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Design and modeling of asymmetric bow-tie VCSELs for 100 GHz and beyond: supplement

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In this supplementary material, we report the proof Eq. (5), which allows for the calculation of the output optical power as a post processing of the time-varying expansion coefficients.

1. CALCULATION OF THE OUTPUT POWER

After solving the dynamical equations (3)-(4), we must calculate the output optical power $P(t)$ emitted by the VCSEL. To this aim, a few preliminary calculations are needed. Let us begin by defining:

$$k_{ij}^{(0)} = \frac{k_{ij}}{1 + i\alpha_H} \tag{S1}$$

and

$$k'_{ij} = \Re\{k_{ij}\}. \tag{S2}$$

As discussed in [1], the modulation bandwidth enhancement in BT-VCSEL comes from two modes interacting to generate a PPR peak, so let us fix $M = 2$ for simplicity.

Identifying the second term of the right-hand side of (4) as the stimulated recombination term,

$$R_{\text{st}}(x,y,t) = v_g \Re\{\tilde{g}(x,y,t)\} S(x,y,t), \tag{S3}$$

let us evaluate its volume integral, as it represents the whole power generated by carriers inside the VCSEL. For carriers, integrating along the longitudinal direction simply implies the multiplication by the AR thickness. Expliciting S according to (10) one obtains (for simplicity, time and space dependencies are implied):

$$\begin{aligned}
&\iiint_{\mathbb{R}^3} R_{\text{st}}\, \mathrm{d}x\mathrm{d}y\mathrm{d}z = N_{\text{QW}} d_{\text{QW}} \iint_{\mathbb{R}^2} R_{\text{st}} \mathrm{d}x\mathrm{d}y = \\
&= N_{\text{QW}} d_{\text{QW}} v_g \iint_{\mathbb{R}^2} \frac{g(N)}{1+\epsilon_{\text{nlg}} S} \left(|e_1|^2 |\Psi_1|^2 + |e_2|^2 |\Psi_2|^2 + 2\Re\left\{ e_1^* e_2 \Psi_1^* \Psi_2 \right\} \right) \mathrm{d}x\mathrm{d}y = \\
&\overset{\text{Eq. (8)}-\text{Eq. (S1)}-\text{Eq. (S2)}}{=} \frac{2 N_{\text{QW}} d_{\text{QW}}}{\Gamma_z} \left(k'_{11} |e_1|^2 + k'_{22} |e_2|^2 + 2\Re\left\{ k_{12}^{(0)} e_1^* e_2 \right\} \right).
\end{aligned} \tag{S4}$$

Consider now:

$$\begin{aligned}
\frac{d|e_1|^2}{dt} &= \frac{de_1}{dt} e_1^* + e_1 \left(\frac{de_1}{dt} \right)^* = 2\Re\left\{ \frac{de_1}{dt} e_1^* \right\} = \\
&\overset{\text{Eq. (3)}}{=} -2\alpha_1 |e_1|^2 + 2k'_{11} |e_1|^2 + 2\Re\left\{ k_{12} e_1^* e_2 \right\}.
\end{aligned} \tag{S5}$$

Following the same procedure:

$$\frac{d|e_2|^2}{dt} = -2\alpha_2 |e_2|^2 + 2k'_{22} |e_2|^2 + 2\Re\left\{ k_{21} e_1 e_2^* \right\}. \tag{S6}$$

Finally, we can express the output power as a power balance. Specifically, the net total power inside the VCSEL can be obtained as:

$$P_{\text{net}} = E_{\text{ph}} \iiint_{\mathbb{R}^3} \frac{\partial S}{\partial t} \mathrm{d}x\mathrm{d}y\mathrm{d}z, \tag{S7}$$

where E_{ph} is the photon energy at our reference wavelength. Note that integrating photons over the longitudinal dimension produces a multiplication by the optical length $L_{\text{opt}} = N_{\text{QW}} d_{\text{QW}} / \Gamma_z$. Substituting Eq. (10) with $M = 2$ in Eq. (S7) allows to identify the terms associated to generation and the ones related to escape. The latter are used to calculate the output power. Equation Eq. (S7) becomes:

$$P_{\text{net}} \overset{\text{Eq. (10)}}{=} E_{\text{ph}} L_{\text{opt}} \iint_{\mathbb{R}^2} \frac{d}{dt} \left(|e_1|^2 |\Psi_1|^2 + |e_2|^2 |\Psi_2|^2 + 2\Re\left\{ e_1^* e_2 \Psi_1^* \Psi_2 \right\} \right) \mathrm{d}x\mathrm{d}y. \tag{S8}$$

By recalling Eq. (S5)–Eq. (S6) and assuming modes normalization in the QW section, one can further define:

$$\xi_{12} = \iint_{\mathbb{R}^2} \Psi_1^* \Psi_2 \, \mathrm{d}x\mathrm{d}y, \tag{S9}$$

so that it is possible to write:

$$P_{\mathrm{net}} = E_{\mathrm{ph}} L_{\mathrm{opt}} \Big[-2\alpha_1 |e_1|^2 + 2k'_{11}|e_1|^2 + 2\Re\{k_{12}e_1^* e_2\} + \\ -2\alpha_2|e_2|^2 + 2k'_{22}|e_2|^2 + 2\Re\{k_{21}e_1e_2^*\} + 2\Re\left\{\frac{d(e_1^*e_2)}{dt}\xi_{12}\right\}\Big]. \tag{S10}$$

Let us further develop the following two terms:

$$2\Re\{k_{12}e_1^*e_2 + k_{21}e_1e_2^*\} = 2\Re\left\{(1+i\alpha_H)\left(k_{12}^{(0)}e_1^*e_2 + k_{21}^{(0)}e_1e_2^*\right)\right\}. \tag{S11}$$

However, from Eq. (8), one can notice that $k_{12}^{(0)} = \left(k_{21}^{(0)}\right)^*$, so that:

$$2\Re\{k_{12}e_1^*e_2 + k_{21}e_1e_2^*\} = 4\Re\left\{k_{12}^{(0)}e_1^*e_2\right\}, \tag{S12}$$

being independent on α_H. Recalling Eq. (S4) and the definition of L_{opt}, the total optical power can be rewritten as:

$$P_{\mathrm{net}} = E_{\mathrm{ph}} \iiint_{\mathbb{R}^3} R_{\mathrm{st}} \, \mathrm{d}x\mathrm{d}y\mathrm{d}z - 2E_{\mathrm{ph}}L_{\mathrm{opt}}\left[\alpha_1|e_1|^2 + \alpha_2|e_2^2| - \Re\left\{\frac{d\left(e_1^*e_2\right)}{dt}\xi_{12}\right\}\right], \tag{S13}$$

where the first term is the generated power, while the second one is the output power. The final formula for $P(t)$ also accounts for the modal outcoupling efficiencies η_1, η_2, which are phenomenologically added to the expansion coefficients [2] as:

$$P(t) = 2E_{\mathrm{ph}}L_{\mathrm{opt}}\left\{\eta_1\alpha_1|e_1(t)|^2 + \eta_2\alpha_2|e_2(t)|^2 - \sqrt{\eta_1\eta_2}\Re\left\{\frac{d\left[e_1^*(t)e_2(t)\right]}{dt}\xi_{12}\right\}\right\}. \tag{S14}$$

The last term represents a cross power, which is zero under stationary conditions ($d/dt = 0$) or for orthogonal modes ($\xi_{12} = 0$).

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