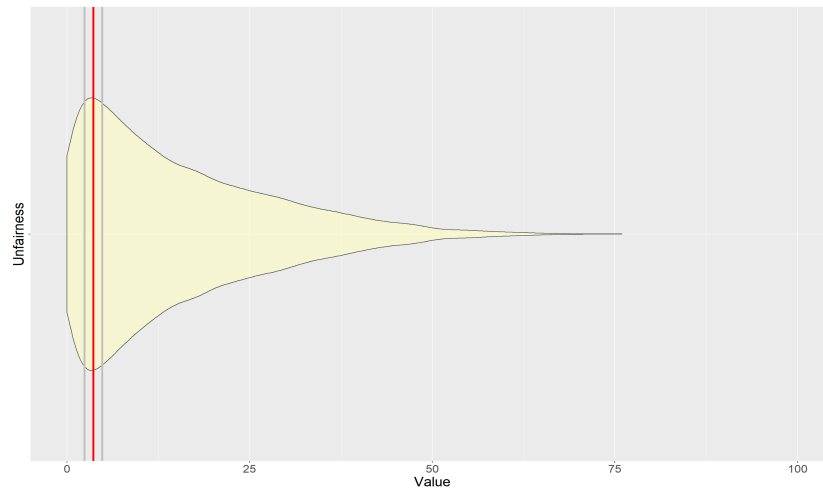


Figure 4: Thresholds Configuration 4.

### Configuration 5

- f\_base: mean between *minimum* and *1st quartile*

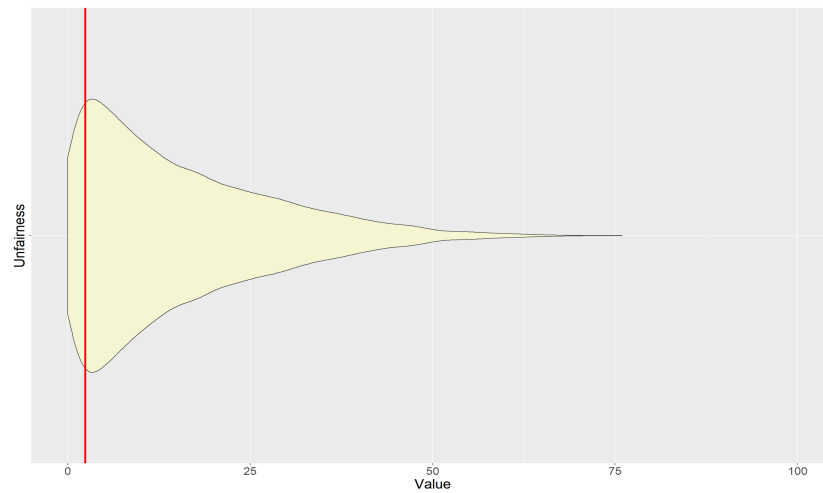


Figure 5: Thresholds Configuration 5.



## APPENDIX C: thresholds tables

In this Appendix, for each combination of balance-unfairness-algorithm we report the best thresholds selected by accuracy, the configuration they correspond to (among the 5 options described in Appendix B), and all the evaluation metrics related to those thresholds. For sake of legibility, we report values for the thresholds of both fairness criteria and balance measures multiplied by 100, i.e. on a scale  $[0, 100]$ . Results are presented in separate tables for the binary and the multiclass cases, grouped by balance measure (Gini, Shannon, Simpson, and IR indexes), and ordered by unfairness measures (Independence, Separation\_TP, Separation\_FP, Sufficiency\_PP, and Sufficiency\_PN criteria); then, in each table results vary according to the algorithm used in the classification task (logistic regression, support vector machine, random forest, k-nearest neighbors). Finally, we remind that the aim of this study was to define two thresholds  $s$  (for balance measures) and  $f$  (for unfairness measures) such that if the balance of the training set is greater than  $s$ , then the unfairness of the classification on the test set is expected to be less than  $f$ .

### Binary attributes

#### *Gini index*

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Gini	Independence	logit	3	97,62	3,20	0,68	0,75	0,21	0,84	0,79
Gini	Independence	svm	5	95,49	1,66	0,68	0,86	0,30	0,75	0,80
Gini	Independence	rf	3	80,59	4,01	0,54	0,76	0,41	0,58	0,66
Gini	Independence	knn	3	96,18	2,33	0,65	0,77	0,21	0,78	0,77

Table 1: Thresholds and evaluation metrics for the combination Gini-Independence in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Gini	Sep_TP	logit	3	94,55	4,72	0,67	0,80	0,33	0,76	0,78
Gini	Sep_TP	svm	3	79,29	3,99	0,63	0,86	0,56	0,64	0,73
Gini	Sep_TP	rf	5	99,99	3,31	0,85	0,88	0,07	0,96	0,92
Gini	Sep_TP	knn	5	99,96	2,06	0,80	0,85	0,11	0,93	0,89

Table 2: Thresholds and evaluation metrics for the combination Gini-Sep\_TP in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Gini	Sep_FP	logit	3	84,68	2,89	0,61	0,76	0,42	0,68	0,72
Gini	Sep_FP	svm	1	38,44	6,77	0,49	0,48	0,58	0,40	0,44
Gini	Sep_FP	rf	1	54,58	7,875	0,61	0,55	0,64	0,59	0,57
Gini	Sep_FP	knn	5	91,50	0,91	0,90	0,74	0,35	0,69	0,81

Table 3: Thresholds and evaluation metrics for the combination Gini-Sep\_FP in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Gini	Suf_PP	logit	5	72,02	2,50	0,58	0,89	0,50	0,59	0,71
Gini	Suf_PP	svm	5	85,00	2,16	0,67	0,91	0,46	0,70	0,79
Gini	Suf_PP	rf	4	96,53	4,46	0,74	0,82	0,29	0,86	0,84
Gini	Suf_PP	knn	3	96,27	6,10	0,66	0,71	0,26	0,85	0,77

Table 4: Thresholds and evaluation metrics for the combination Gini-Suf\_PP in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Gini	Suf_PN	logit	5	95,77	2,17	0,71	0,88	0,21	0,78	0,82
Gini	Suf_PN	svm	5	95,80	1,92	0,72	0,88	0,25	0,79	0,83
Gini	Suf_PN	rf	1	67,60	7,84	0,62	0,60	0,53	0,71	0,65
Gini	Suf_PN	knn	5	96,41	2,47	0,73	0,85	0,25	0,82	0,84

Table 5: Thresholds and evaluation metrics for the combination Gini-Suf\_PN in the case of binary attributes.

### *Shannon index*

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Shannon	Independence	logit	3	98,28	3,20	0,68	0,75	0,21	0,84	0,79
Shannon	Independence	svm	5	96,72	1,66	0,68	0,86	0,30	0,75	0,80
Shannon	Independence	rf	3	85,51	4,01	0,54	0,76	0,41	0,58	0,66
Shannon	Independence	knn	3	97,232,33	2,33	0,65	0,77	0,21	0,78	0,77

Table 6: Thresholds and evaluation metrics for the combination Shannon-Independence in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Shannon	Sep_TP	logit	3	96,03	4,72	0,67	0,80	0,33	0,76	0,78
Shannon	Sep_TP	svm	3	83,03	3,99	0,62	0,87	0,58	0,63	0,73
Shannon	Sep_TP	rf	5	99,99	99,99	0,85	0,88	0,08	0,95	0,92
Shannon	Sep_TP	knn	5	99,97	2,06	0,80	0,85	0,12	0,93	0,89

Table 7: Thresholds and evaluation metrics for the combination Shannon-Sep\_TP in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Shannon	Sep_FP	logit	3	88,65	2,89	0,61	0,76	0,41	0,68	0,72
Shannon	Sep_FP	svm	1	46,58	6,77	0,49	0,49	0,63	0,36	0,41
Shannon	Sep_FP	rf	1	63,19	7,88	0,61	0,55	0,64	0,58	0,57
Shannon	Sep_FP	knn	5	93,68	0,91	0,69	0,90	0,35	0,74	0,81

Table 8: Thresholds and evaluation metrics for the combination Shannon-Sep\_FP in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Shannon	Suf_PP	logit	5	77,06	2,50	0,57	0,89	0,50	0,58	0,70
Shannon	Suf_PP	svm	5	88,89	2,16	0,67	0,91	0,45	0,910,69	0,79
Shannon	Suf_PP	rf	4	97,48	4,46	0,74	0,82	0,29	0,86	0,84
Shannon	Suf_PP	knn	3	97,30	6,10	0,66	0,71	0,26	0,84	0,77

Table 9: Thresholds and evaluation metrics for the combination Shannon-Suf\_PP in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Shannon	Suf_PN	logit	5	96,93	2,17	0,71	0,88	0,21	0,78	0,82
Shannon	Suf_PN	svm	5	96,90	1,92	0,72	0,88	0,25	0,78	0,83
Shannon	Suf_PN	rf	1	73,07	7,84	0,62	0,61	0,56	0,68	0,64
Shannon	Suf_PN	knn	5	97,39	2,47	0,73	0,85	0,25	0,82	0,84

Table 10: Thresholds and evaluation metrics for the combination Shannon-Suf\_PN in the case of binary attributes.

### *Simpson index*

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Simpson	Independence	logit	3	95,35	3,20	0,68	0,75	0,21	0,84	0,79
Simpson	Independence	svm	5	91,37	1,66	0,68	0,86	0,30	0,75	0,80
Simpson	Independence	rf	3	67,50	4,01	0,54	0,76	0,41	0,58	0,66
Simpson	Independence	knn	3	92,65	2,33	0,65	0,77	0,21	0,770,78	0,77

Table 11: Thresholds and evaluation metrics for the combination Simpson-Independence in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Simpson	Sep_TP	logit	3	89,66	4,72	0,67	0,80	0,33	0,76	0,78
Simpson	Sep_TP	svm	3	73,11	3,99	0,65	0,85	0,47	0,70	0,77
Simpson	Sep_TP	rf	5	99,99	3,31	0,85	0,88	0,04	0,96	0,92
Simpson	Sep_TP	knn	5	99,93	2,06	0,80	0,85	0,11	0,93	0,89

Table 12: Thresholds and evaluation metrics for the combination Simpson-Sep\_TP in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Simpson	Sep_FP	logit	3	73,43	2,89	0,61	0,76	0,41	0,68	0,72
Simpson	Sep_FP	svm	1	29,43	6,77	0,49	0,49	0,56	0,43	0,46
Simpson	Sep_FP	rf	1	40,32	7,88	0,61	0,55	0,63	0,59	0,57
Simpson	Sep_FP	knn	5	85,47	0,91	0,70	0,90	0,35	0,74	0,81

Table 13: Thresholds and evaluation metrics for the combination Simpson-Sep\_FP in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Simpson	Suf_PP	logit	5	63,82	2,50	0,58	0,89	0,48	0,60	0,72
Simpson	Suf_PP	svm	5	73,91	2,16	0,67	0,91	0,45	0,69	0,79
Simpson	Suf_PP	rf	4	93,36	4,46	0,74	0,82	0,29	0,86	0,84
Simpson	Suf_PP	knn	3	92,82	6,10	0,66	0,71	0,26	0,84	0,77

Table 14: Thresholds and evaluation metrics for the combination Simpson-Suf\_PP in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Simpson	Suf_PN	logit	5	91,88	2,17	0,71	0,88	0,21	0,78	0,82
Simpson	Suf_PN	svm	5	92,54	1,92	0,74	0,88	0,23	0,81	0,84
Simpson	Suf_PN	rf	1	60,61	7,84	0,63	0,61	0,52	0,74	0,67
Simpson	Suf_PN	knn	5	93,07	2,47	0,73	0,85	0,25	0,82	0,84

Table 15: Thresholds and evaluation metrics for the combination Simpson-Suf\_PN in the case of binary attributes.

### *Imbalance Ratio index*

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
IR	Independence	logit	3	73,27	3,20	0,68	0,75	0,21	0,84	0,79
IR	Independence	svm	5	64,96	1,66	0,68	0,86	0,30	0,75	0,80
IR	Independence	rf	3	38,84	4,01	0,54	0,76	0,41	0,58	0,66
IR	Independence	knn	3	67,32	2,33	0,65	0,77	0,21	0,78	0,77

Table 16: Thresholds and evaluation metrics for the combination IR-Independence in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
IR	Sep_TP	logit	3	62,14	4,72	0,67	0,80	0,33	0,76	0,78
IR	Sep_TP	svm	4	51,20	3,00	0,70	0,89	0,43	0,74	0,81
IR	Sep_TP	rf	5	98,29	3,31	0,85	0,88	0,04	0,96	0,92
IR	Sep_TP	knn	5	96,28	2,06	0,80	0,85	0,11	0,93	0,89

Table 17: Thresholds and evaluation metrics for the combination IR-Sep\_TP in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
IR	Sep_FP	logit	3	43,74	2,89	0,61	0,76	0,41	0,68	0,72
IR	Sep_FP	svm	1	19,31	6,77	0,49	0,49	0,56	0,43	0,46
IR	Sep_FP	rf	1	22,13	7,88	0,62	0,55	0,63	0,60	0,57
IR	Sep_FP	knn	5	70,24	0,91	0,76	0,89	0,19	0,84	0,86

Table 18: Thresholds and evaluation metrics for the combination IR-Sep\_FP in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
IR	Suf_PP	logit	5	43,87	2,50	0,65	0,89	0,43	0,68	0,77
IR	Suf_PP	svm	5	44,16	2,16	0,67	0,91	0,45	0,69	0,79
IR	Suf_PP	rf	4	70,25	4,46	0,74	0,82	0,26	0,87	0,84
IR	Suf_PP	knn	3	67,64	6,10	0,66	0,71	0,26	0,84	0,77

Table 19: Thresholds and evaluation metrics for the combination IR-Suf\_PP in the case of binary attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
IR	Suf_PN	logit	5	65,88	2,17	0,71	0,88	0,21	0,78	0,82
IR	Suf_PN	svm	5	82,40	1,92	0,78	0,88	0,16	0,87	0,88
IR	Suf_PN	rf	1	52,13	7,84	0,58	0,55	0,35	0,81	0,66
IR	Suf_PN	knn	5	8,13	2,47	0,73	0,85	0,25	0,82	0,84

Table 20: Thresholds and evaluation metrics for the combination IR-Suf\_PN in the case of binary attributes.

## Multiclass attributes

### *Gini index*

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Gini	Independence	logit	1	88,54	8,45	0,44	0,43	0,55	0,33	0,38
Gini	Independence	svm	1	93,73	10,08	0,42	0,40	0,37	0,47	0,43
Gini	Independence	rf	5	91,09	3,61	0,47	0,92	0,50	0,47	0,62
Gini	Independence	knn	5	93,71	1,69	0,52	0,86	0,38	0,55	0,67

Table 21: Thresholds and evaluation metrics for the combination Gini-Independence in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Gini	Sep_TP	logit	1	92,23	10,04	0,51	0,65	0,54	0,49	0,56
Gini	Sep_TP	svm	2	94,64	8,21	0,56	0,80	0,44	0,59	0,68
Gini	Sep_TP	rf	5	99,83	4,55	0,77	0,89	0,14	0,84	0,87
Gini	Sep_TP	knn	1	93,50	9,98	0,53	0,64	0,55	0,52	0,57

Table 22: Thresholds and evaluation metrics for the combination Gini-Sep\_TP in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Gini	Sep_FP	logit	1,00	87,58	8,32	0,43	0,53	0,56	0,34	0,41
Gini	Sep_FP	svm	5,00	92,02	1,46	0,48	0,96	0,51	0,48	0,63
Gini	Sep_FP	rf	1,00	91,96	9,55	0,48	0,55	0,50	0,47	0,51
Gini	Sep_FP	knn	1,00	94,10	8,79	0,47	0,49	0,35	0,58	0,53

Table 23: Thresholds and evaluation metrics for the combination Gini-Sep\_FP in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Gini	Suf_PP	logit	1	87,96	13,29	0,48	0,56	0,59	0,40	0,46
Gini	Suf_PP	svm	1	89,61	11,99	0,47	0,57	0,56	0,41	0,47
Gini	Suf_PP	rf	5	95,16	4,85	0,59	0,93	0,34	0,61	0,73
Gini	Suf_PP	knn	1	86,83	12,88	0,55	0,60	0,72	0,39	0,47

Table 24: Thresholds and evaluation metrics for the combination Gini-Suf\_PP in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Gini	Suf_PN	logit	1	86,67	12,04	0,48	0,53	0,69	0,30	0,38
Gini	Suf_PN	svm	3	98,92	7,60	0,59	0,80	0,31	0,66	0,72
Gini	Suf_PN	rf	2	94,68	7,72	0,52	0,76	0,35	0,57	0,65
Gini	Suf_PN	knn	2	93,05	8,58	0,53	0,87	0,62	0,51	0,64

Table 25: Thresholds and evaluation metrics for the combination Gini-Suf\_PN in the case of multiclass attributes.

### *Shannon index*

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Shannon	Independence	logit	3	84,89	5,19	0,48	0,76	0,54	0,46	0,57
Shannon	Independence	svm	3	85,52	6,54	0,43	0,68	0,42	0,44	0,53
Shannon	Independence	rf	5	87,27	3,61	0,55	0,90	0,27	0,58	0,70
Shannon	Independence	knn	5	86,10	1,69	0,53	0,86	0,36	0,55	0,67

Table 26: Thresholds and evaluation metrics for the combination Shannon-Independence in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Shannon	Sep_TP	logit	4	87,50	5,14	0,57	0,82	0,41	0,60	0,69
Shannon	Sep_TP	svm	2	93,80	8,21	0,60	0,79	0,35	0,66	0,72
Shannon	Sep_TP	rf	5	99,79	4,55	0,79	0,89	0,14	0,87	0,88
Shannon	Sep_TP	knn	1	92,80	9,98	0,54	0,61	0,33	0,67	0,64

Table 27: Thresholds and evaluation metrics for the combination Shannon-Sep\_TP in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Shannon	Sep_FP	logit	1	83,97	8,32	0,47	0,57	0,51	0,44	0,49
Shannon	Sep_FP	svm	5	89,82	1,46	0,58	0,95	0,23	0,60	0,73
Shannon	Sep_FP	rf	3	86,44	4,36	0,52	0,74	0,33	0,58	0,65
Shannon	Sep_FP	knn	3	87,54	4,23	0,52	0,71	0,31	0,59	0,64

Table 28: Thresholds and evaluation metrics for the combination Shannon-Sep\_FP in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Shannon	Suf_PP	logit	1	84,93	13,29	0,53	0,61	0,57	0,50	0,55
Shannon	Suf_PP	svm	2	84,67	8,20	0,52	0,81	0,58	0,50	0,62
Shannon	Suf_PP	rf	2	93,24	8,72	0,61	0,80	0,34	0,68	0,74
Shannon	Suf_PP	knn	1	82,46	12,88	0,56	0,60	0,65	0,48	0,53

Table 29: Thresholds and evaluation metrics for the combination Shannon-Suf\_PP in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Shannon	Suf_PN	logit	1	81,04	12,04	0,46	0,50	0,64	0,31	0,38
Shannon	Suf_PN	svm	3	97,16	7,60	0,59	0,80	0,31	0,66	0,72
Shannon	Suf_PN	rf	2	94,02	7,72	0,59	0,79	0,34	0,66	0,72
Shannon	Suf_PN	knn	2	89,19	8,58	0,59	0,85	0,46	0,61	0,71

Table 30: Thresholds and evaluation metrics for the combination Shannon-Suf\_PN in the case of multiclass attributes.

### *Simpson index*

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Simpson	Independence	logit	1	62,19	8,45	0,50	0,51	0,54	0,47	0,49
Simpson	Independence	svm	3	66,97	6,54	0,43	0,68	0,42	0,44	0,53
Simpson	Independence	rf	4	66,00	5,42	0,47	0,85	0,50	0,47	0,60
Simpson	Independence	knn	5	74,88	1,69	0,53	0,86	0,36	0,55	0,67

Table 31: Thresholds and evaluation metrics for the combination Simpson-Independence in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Simpson	Sep_TP	logit	3	77,24	6,85	0,55	0,76	0,39	0,60	0,67
Simpson	Sep_TP	svm	2	84,92	8,21	0,60	0,79	0,35	0,66	0,72
Simpson	Sep_TP	rf	5	99,14	4,55	0,79	0,89	0,15	0,87	0,88
Simpson	Sep_TP	knn	1	83,05	9,98	0,54	0,62	0,40	0,63	0,62

Table 32: Thresholds and evaluation metrics for the combination Simpson-Sep\_TP in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Simpson	Sep_FP	logit	1	63,30	8,32	0,48	0,58	0,51	0,47	0,51
Simpson	Sep_FP	svm	5	76,82	1,46	0,58	0,95	0,23	0,59	0,73
Simpson	Sep_FP	rf	3	69,14	4,36	0,48	0,76	0,50	0,48	0,59
Simpson	Sep_FP	knn	3	73,71	4,23	0,48	0,73	0,48	0,48	0,58

Table 33: Thresholds and evaluation metrics for the combination Simpson-Sep\_FP in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Simpson	Suf_PP	logit	1	64,35	13,29	0,53	0,61	0,57	0,51	0,55
Simpson	Suf_PP	svm	3	66,23	8,84	0,52	0,78	0,57	0,50	0,61
Simpson	Suf_PP	rf	2	87,44	8,72	0,61	0,80	0,34	0,68	0,74
Simpson	Suf_PP	knn	1	60,97	12,88	0,56	0,58	0,57	0,55	0,56

Table 34: Thresholds and evaluation metrics for the combination Simpson-Suf\_PP in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
Simpson	Suf_PN	logit	1	53,71	12,04	0,47	0,51	0,71	0,27	0,35
Simpson	Suf_PN	svm	4	84,02	5,70	0,64	0,94	0,28	0,66	0,77
Simpson	Suf_PN	rf	2	85,30	7,72	0,59	0,79	0,34	0,66	0,72
Simpson	Suf_PN	knn	3	68,87	9,54	0,53	0,81	0,59	0,51	0,63

Table 35: Thresholds and evaluation metrics for the combination Simpson-Suf\_PN in the case of multiclass attributes.

### *Imbalance Ratio index*

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
IR	Independence		3	18,61	5,19	0,59	0,76	0,35	0,67	0,71
IR	Independence	svm	4	10,92	4,91	0,56	0,83	0,27	0,61	0,70
IR	Independence	rf	5	32,69	3,61	0,63	0,91	0,27	0,66	0,77
IR	Independence	knn	4	12,08	2,54	0,57	0,77	0,30	0,64	0,70

Table 36: Thresholds and evaluation metrics for the combination IR-Independence in the case of multiclass attributes.



Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
IR	Sep_TP	logit	5	24,12	3,42	0,65	0,89	0,41	0,68	0,77
IR	Sep_TP	svm	5	42,71	4,44	0,64	0,91	0,36	0,66	0,77
IR	Sep_TP	rf	5	80,85	4,55	0,80	0,89	0,13	0,88	0,89
IR	Sep_TP	knn	4	22,47	4,86	0,61	0,82	0,34	0,67	0,74

Table 37: Thresholds and evaluation metrics for the combination IR-Sep\_TP in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
IR	Sep_FP	logit	2	16,07	4,83	0,56	0,68	0,32	0,67	0,67
IR	Sep_FP	svm	5	56,18	1,46	0,65	0,95	0,23	0,67	0,78
IR	Sep_FP	rf	4	20,81	3,27	0,64	0,88	0,34	0,68	0,77
IR	Sep_FP	knn	4	18,33	3,17	0,59	0,79	0,28	0,88	0,72

Table 38: Thresholds and evaluation metrics for the combination IR-Sep\_FP in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
IR	Suf_PP	logit	1	30,65	13,29	0,54	0,58	0,35	0,68	0,63
IR	Suf_PP	svm	4	15,57	6,63	0,63	0,86	0,35	0,68	0,76
IR	Suf_PP	rf	5	18,36	4,85	0,66	0,94	0,34	0,68	0,79
IR	Suf_PP	knn	3	16,39	9,40	0,58	0,72	0,34	0,68	0,70

Table 39: Thresholds and evaluation metrics for the combination IR-Suf\_PP in the case of multiclass attributes.

Balance	Unfairness	Algorithm	Configuration	$s$	$f$	Accuracy	Precision	Specificity	Sensitivity	F1-score
IR	Suf_PN	logit	2	17,71	8,11	0,59	0,79	0,30	0,66	0,72
IR	Suf_PN	svm	4	25,90	5,70	0,64	0,94	0,28	0,66	0,77
IR	Suf_PN	rf	5	46,74	4,11	0,65	0,95	0,31	0,66	0,78
IR	Suf_PN	knn	4	19,22	7,16	0,63	0,88	0,35	0,67	0,76

Table 40: Thresholds and evaluation metrics for the combination IR-Suf\_PN in the case of multiclass attributes.