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Interpolatory Curl-Conforming Pyramidal Elements: Progress and Results

Roberto D. Graglia*, Paolo Petrini, and Damiano Franzò
Politecnico di Torino, Torino, Italy; e-mail: roberto.graglia@polito.it; paolo.petrini@polito.it;
damiano.franzo@polito.it

Advanced finite element codes for the analysis and synthesis of three-dimensional electromagnetic structures should be able to use hybrid meshes composed of cells of different shapes, namely cuboids, tetrahedra, prisms and pyramids. For all these elements, with the exception of pyramids, it has long been known how to define divergence-conforming or curl-conforming higher-order basis functions, whether interpolatory or hierarchical [1]. We know that for all elements it is better to work in a parent space where a parent cell is defined. The cells of the observer domain (otherwise called child domain) are obtained using appropriate nodal shape functions which, like the vector basis functions, are polynomials of the parent variables. Unfortunately, things are not so simple for the pyramid because it has four edges converging on its tip. Within a geometrically continuous hybrid mesh, i.e. without gaps between cells, the tangential or normal continuity of the vector basis functions on the boundaries of the pyramids is guaranteed only by accepting that the shape functions and the basis functions of the pyramid are fractional functions with a tip singularity. This meant that for a long time it was not possible to properly define the order of the bases nor was it possible to demonstrate the completeness of the set of shape functions and vector bases of the pyramidal element. As demonstrated in [2, 3], shape functions and vector bases take polynomial form if defined in a grandparent space, different from the parent one. In the grandparent space the pyramid has the shape of a cube (see Fig. 1). Therefore, using grandparent variables we can clearly define the order of the bases (since we work with sets of polynomials) while ensuring the required continuity from element to element.
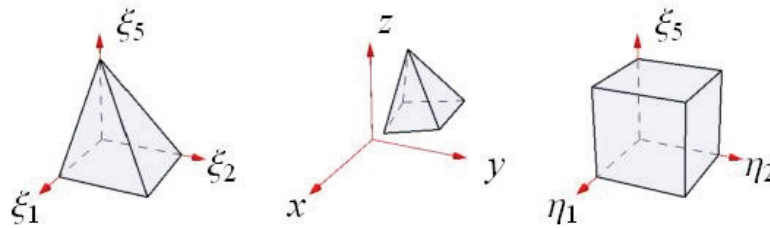

Figure 1. The parent pyramid on the left is mapped onto the child pyramid in the center by appropriate nodal shape functions. In the grandparent space the pyramid has the shape of a cube and the basis functions take polynomial form.

The hierarchical bases of the pyramid are defined and tested in [2, 4]. They are more convenient when using *p*-adaptive techniques. At the conference we will instead show a very simple technique for obtaining interpolatory curl-conforming bases and some results obtained with these. The bases in question complete the interpolatory families of the other cells of different shapes reported in [1] and are easily installed on a code that uses zero-order bases simply by adding a few subroutines.

1. R. D. Graglia, and A. F. Peterson, *Higher-order Techniques in Computational Electromagnetics*, SciTech Publishing/IET, Edison, NJ, 2016.

2 R. D. Graglia and P. Petrini, "Hierarchical curl-conforming vector bases for pyramid cells," *IEEE Trans. Antennas Propagat.,* vol. 70, no. 7, pp. 5623-5635, July 2022, doi: 10.1109/TAP.2022.3145430.

3. R. D. Graglia, "Hierarchical divergence-conforming vector bases for pyramid cells," *IEEE Trans. Antennas Propagat.,* vol. 71, no. 12, pp. 9334-9343, Dec. 2023, doi: 10.1109/TAP.2023.3241443.

4. R. D. Graglia and P. Petrini, "Assessing curl-conforming bases for pyramid cells," *IEEE Journal on Multiscale and Multiphysics Computational Techniques*, vol. 9, pp. 20-26, 2024, doi: 10.1109/JMMCT.2023.3333563.