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Dynamic Analysis of Sandwich Beams with Adhesive Layers Using the Mixed Refined Zigzag Theory

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Abstract. In this paper, a dynamic analysis of the free vibration of sandwich beams is performed using the mixed Refined Zigzag Theory (RZT(m)). This recently developed theory has been demonstrated to be very accurate in predicting transverse displacements, fundamental frequencies, buckling loads, and local through-the-thickness quantities such as in-plane displacements and stresses. The beam is modelled using the quadrilateral finite element formulated by Refined Zigzag Theory for plates and the Reissner's Mixed Variational Theorem in order to evaluate both flexural and torsional modes. Results of numerical models are compared with high-fidelity finite elements and those obtained with an experimental hammer test on sandwich beam specimens. It is concluded that the effect of the adhesive layers is crucial to assess the dynamic behavior of multilayered sandwich structures correctly.

INTRODUCTION

In the last decades, sandwich structures are widely used in many engineering fields due to their excellent properties (high strength to weight ratio, impact energy absorption and noise reduction). A sandwich structure is typically made of two stiff face-sheets connected with a compliant layer, called core. Face-sheets can be metallic or laminated composite, whereas the core can be made of different solutions, such as foams, honeycomb or corrugated.

Although they exhibit these excellent properties, it is difficult to predict their structural behaviour due to the high transverse anisotropy. Therefore, it is common to study these structures using high-fidelity three-dimensional finite element models. However, it involves high computational costs that become prohibitive for complex structures. An alternative approach is represented by the finite elements formulated on the Equivalent Single Layer (ESL) theories [1]. The ESL theories assume a displacement field for the whole laminate. As a result, they are quite accurate in predicting maximum displacements and fundamental frequencies. However, they give poor results for stresses and dynamic responses for high-frequency modes. On the other hand, the Layer-Wise (LW) theories, thanks to their displacement field assumed independently for each layer, can give more realistic results, but the computational cost is even higher [2]. A compromise is represented by the Zigzag theories (ZZ), where the kinematic field is based on the superposition of a global contribution and a local zigzag kinematic refinement.

Among the zigzag theories, the recently developed Refined Zigzag Theory (RZT) seems to be very accurate in describing the structural behaviour of composite sandwich beams [3], plates [4] and shells [5]. The accuracy of RZT can be further improved using Reissner's Mixed Variational Theorem (RMVT) [6]. In this approach, the transverse shear stress distributions can be assumed independently from the kinematic field and a-priori using the indefinite equilibrium equations. Various authors have been applied the mixed-RZT (RZT(m)) for beam and plate structures [7–9].

An interesting aspect of the RZT/RZT(m) is that it requires only C0-continuity of the kinematic variables, thus simple and efficient finite elements can be formulated. Many authors have formulated beam [8,10–12], plate [13–18] and shell [19] finite elements. Recently, the RZT has been also implemented with other numerical methods such as the Higher Order Haar Wavelet Method [20] and the Isogeometric method [21,22].

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In the previous works, the face-sheets are perfectly bonded to the core layer, and the effect of the adhesive layer is neglected. Generally, this assumption is not entirely valid since the mechanical properties of cured adhesive can contribute to the stiffness of the structure, as demonstrated by Gherlone [23].

In order to investigate and assess the RZT(m) model for the dynamic analysis of sandwich beams, the related finite elements are formulated and implemented in MATLAB®. This work aims to model and study the dynamic behaviour of sandwich beams with four node plate elements (QUAD4) developed to investigate both flexural and torsional modes, including the effect of the adhesive layer. Results with numerical models are compared with those obtained by experimental data.

FORMULATION

In this section, the basic kinematic assumptions of RZT(m) for laminate composite and sandwich plates are shown. We consider a plate made of N perfectly bonded orthotropic layers referred to a cartesian coordinate system $\mathbf{X}=\left(x_1,x_2,x_3\right)$, where $x_3 \in\left[-h/2,+h/2\right]$ is the thickness coordinate and $\mathbf{x}=\left(x_1.x_2\right)$ are the in-plane coordinate of the reference plane. L is the length and b is the width. V is the volume of the plate. The displacement field of RZT$^{(m)}$ [15] can be written as

$$\begin{aligned} u_1^{(k)}(\mathbf{X},t) &= u_1^{(0)}(\mathbf{x},t)+x_3\theta_1(\mathbf{x},t)+\phi_1^{(k)}(x_3)\psi_1(\mathbf{x},t) \\ u_2^{(k)}(\mathbf{X},t) &= u_2^{(0)}(\mathbf{x},t)+x_3\theta_2(\mathbf{x},t)+\phi_2^{(k)}(x_3)\psi_2(\mathbf{x},t) \\ u_3^{(k)}(\mathbf{X},t) &= u_3^{(0)}(\mathbf{x},t) \end{aligned} \tag{1}$$

Where $u_i^{(0)}$ (i=1,2,3) and θ_α ($\alpha = 1{,}2$) are the global uniform displacement, and rotations of the normal, respectively, to the reference plane, ψ_α are the unknown spatial amplitudes of the $\phi_\alpha^{(k)}(x_3)$ zigzag functions (assumed continuous piecewise linear vanishing to top and bottom external surfaces). The detailed derivation of zigzag functions is reported in Ref. [24].

The governing equations of RZT(m) are derived using RMVT, assuming $\sigma_{33}=0$

$$\int_V \left(\delta\boldsymbol{\varepsilon}^T\boldsymbol{\sigma}+\delta\boldsymbol{\gamma}_t{}^T\hat{\boldsymbol{\tau}}_t+\delta\hat{\boldsymbol{\tau}}_t\left(\boldsymbol{\gamma}_t-\hat{\boldsymbol{\gamma}}_t\right)\right)dV-\delta W_{in}=0 \tag{2}$$

where δ is the virtual variation, W_{in} is the work of inertial loads, $\boldsymbol{\varepsilon}^T=\left[\varepsilon_{11} \quad \varepsilon_{22} \quad \gamma_{12}\right]$; $\boldsymbol{\sigma}^T=\left[\sigma_{11} \quad \sigma_{22} \quad \tau_{12}\right]$; $\hat{\boldsymbol{\tau}}_t{}^T=\left[\hat{\tau}_{13} \quad \hat{\tau}_{23}\right]$ is the vector of assumed transverse shear stresses and $\hat{\boldsymbol{\gamma}}_t{}^T=\left[\hat{\gamma}_{13} \quad \hat{\gamma}_{23}\right]$ is the vector of corresponding transverse shear strains.

The assumed transverse shear stresses are chosen to be equilibrated with the in-plane stresses, continuous through-the-thickness and satisfy the traction equilibrium conditions on the external surfaces of the structure. The derivation of the assumed transverse shear stresses involves the in-plane indefinite equilibrium equations (without body forces). Under the assumption of cylindrical bending to avoid over-fitting problems, the expression $\hat{\boldsymbol{\tau}}_t$ can be obtained. Further details on their expressions can be found in Ref. [15].

Since the expression of assumed transverse shear stresses are independent of the RZT kinematic variables, the variational statement (2) can be rewritten into two separate and independent contributes

$$\begin{aligned} &\int_V \delta\hat{\boldsymbol{\tau}}_t\left(\boldsymbol{\gamma}_t-\hat{\boldsymbol{\gamma}}_t\right)dV=0 \\ &\int_V \left(\delta\boldsymbol{\varepsilon}^T\boldsymbol{\sigma}+\delta\boldsymbol{\gamma}_t{}^T\hat{\boldsymbol{\tau}}_t+\right)dV-\delta W_{in}=0 \end{aligned} \tag{3}$$

The corresponding finite elements are formulated considering the anisoparametric constrained interpolation strategy that alleviates the shear locking problem [25] when they are used for thin plates. The discretized equations of motions are obtained by substituting the approximate kinematic field into the strain and stress vectors and applying the second term of Eq. (3).

RESULTS

A numerical comparison has been made in this section between results for frequencies obtained by experimental data and those coming from a numerical model with QUAD4 RZT(m) elements implemented in a MATLAB® code. In the numerical models, two cases are here reported, without and with the adhesive layers. A cantilever beam has been considered, the measured length is 960 mm, the width is 72.1 mm, and the core thickness is 20.1 mm. The thickness of the adhesive layers between core and face-sheets is assumed to be 0.2 mm.

The face-sheets are made of ERGAL (SAE 7075 – E=69.57GPa ν=0.35 ρ=2849 kg/m3) with 2 mm of thickness each, and the core is made in Rohacell® WF110 (E=196 MPa ν=0.49 ρ=112 kg/m3). The adhesive layer (3M® AF 163-2 K) is an epoxy adhesive modelled as isotropic material (E=4100 MPa ν=0.37 ρ=1270 kg/m3).

The experimental activity has been conducted at the LAQ-AERMEC laboratory of the Department of Mechanical and Aerospace Engineering of Politecnico di Torino. The experimental clamped edge has been obtained by inserting the beam between two heavy steel blocks joined each other by bolted connections. In order to prevent the core crushing due to this configuration, between faces, that are a little bit longer than the measured length, a wood block of the same thickness of the core is inserted. The natural frequencies and corresponding modal shapes have been evaluated using the experimental modal analysis procedure. An impulse hammer (Bruel & Kjaer 8202 with force transducer 8200) exited the beam. The acceleration responses were measured by eight accelerometers (Bruel & Kjaer 4518) placed along with the beam specimen. The experimental data signals have been acquired and processed by LMS-Siemens SCADAS III.

In Table 1 are compared the experimental results with the first flexural and torsional frequencies.

The beam has been modelled using the MATLAB® FEM code with 7500 RZT(m) plate elements, 300 elements along x1 direction and 25 along x2 direction. The mesh is refined enough to provide converged results for the first two frequencies.

TABLE 1. Natural frequencies (in Hz) for cantilever sandwich beam (F: flexural, T: torsional)

Mode	Experimental	RZT$^{(m)}$ with adhesive layer	RZT$^{(m)}$ without adhesive layer
1F	27.36	28.57	28.96
1T	177.46	186.77	188.42

It is evident from Table 1, that the frequencies of the model with adhesive layer are lower than those in which the effect is neglected. Moreover, the fundamental frequency computed with the effect of adhesive has a lower error than without considering it, with respect to the experimental one.

CONCLUSIONS

In this work, the effect of adhesive layer in the dynamic analysis of sandwich beam is assessed using the mixed-Refined Zigzag Theory (RZT$^{(m)}$). The sandwich beam specimen is excited with an impulse hammer, and the experimental data have been collected to evaluate the first frequencies corresponding to the flexural and torsional modes. The experimental results are compared with the responses obtained by a numerical model in which the shear locking free QUAD4 RZT$^{(m)}$ element have been adopted. The results indicate that the inclusion of the model's adhesive layers is important to evaluate sandwich structures behaviour correctly.

REFERENCES

[1] S. Abrate and M. Di Sciuva, Composite Structures **179**, pp. 482-494 (2017).
[2] A.S. Sayyad and Y.M. Ghugal, Composite Structures C **171**, pp. 486-504 (2017).
[3] A. Tessler, M. Di Sciuva, and M. Gherlone, NASA/TP-2007-215086 1 (2007).
[4] A. Tessler, M. Di Sciuva, and M. Gherlone, NASA/TP-2009-215561 1 (2009).
[5] M. Gherlone, D. Versino, and V. Zarra, Composite Structures **233,** 111629 (2019).

[6] E. Reissner, International Journal for Numerical Methods in Engineering **20**, pp. 1366-1368 (1984).
[7] L. Iurlaro, M. Gherlone, M. Di Sciuva, and A. Tessler, Composite Structures **133**, pp. 809-817 (2015).
[8] L. Iurlaro, M. Gherlone, and M. Di Sciuva, International Journal of Solids and Structures **73–74**, pp. 1-19 (2015).
[9] A. Tessler, Meccanica **50**, pp. 2621-2648 (2015).
[10] M. Di Sciuva, M. Gherlone, L. Iurlaro, and A. Tessler, Composite Structures **132**, pp. 784-803 (2015).
[11] M. Gherlone, A. Tessler, and M. Di Sciuva, Composite Structures **93**, pp. 2882-2894 (2011).
[12] E. Oñate, A. Eijo, and S. Oller, Computer Methods in Applied Mechanics and Engineering **213–216**, pp. 362-382 (2012).
[13] A. Eijo, E. Oñate, and S. Oller, International Journal for Numerical Methods in Engineering **95**, pp. 631-660 (2013).
[14] M. Di Sciuva and M. Sorrenti, Journal of Composites Science **3**, pp. 1-31 (2019).
[15] M. Gherlone, Composite Structures **220**, pp. 510-520 (2019).
[16] M. Sorrenti, M. Di Sciuva, and A. Tessler, Computers & Structures **242**, pp. 1-22 (2021).
[17] A. Barut, E. Madenci, and A. Tessler, Composite Structures **106**, pp. 835-853 (2013).
[18] D. Versino, M. Gherlone, M.C. Mattone, M. Di Sciuva, and A. Tessler, Composites Part B: Engineering **44**, pp. 218-230 (2013).
[19] D. Versino, M. Gherlone, and M. Di Sciuva, Composite Structures **118**, pp. 392-402 (2014).
[20] M. Sorrenti, M. Di Sciuva, J. Majak, and F. Auriemma, Mech Compos Mater **57**, pp. 1-18 (2021).
[21] K.A. Hasim, Composite Structures **186**, pp. 365-374 (2018).
[22] K.A. Hasim, A. Kefal, and E. Madenci, Composite Structures **222**, 110931 (2019).
[23] M. Gherlone, in *14th U.S. National Congress on Computational Mechanics* (Montreal, Quebec, Canada, 2017).
[24] A. Tessler, M. Di Sciuva, and M. Gherlone, Journal of Mechanics of Materials and Structures **5**, pp. 341-367 (2010).
[25] A. Tessler and T.J.R. Hughes, Computer Methods in Applied Mechanics and Engineering **39**, pp. 311-335 (1983).