DEFINITION OF THE FLEXIBLE AIRCRAFT LONGITUDINAL MODEL FOR A PRELIMINARY CONTROL DESIGN

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DEFINITION OF THE FLEXIBLE AIRCRAFT LONGITUDINAL MODEL FOR A PRELIMINARY CONTROL DESIGN

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ABSTRACT

Developing control system of high aspect ratio aircraft can be challenging due to flexibility involved in the control loop design. A model based approach can be straightforward to tune the control system parameters and, to this aim, a reliable aircraft flexible model is necessary. This paper aims to present the approach followed to design the longitudinal control strategy considering the aircraft simulator in the loop. The elastic modes are calculated from the lumped mass geometrical model and an aerodynamic properties from a reference aircraft. The approach and the model validation have been done in partnership with Leonardo Aircraft, as a thesis topic. Beginning with verification of the trim conditions, the flexible dynamic modes are compared to the rigid ones in order to highlight the relevant changes in the aircraft modes. A preliminary design of the longitudinal control strategy is herein proposed to achieve the dynamic response objectives.

Keywords: modelling, flexible, control, flight dynamics

1 INTRODUCTION

A suitable aircraft model is a key-to-success to design a control system for unconventional vehicle configuration. Some issues can arise when the frequency separation between natural modes of the aircraft dominated by the rigid-body degrees of freedom and those dominated by the elastic ones.
In fact, when structural displacements occur at frequencies that are comparable to those of the rigid body, the coupling between the rigid body motion and the flexibility of the structure cannot be neglected.
The present work deals with a preliminary evaluation of a control strategy for a slender body with aspect ratio higher than 10. The aircraft characteristics are not published here because of company confidentiality.
The structural displacements are considered small in order to consider the linear elastic theory still applicable. The elastic modes are derived using a lumped mass geometrical model and a commercial code for finite element analysis.
The paper introduces the notations and the reference frames used to derive the elastic contributions to the equations of motion. The Lagrange's formulation is adopted to develop the governing equations of the elastic body using the generalised forces that are expressed as function of the aerodynamic and propulsive coefficients. The latter definition of the generalised forces introduces the coupling between the rigid-body and elastic degrees of freedom. The described approach is used to derive a longitudinal model of the elastic aeroplane with the aim to study the elastic modes and to evaluate possible control strategy design to be tuned on the flexible aircraft model.

2 NOTATIONS AND REFERENCE FRAMES

In this work, vectors are indicated with bold lower-case letters (e.g. $\boldsymbol{v}$) and lower case letters (e.g. v) are used for vector components, whereas the matrices are in bold capital letters (e.g. $\boldsymbol{A}$). An inertial reference frame $\mathcal{F}_I = \{X_I, Y_I, Z_I\}$ is considered. A body non-inertial reference frame $\mathcal{F}_B = \{X_B, Y_B, Z_B\}$ is centred in the aircraft centre of gravity (CG) has axes oriented along fixed directions on board as described in Figure 1. The X-axis is contained in the symmetry plane and it is positive towards the aircraft nose, the Z-axis is contained in the symmetry plane and directed downward (i.e. from the upper to the lower wind surface), the Y-axis is defined to complete a left-handed coordinate system (i.e. towards the pilot's right hand side) [4]. The position of the generic mass element ρdV of Figure 1(a) can be written as

$$\boldsymbol{r}_I = \boldsymbol{r}_{CG} + \boldsymbol{p} \tag{1}$$

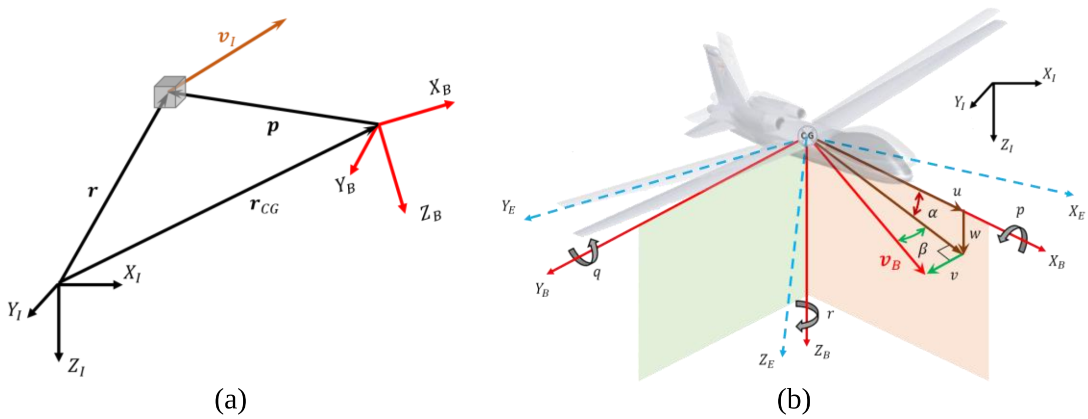
If each point can be treated as a point of mass and assuming that $\mathcal{F}_B$ is rotating with angular speed $\boldsymbol{\omega}$ in the inertial reference system $\mathcal{F}_I$, the velocity of the point of mass can be determined as

$$\frac{d\boldsymbol{r}_I}{dt} = \boldsymbol{v}_I = \frac{d\boldsymbol{r}_{CG}}{dt} + \frac{d\boldsymbol{p}}{dt} = \frac{d\boldsymbol{r}_{CG}}{dt} + \frac{\delta \boldsymbol{p}}{\delta t} + \omega \times \boldsymbol{p} \tag{2}$$

where the $\delta(\cdot)/\delta t$ is the time derivative operator with respect to the no-inertial reference frame and $d(\cdot)/dt$ is the time derivative operator in the inertial reference system
Dealing with flexible aircraft, it is convenient to introduce the aircraft mean axes that represent a non-inertial and time varying reference system $\mathcal{F}_E = \{X_E, Y_E, Z_E\}$ is centred in the current aircraft centre of gravity (CG) [1]. Considering the rigid aircraft body, the $\mathcal{F}_E \equiv \mathcal{F}_B$, when the vehicle is deformed, the mean axes are anchored to the aircraft structure and, therefore, they follow the new aircraft configuration. In fact, the mean axis is defined in order to have null linear and angular momenta due to elastic deformation at every instant. The latter definitions are based on the conditions of Eq. (3).

$$\int_V \frac{\delta \boldsymbol{p}}{\delta t} \rho dV = \int_V \boldsymbol{p} \times \frac{\delta \boldsymbol{p}}{\delta t} \rho dV = 0 \tag{3}$$

Even though the exact constraints of Eq. (3) are not always easy to be applied, "practical" constraints can be derived [1].



(a) (b)

Figure 1: Position of the mass element (a) and representation (b) of the body (solid) and mean axes (dashed) reference frames with positive aerodynamic angles (α, β), linear relative velocities (u, v, w) and angular rates (p, q, r)

The linear displacements are denoted using the variable ξ, whereas the rotational displacements are denoted using the variable θ. For example, the vertical (along Z_B direction) displacements of the left wing are denoted as $\xi_{wZ,l}$, the right wing rotation along the Y_B axis is denoted as $\theta_{wY,r}$ whereas the only possible fuselage rotation around the X_B axis is denoted as θ_f. Moreover, the position of the point of mass $\boldsymbol{p}$ can be written as the sum of the undeformed position $\boldsymbol{s}(x,y,z)$ and its deformation $\boldsymbol{d}(x,y,z)$.

3 EQUATIONS OF THE FLEXIBLE AIRCRAFT

3.1 Dynamics of the Unconstrained Flexible Body

Considering Eq.s (1), (2), the kinetic energy of the body, can be written as

$$T = \frac{1}{2}\int_V \frac{d\boldsymbol{r}_I}{dt}\cdot\frac{d\boldsymbol{r}_I}{dt}\rho dV = \frac{1}{2}\int_V \left[\frac{d\boldsymbol{r}_{CG}}{dt}\cdot\frac{d\boldsymbol{r}_{CG}}{dt} + 2\frac{d\boldsymbol{r}_{CG}}{dt}\cdot\frac{\delta\boldsymbol{p}}{\delta t} + \frac{\delta\boldsymbol{p}}{\delta t}\cdot\frac{\delta\boldsymbol{p}}{\delta t} + 2*\frac{\delta\boldsymbol{p}}{\delta t}\cdot(\boldsymbol{\omega}\times\boldsymbol{p}) + (\boldsymbol{\omega}\times\boldsymbol{p})\cdot(\boldsymbol{\omega}\times\boldsymbol{p}) + 2(\boldsymbol{\omega}\times\boldsymbol{p})\cdot\frac{d\boldsymbol{r}_{CG}}{dt}\right]\rho dV \tag{4}$$

Considering to apply Eq. (4) to the mean axes, the simplifications of Eq. (3) can be applied and the kinetic energy can be rewritten as

$$T = \frac{1}{2}\int_V \left[\frac{d\boldsymbol{r}_{CG}}{dt}\cdot\frac{d\boldsymbol{r}_{CG}}{dt} + \frac{\delta\boldsymbol{p}}{\delta t}\cdot\frac{\delta\boldsymbol{p}}{\delta t} + (\boldsymbol{\omega}\times\boldsymbol{p})\cdot(\boldsymbol{\omega}\times\boldsymbol{p}) + 2(\boldsymbol{\omega}\times\boldsymbol{p})\cdot\frac{d\boldsymbol{r}_{CG}}{dt}\right]\rho dV \tag{5}$$

The gravitational potential energy can be written as

$$U = -\int_V -\boldsymbol{g}\cdot(\boldsymbol{r}_{CG}+\boldsymbol{p})\rho dV \tag{6}$$

The potential energy due to the elastic deformation is due to the work done on the structure from the undeformed reference shape to the deformed one. Considering the D'Alambert principle, the potential strain energy can be written as

$$U = -\frac{1}{2}\int_V \frac{\delta^2\boldsymbol{d}}{\delta^2 t}\cdot\boldsymbol{d}\,\rho dV \tag{7}$$

3.1.1 Free vibration modes

The elastic deformation $\boldsymbol{d}$ of the body at a defined location (x,y,z) can be expressed as function of the mode shapes $\boldsymbol{v}(x,y,z)$ and the generalised coordinates $\eta(t)$

$$\boldsymbol{d} = \sum_{i=1}^{\infty}\boldsymbol{v}(x,y,z)\eta(t) \tag{8}$$

The aerodynamic force and moments can be split into three independent contributions: rigid, propulsive and elastic, e.g. the longitudinal external $M = M_{AR} + M_P + M_{AE}$. The elastic contribution can be expanded from an initial point as $M_{AE} = \sum_{i=1}^{\infty}\left(\frac{\partial M_E}{\partial\eta_i}\eta_i + \frac{\partial M_E}{\partial\dot{\eta}_i}\dot{\eta}_i\right)$.
Moreover, introducing the generalised force Q_i of the i-th vibration mode, the governing equation of the i-th elastic free mode is

$$\ddot{\eta}_i + 2\zeta_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = \frac{Q_i}{M_i} \tag{9}$$

where ω_i is the natural frequency of the i-th vibration mode, ζ_i the (assumed) damping ratio of the i-th mode, M_i the generalised mass of the i-th mode. The generalised force can be expressed as function of the aerodynamic coefficients and the vibration modes [2].
Considering the longitudinal dynamics of a flexible aircraft, the governing system of equations can be written as

$$\begin{cases} \dot{u} = -qw - g\sin\vartheta + \dfrac{F_{AX,R} + F_{AX,E} + F_{PX}}{m} \\ \dot{w} = qu + g\cos\vartheta + \dfrac{F_{AZ,R} + F_{AZ,E} + F_{PZ}}{m} \\ \dot{q} = \dfrac{1}{I_{YY}}\left(M_{A,R} + M_{A,E} + M_P\right) \\ \ddot{\eta}_i + 2\zeta_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = \dfrac{Q_i}{M_i} \end{cases} \tag{10}$$

Considering a single vibration mode, Eq. (10) can be rearranged using the state-space notations

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \tag{11}$$

where the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ include both rigid and elastic contributions and the state vector is $\boldsymbol{x} = \{u, w, \vartheta, q, \eta, \dot{\eta}\}$.

4 NUMERICAL RESULTS

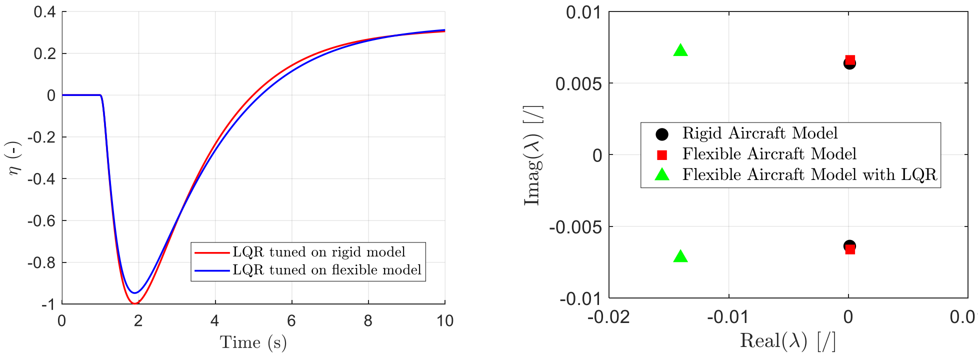
Data used to support the findings of this study are not made available because of company confidentiality and, therefore, the numerical results presented here are non-dimensional. Considering a specific trim condition, the modal analysis highlights that the aircraft modes are slightly affected by the elastic deformation and the phugoid mode is unstable. The short period is more damped (about 5%), whereas the phugoid is much more unstable (about 40%).

Eigenvalue	Rigid Model	Elastic Model
$\boldsymbol{\lambda_1}$(short period) [/]	$-0.042 + i0.070$	$-0.044 + i0.068$
$\boldsymbol{\lambda_2}$ (phugoid) [/]	$0.00010 + i0.0064$	$0.00014 + i0.0066$

Table 1: Eigenvalues of the rigid and flexible aircraft model

4.1 Preliminary Control Design

A preliminary controller is based on the Linear Quadratic Regulator (LQR) [3] aiming to reduce the energy associated to the vibration modes and to stabilise the phugoid mode. The controller is designed using both the rigid and flexible aircraft model, whereas it is tested on the elastic model in order to evaluate the wing deformation. The LQR is designed considering the input vector $\boldsymbol{x}_s = [\vartheta, q, \eta, \dot{\eta}]$ and the control vector is made up of the elevator deflection δ_e, whereas the rigid input vector does not contain the elastic variables η and $\dot{\eta}$. The control parameters of R (input matrix) and Q (control matrix) are defined according to a trial-and-error approach for the scope of the work till a satisfactory dynamic response is obtained.



(a) (b)

Figure 2: Generalised coordinate η at the wing tip. (b) Phugoid eigenvalues without and with the LQR controller

The numerical results presented here are obtained considering a step manoeuvre on the pitch angle. In Figure 2(a) it is plotted the elastic deformation at the wing tip that is obtained with the two controllers. From Figure 2(a) the benefit of the controller designed on the elastic model can be noted in 5% reduction of the maximum displacement. In Figure 2(b) the effect of LQR controller designed on the flexible model can be evaluated comparing the phugoid mode eigenvalues with respect to uncontrolled rigid end elastic models.

5 CONCLUDING REMARKS

Dealing with high aspect ratio vehicles, the flexible structure cannot be neglected. The work introduces a flexible aircraft longitudinal model useful to tune a suitable controller in the frame of a model-based design approach. Notations used in the present work are presented before introducing the analytical model. Eigenvalue analysis is presented for the short period and phugoid mode. The coupling between the aircraft rigid body motion and aircraft structure vibrations can be noted. The elastic structure plays a significant role in the aircraft dynamics as shown by the preliminary results of the present work. In fact, the proposed optimal controller must be tuned using the flexible aircraft model in order to control the aircraft and, at the same time, to reduce the structural displacements of the flexible structure.

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